Problem of the Week
Problem D and Solution
Small Change

Problem

Carroll and Arthur cleaned their house and found a total of 33 coins. The coins were either nickels (5 cent coins), dimes (10 cent coins), or quarters (25 cent coins). There were twice as many quarters as dimes, and the total value of all the coins they found was $5.25.

How many of each type of coin did they find?

NOTE: In Canada, 100 cents is equal to $1.

Solution

Let n be the number of nickels, d be the number of dimes, and q be the number of quarters.

From the total number of coins we get the equation

$$n + d + q = 33 \tag{1}$$

From the value of the coins we get the equation

$$5n + 10d + 25q = 525 \tag{2}$$

We also know that $q = 2d$.

Substituting $q = 2d$ into equation (1) and simplifying, we get

$$n + d + 2d = 33$$
$$n + 3d = 33 \tag{3}$$

Substituting $q = 2d$ into equation (2) and simplifying, we get

$$5n + 10d + 25(2d) = 525$$
$$5n + 60d = 525$$
$$n + 12d = 105 \tag{4}$$

We can isolate n in equation (3) to get $n = 33 - 3d$.
Similarly, we can isolate n in equation (4) to get $n = 105 - 12d$.

Since $n = n$, it follows that

$$33 - 3d = 105 - 12d$$
$$-3d + 12d = 105 - 33$$
$$9d = 72$$
$$d = 8$$

Substituting $d = 8$ into $n = 33 - 3d$, it follows that $n = 33 - 3(8) = 33 - 24 = 9$.

Finally, we substitute $d = 8$ into $q = 2d$, to find $q = 2(8) = 16$.

Therefore, they found 9 nickels, 8 dimes, and 16 quarters.