Problem of the Week
Problem D and Solution
More Chips

Problem

Mrs. Chips has 400 bingo chips. Each chip is either blue, yellow, red, green, or purple.

The ratio of the number of blue chips to the number of green chips to the number of red chips is $1:2:4$. The ratio of the number of green chips to the number of yellow chips to the number of purple chips is $1:3:6$.

How many chips are there of each colour?

Solution

Let b represent the number of blue chips, g represent the number of green chips, r represent the number of red chips, y represent the number of yellow chips, and p represent the number of purple chips. From here, we present two different solutions.

Solution 1

We are given that $b:g:r = 1:2:4$ and that $g:y:p = 1:3:6$. Multiplying by 2, the ratio $1:3:6$ is equivalent to the ratio $2:6:12$. Thus, $g:y:p = 2:6:12$.

We chose to scale this ratio by a factor of 2 so that the only colour common to the two given ratios, green, now has the same number in both of these ratios. We can now combine these to form a single ratio, $b:g:r:y:p = 1:2:4:6:12$.

This ratio tells us that for every blue chip, there are 2 green, 4 red, 6 yellow, and 12 purple chips. Thus, if there was only 1 blue chip, then there would be $1+2+4+6+12 = 25$ chips in total. However, we are given that the box contains 400 chips in total. Therefore, there are $\frac{400}{25} = 16$ blue chips. Multiplying by 16, the ratio $b:g:r:y:p$ becomes $16:32:64:96:192$.

Therefore, there are 16 blue, 32 green, 64 red, 96 yellow, and 192 purple chips.
(Note that there are $16+32+64+96+192 = 400$ chips in total.)

Solution 2

We are given that $b:g:r = 1:2:4$ and that $g:y:p = 1:3:6$.
The first ratio tells us that $\dfrac{b}{g} = \dfrac{1}{2}$, and so $b = \dfrac{g}{2}$. It also tells us that $\dfrac{g}{r} = \dfrac{2}{4}$, and so $r = 2g$.
The second ratio tells us that $\dfrac{g}{y} = \dfrac{1}{3}$, and so $y = 3g$. It also tells us that $\dfrac{g}{p} = \dfrac{1}{6}$, and so $p = 6g$.

We are given that there are a total of 400 chips. That is, $b+g+r+y+p = 400$.

Substituting $b = \frac{g}{2}, r = 2g, y = 3g$, and $p = 6g$, this becomes

$$\begin{aligned} \frac{g}{2} + g + 2g + 3g + 6g &= 400 \\ \frac{25g}{2} &= 400 \\ g &= 32 \end{aligned}$$

Thus, $b = \frac{g}{2} = 16$, $r = 2g = 64$, $y = 3g = 96$, and $p = 6g = 192$.

Therefore, there are 16 blue, 32 green, 64 red, 96 yellow, and 192 purple chips.