Problem of the Week
Problem C and Solution
Where Does the Year Go?

Problem

The positive integers are written consecutively in rows, with seven integers in each row. That is, the first row contains the integers 1, 2, 3, 4, 5, 6, and 7. The second row contains the integers 8, 9, 10, 11, 12, 13, and 14. The third row contains the integers 15, 16, 17, 18, 19, 20, and 21, and so on.

Determine the row and the column that the integer 2024 is in.

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

Solution

The last number in row 1 is 7, the last number in row 2 is 14, and the last number in row 3 is 21. Observe that the last number in each row is a multiple of 7. Furthermore, the last number in row n is $7 \times n$. So, we will find the largest multiple of 7 that is less than 2024.

We solve the equation $7 \times n = 2024$ to get $n \approx 289.14$.

Therefore, the largest multiple of 7 that is less than 2024 is $289 \times 7 = 2023$. This means that 2023 is the last number in row 289. Thus, 2024 will be the first number in in row 290.

Therefore, 2024 is in row 290 and column 1.