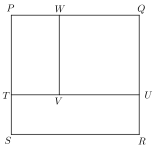


Problem of the Week
Problem D and Solution
Square Parts

Problem

Square $PQRS$ has W on PQ, U on QR, T on PS, and V on TU such that $QUVW$ is a square, and $PWVT$ and $RSTU$ are rectangles.

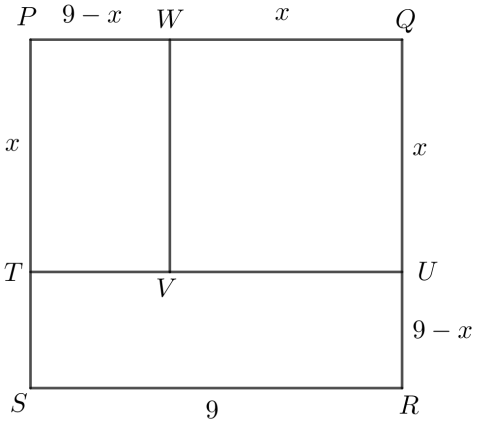
The side length of square $PQRS$ is 9 cm, and

$$\text{area of } QUVW - \text{ area of } RSTU = \text{ area of } RSTU - \text{ area of } PWVT$$

If square $QUVW$ has side length equal to x cm, determine the value of x and the areas of rectangles $PWVT$ and $RSTU$.

Solution

We know $SR = PQ = 9$ cm and $WQ = QU = x$ cm.
Therefore, $PW = PQ - WQ = (9 - x)$ cm. Similarly, $UR = (9 - x)$ cm.



Thus, we have that the area of $QUVW$ is equal to x^2 cm^2, the area of $RSTU$ is equal to $9(9 - x)$ cm^2, and the area of $PWVT$ is equal to $x(9 - x)$ cm^2.

Therefore, we know that

$$\begin{aligned}
\text{area of } QUVW - \text{ area of } RSTU &= \text{ area of } RSTU - \text{ area of } PWVT \\
x^2 - 9(9 - x) &= 9(9 - x) - x(9 - x) \\
x^2 - 81 + 9x &= 81 - 9x - 9x + x^2 \\
27x &= 162 \\
x &= 6
\end{aligned}$$

Therefore, $x = 6$ cm, the area of $PWVT$ is equal to $x(9 - x) = 6(9 - 6) = 18$ cm^2, and the area of $RSTU = 9(9 - x) = 9(9 - 6) = 27$ cm^2.