Problem of the Week
Problem D and Solution
Head Start

Problem

Gabi and Silvio are training for a cycling race. They live on the same street, but Silvio's house is 2 km east of Gabi's. On Sunday morning at 7 a.m. they each start biking east from their house. If Gabi bikes at a constant speed of 24 km/h and Silvio bikes at a constant speed of 18 km/h, at what time will Gabi catch up to Silvio?

Solution

For the first two solutions we will use the formula: time $= \frac{\text{distance}}{\text{speed}}$.

For the third solution we will use the formula: distance $=$ speed $\times$ time.

Solution 1

Since Gabi bikes at 24 km/h and Silvio bikes at 18 km/h, then Gabi gains 6 km/h on Silvio.

Since Silvio starts 2 km east of of Gabi, then it takes Gabi $\frac{2}{6} = \frac{1}{3}$ of an hour or $\frac{1}{3} \times 60 = 20$ minutes to catch up to Silvio. Since they started biking at 7 a.m., Gabi will catch up to Silvio at 7:20 a.m.

Solution 2

Silvio bikes at 18 km/h or $\frac{18}{60} = \frac{3}{10}$ km/min. Gabi bikes at 24 km/h or $\frac{24}{60} = \frac{2}{5}$ km/min. Therefore Gabi gains $\frac{2}{5} - \frac{3}{10} = \frac{1}{10}$ km/min on Silvio.

Since Silvio started 2 km east of Gabi, then it takes Gabi $2 \div \frac{1}{10} = 20$ minutes to catch Silvio. Since they started biking at 7 a.m., Gabi will catch up to Silvio at 7:20 a.m.

Solution 3

Suppose it takes t hours for Gabi to catch up to Silvio. Then Silvio has biked 18 km/h $\times$ t h $= 18t$ km, and Gabi has biked 24 km/h $\times$ t h $= 24t$ km.

Since Silvio starts 2 km east of Gabi, then when they meet, Gabi will have travelled 2 km further than Silvio. That is,
$$24t = 18t + 2$$
$$6t = 2$$
$$t = \tfrac{1}{3}$$

Therefore, it takes Gabi $\frac{1}{3}$ of an hour, or 20 minutes to catch up to Silvio. Since they started biking at 7 a.m., Gabi will catch up to Silvio at 7:20 a.m.