Problem of the Week
Problem D and Solution
Boxes of Doughnuts

Problem

A bakery is famous for its specialty doughnuts. One weekend they had three flavours available, each packaged in boxes. Peach caramel doughnuts were sold in small boxes with 3 doughnuts per box, chocolate fudge doughnuts were sold in medium boxes with 4 doughnuts per box, and rainbow doughnuts were sold in large boxes with 8 doughnuts per box.

At the end of the weekend, the owner calculated that they sold 68 boxes of doughnuts in total. Also an equal number of doughnuts of each flavour were sold. How many doughnuts did they sell in total?

Solution

Solution 1

Let n represent the number of doughnuts of each flavour sold. Since there were 3 peach caramel doughnuts per box, then n must be divisible by 3. Since there were 4 chocolate fudge doughnuts per box, then n must be divisible by 4. Since there were 8 rainbow doughnuts per box, then n must be divisible by 8. Therefore, n must be divisible by 3, 4, and 8. The smallest number divisible by 3, 4, and 8 is 24. (This number is called the *least common multiple* or *LCM*).

If there were 24 doughnuts of each flavour sold, there would have been $24 \div 3 = 8$ boxes of peach caramel doughnuts, $24 \div 4 = 6$ boxes of chocolate fudge doughnuts, and $24 \div 8 = 3$ boxes of rainbow doughnuts. This would mean $8 + 6 + 3 = 17$ boxes of doughnuts would have been sold in total. However, we know that 68 boxes of doughnuts were sold, and since $68 = 17 \times 4$, it follows that 4 times as many doughnuts were sold. Therefore, $24 \times 4 = 96$ doughnuts of each flavour were sold. Thus, the total number of doughnuts sold was $96 \times 3 = 288$.

We can check the correctness of this solution. Since there were 3 peach caramel doughnuts per box, then there were $96 \div 3 = 32$ boxes of peach doughnuts sold. Since there were 4 chocolate fudge doughnuts per box, then there were $96 \div 4 = 24$ boxes of chocolate fudge doughnuts sold. Since there were 8 rainbow doughnuts per box, then there were $96 \div 8 = 12$ boxes of rainbow doughnuts sold. The total number of boxes sold was therefore $32 + 24 + 12 = 68$, as expected.

Solution 2

This solution uses algebra and equation solving. Let n represent the number of doughnuts of each flavour sold. Since there were 3 peach caramel doughnuts per box, then $\frac{n}{3}$ boxes of peach caramel doughnuts were sold. Since there were 4

chocolate fudge doughnuts per box, then $\frac{n}{4}$ boxes of chocolate fudge doughnuts were sold. Since there were 8 rainbow doughnuts per box, then $\frac{n}{8}$ boxes of rainbow doughnuts were sold. Since 68 boxes of doughnuts were sold in total,

$$\frac{n}{3} + \frac{n}{4} + \frac{n}{8} = 68$$
$$\frac{8n}{24} + \frac{6n}{24} + \frac{3n}{24} = 68$$
$$\frac{17n}{24} = 68$$
$$17n = 68 \times 24$$
$$17n = 1632$$
$$n = \frac{1632}{17} = 96$$

Therefore, 96 doughnuts of each flavour were sold. Thus, the total number of doughnuts sold was $96 \times 3 = 288$.

Solution 3

This solution uses ratios. Let n represent the number of doughnuts of each flavour sold. The ratio of the number of boxes of rainbow doughnuts to peach caramel doughnuts sold is

$$\frac{n}{8} : \frac{n}{3} = \frac{3n}{24} : \frac{8n}{24} = 3n : 8n = 3 : 8$$

Similarly, the ratio of the number of boxes of peach caramel doughnuts to chocolate fudge doughnuts sold is $4 : 3 = 8 : 6$. So the ratio of the number of boxes of rainbow doughnuts to peach caramel doughnuts to chocolate fudge doughnuts sold is $3 : 8 : 6$. Let the number of boxes of rainbow doughnuts be $3k$, the number of boxes of peach caramel doughnuts be $8k$, and the number of boxes of chocolate fudge doughnuts be $6k$. Since 68 boxes of doughnuts were sold in total,

$$3k + 8k + 6k = 68$$
$$17k = 68$$
$$k = \frac{68}{17} = 4$$

It follows that the number of boxes of rainbow doughnuts sold was $3 \times 4 = 12$, so the number of rainbow doughnuts sold was $12 \times 8 = 96$. Therefore $n = 96$, so 96 doughnuts of each flavour were sold. Thus, the total number of doughnuts sold was $96 \times 3 = 288$.