Problem of the Week
Problem C and Solution
Cycles of Eclipses

Problem

A planet in a distant solar system has a moon and a sun. On this planet, there is a total solar eclipse whenever the following is true.

- There is a full moon,

- the moon is at its closest point to the planet, and

- the centre of the moon is in line with the centres of the planet and the sun.

On this planet, there is a full moon every 16 days. Also, every 12 days, the moon is at its closest point to the planet. As well, every n days the centre of the moon is in line with the centres of the planet and the sun.

If n is greater than 10 but less than 20, and total solar eclipses happen on this planet every 240 days, determine the value of n.

Solution

Since total solar eclipses happen every 240 days on this planet, it follows that 240 is the *least common multiple* (LCM) of 16, 12, and n.

To determine the value of n, we will rewrite each of 16, 12, and 240 as a product of prime numbers. This is known as *prime factorization.*

$$16 = 2 \times 2 \times 2 \times 2$$
$$12 = 2 \times 2 \times 3$$
$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

The LCM is calculated by determining the greatest number of each prime number in any of the factorizations, and then multiplying these numbers together. From the prime factorizations of 16 and 12, we can determine that their LCM is equal to $2 \times 2 \times 2 \times 2 \times 3 = 48$. Since 240 has an extra factor of 5, and 240 is the LCM of 16, 12, and n, it follows that 5 must be a factor of n. The only number with a factor of 5 that is greater than 10 but less than 20 is 15.

Since the prime factorization of 15 is $15 = 3 \times 5$, we can conclude that the LCM of 16, 12, and 15 is 240, as desired. Therefore $n = 15$.

EXTENSION: Research what conditions must occur for there to be a total solar eclipse on Earth. How often do total solar eclipses occur on Earth?