Problem of the Week
Problem D and Solution
Mangoes and Oranges

Problem

At POTW's Supermarket, Livio stocks mangoes and Dhruv stocks oranges. One day they noticed that an equal number of mangoes and oranges were rotten. Also, $\frac{2}{3}$ of the mangoes were rotten and $\frac{3}{4}$ of the oranges were rotten. What fraction of the total number of mangoes and oranges was rotten?

Solution

Solution 1:

Let the total number of mangoes be represented by a and the total number of oranges be represented be b. Since there were an equal number of rotten mangoes and rotten oranges, then $\frac{2}{3}a = \frac{3}{4}b$, so $b = \frac{4}{3}(\frac{2}{3}a) = \frac{8}{9}a$.

Therefore, there were a total of $a + b = a + \frac{8}{9}a = \frac{17}{9}a$ mangoes and oranges.

Also, the total amount of rotten fruit was $2(\frac{2}{3}a) = \frac{4}{3}a$.

Therefore, $\frac{\frac{4}{3}a}{\frac{17}{9}a} = \frac{4}{3}\left(\frac{9}{17}\right) = \frac{12}{17}$ of the total number of mangoes and oranges was rotten.

Solution 2:

Since $\frac{2}{3}$ of the mangoes were rotten, $\frac{3}{4}$ of the oranges were rotten, and the number of rotten mangoes equaled the number of rotten oranges, suppose there were 6 rotten mangoes. (We choose 6 as it is a multiple of the numerator of each fraction.) Then the number of rotten oranges will also be 6.

If there were 6 rotten mangoes, then there were a total of $6 \div \frac{2}{3} = 6\left(\frac{3}{2}\right) = 9$ mangoes.

If there were 6 rotten oranges, then there were a total of $6 \div \frac{3}{4} = 6\left(\frac{4}{3}\right) = 8$ oranges.

Therefore, there were $9 + 8 = 17$ pieces if fruit in total, of which $6 + 6 = 12$ were rotten.

Thus, $\frac{12}{17}$ of the total number of mangoes and oranges was rotten.

NOTE: In Solution 2 we could have used any multiple 6 for the number of rotten mangoes and thus the number of rotten oranges. The final fraction would always reduce to $\frac{12}{17}$. We will show this in general in Solution 3.

Solution 3:

According to the problem, there were an equal number of rotten mangoes and rotten oranges. Let the number of rotten mangoes and rotten oranges each be $6x$, for some positive integer x.

The total number of mangoes was thus $6x \div \frac{2}{3} = 9x$.

The total number of oranges was thus $6x \div \frac{3}{4} = 8x$.

Therefore, the total number of mangoes and oranges was $9x + 8x = 17x$.

Also, the total number of rotten mangoes and rotten oranges was $6x + 6x = 12x$.

Therefore, $\frac{12x}{17x} = \frac{12}{17}$ of the total number of mangoes and oranges was rotten.