Problem of the Week
Problem C and Solution
Sharing Grapes

Problem

Jessica has some grapes. She gives one-third of her grapes to Callista. She then gives 4 grapes to Monica. Finally, she gives one-half of her remaining grapes to Peter. If Jessica then has 16 grapes left, how many grapes did Jessica begin with?

Solution

Solution 1:

We work backwards from the last piece of information given.

Jessica has 16 grapes left after giving one-half of her remaining grapes to Peter. This means that she had $2 \times 16 = 32$ grapes immediately before giving grapes to Peter.

Immediately before giving grapes to Peter, she gave 4 grapes to Monica, which means that she had $32 + 4 = 36$ grapes immediately before giving 4 grapes to Monica.

Immediately before giving the 4 grapes to Monica, she gave one-third of her grapes to Callista, which would have left her with two-thirds of her original amount.

Since two-thirds of her original amount equals 36 grapes, then one-third equals one half of 36 or $\frac{36}{2} = 18$ grapes.

Thus, she gave 18 grapes to Callista, and so Jessica began with $36 + 18 = 54$ grapes.

Solution 2:

Suppose Jessica started with x grapes.

She gives $\frac{1}{3}x$ grapes to Callista, leaving her with $1 - \frac{1}{3}x = \frac{2}{3}x$ grapes.

She then gives 4 grapes to Monica, leaving her with $\frac{2}{3}x - 4$ grapes.

Finally, she gives away one-half of what she has left to Peter, which means that she keeps one-half of what she has left, and so she keeps $\frac{1}{2}(\frac{2}{3}x - 4)$ grapes.

Simplifying this expression, we obtain $\frac{2}{6}x - \frac{4}{2} = \frac{1}{3}x - 2$ grapes.

Since she has 16 grapes left, then $\frac{1}{3}x - 2 = 16$ and so $\frac{1}{3}x = 18$ or $x = 54$.

Therefore, Jessica began with 54 grapes.