Problem of the Week
Problem D and Solution
A Weightier Problem

Problem

George has three objects, each of a different mass. He weighs the objects in pairs and records the mass of each pair of objects. Later, he realizes that he forgot to weigh the objects individually, but no longer has access to a scale. The three recorded masses are 2986 g, 3464 g, and 3550 g. Determine the mass of the heaviest object.

Solution

Let a represent the mass, in grams, of the lightest object
Let c represent the mass, in grams, of the heaviest object.
Let b represent the mass, in grams, of the third object.

The smallest recorded mass is created by adding the masses of the two lightest objects together. Therefore,

$$a + b = 2986 \tag{1}$$

The largest recorded mass is created by adding the masses of the two heaviest objects together. Therefore,

$$b + c = 3550 \tag{2}$$

Thus, it must be the case that

$$a + c = 3464 \tag{3}$$

At this point we could solve a system of equations involving three equations and three unknowns. Instead, we will add equations (1), (2), and (3) together.

$$(a + b) + (b + c) + (a + c) = 2986 + 3550 + 3464$$
$$2a + 2b + 2c = 10\,000$$
$$2(a + b + c) = 10\,000$$
$$a + b + c = 5000 \tag{4}$$

From equation (4), we know that the total mass of the three objects is 5000 g. But from equation (1), the mass of the two lighter objects is 2986 g. We can subtract equation (1) from equation (4) to obtain the mass of the heaviest object.

$$(a + b + c) - (a + b) = 5000 - 2986$$
$$a + b + c - a - b = 2014$$
$$c = 2014$$

Therefore, the heaviest object has a mass of 2014 g. Although we are not asked to, from here, we could determine that the other objects have mass of 1450 g and mass of 1536 g.