# Problem of the Week
Problem C and Solution
A Weighty Problem

Problem

Ron has ten identical spheres and ten identical cubes. He was asked to determine the combined mass of the ten spheres and ten cubes, but he did not have a reliable weigh scale. However, he was given the following information:

- Two spheres and three cubes have a mass of 21 g.

- Three spheres and two cubes have a mass of 19 g.

Your task is to determine the combined mass of the ten spheres and ten cubes.
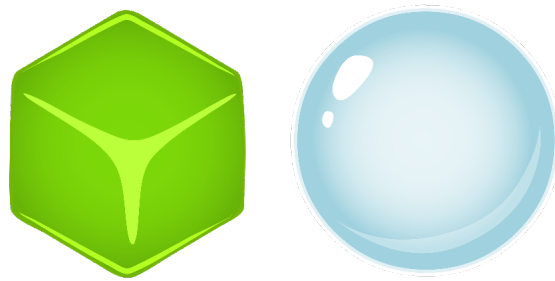
Solution

Solution 1

Since two spheres and three cubes have a mass of 21 g and three spheres and two cubes have a mass of 19 g, then, by combining the two pieces of information, we know that five spheres and five cubes have a combined mass of $21 + 19 = 40$ g. If we double this, we find that ten spheres and ten cubes have a combined mass of $40 \times 2 = 80$ g.

Solution 2

From the first piece of given information to the second, the number of spheres has been increased by one and the number of cubes has been decreased by one. This causes a mass decrease of 2 g. Therefore, the mass of the cube is 2 g more than the mass of the sphere.

Let s represent the mass, in grams, of one sphere.
Therefore, the mass, in grams, of one cube is $s + 2$.

From the first piece of information, we know $2s + 3(s + 2) = 21$. We now solve this equation.

$$2s + 3(s + 2) = 21$$
$$2s + 3s + 6 = 21$$
$$5s + 6 = 21$$
$$5s = 15$$
$$s = 3$$

Therefore, one sphere has a mass of 3 g and one cube has a mass $3 + 2 = 5$ g.

Thus, the combined mass of ten spheres and ten cubes is $10 \times 3 + 10 \times 5 = 30 + 50 = 80$ g.

Solution 3

This solution uses algebra that is learned in future mathematics courses.

Let s represent the mass, in grams, of a sphere.
Let c represent the mass, in grams, of a cube.

Using the given information, we obtain the following system of equations.

$$2s + 3c = 21 \tag{1}$$
$$3s + 2c = 19 \tag{2}$$

We will now use elimination to solve for s and c.

First, multiplying equation (1) by 2, we get

$$4s + 6c = 42 \tag{3}$$

Multiplying equation (2) by 3, we get

$$9s + 6c = 57 \tag{4}$$

Subtracting equation (3) from equation (4), we get $5s = 15$, and so $s = 3$.

Substituting $s = 3$ into equation (1), we have

$$2(3) + 3c = 21$$
$$6 + 3c = 21$$
$$3c = 15$$
$$c = 5$$

Therefore, one sphere has a mass of 3 g and one cube has a mass 5 g.

Thus, the combined mass of ten spheres and ten cubes is $10 \times 3 + 10 \times 5 = 30 + 50 = 80$ g.