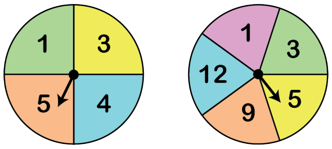

Problem of the Week

Problem C and Solution

Spin to Win

Problem

Esa has created a game for his math fair using two spinners. One spinner is divided into four equal sections labeled 1, 3, 4, and 5. The other spinner is divided into five equal sections labeled 1, 3, 5, 9, and 12. To play the game, a player spins each spinner once and then multiplies the two numbers the spinners land on. If this product is a perfect square, the player wins. What is the probability of winning the game?

NOTE: A square of any integer is called a *perfect square*. For example, the number 25 is a perfect square since it can be expressed as 5^2 or 5×5.

Solution

In order to determine the probability, we must determine the number of ways to obtain a perfect square and divide it by the total number of possible combinations of spins. To do so, we will create a table where the rows show the possible results for Spinner 1, the spinner with four sections, the columns show the possible results for Spinner 2, the spinner with five sections, and each cell in the body of the table gives the product of the numbers on the corresponding spinners.

		Spinner 2				
		1	**3**	**5**	**9**	**12**
Spinner 1	**1**	1	3	5	9	12
	3	3	9	15	27	36
	4	4	12	20	36	48
	5	5	15	25	45	60

From the table, we see that there are 20 possible combinations of spins. Of these, the following result in products that are perfect squares:

- The number 1 is a perfect square ($1 = 1 \times 1$), and it occurs one time.
- The number 4 is a perfect square ($4 = 2 \times 2$), and it occurs one time.
- The number 9 is a perfect square ($9 = 3 \times 3$), and it occurs two times.
- The number 25 is a perfect square ($25 = 5 \times 5$), and it occurs one time.
- The number 36 is a perfect square ($36 = 6 \times 6$), and it occurs two times.

Thus, 7 of the 20 products are perfect squares. Therefore, the probability of winning the game is $\frac{7}{20}$, or 35%.

EXTENSION: A game is considered *fair* if the probability of winning is 50%. Can you modify this game so that it is fair?