Problem of the Week

Problem B and Solution

Gamer!

Problem

Geoff plays a game using two standard six-sided dice: a black one and a white one. To win the game, Geoff must roll the dice and have the numbers on the two top faces sum to 11.

(a) What is the probability that he rolls a 7 with just the black die?

(b) What is the theoretical probability that he rolls a 1 on the black die and a 6 on the white die?

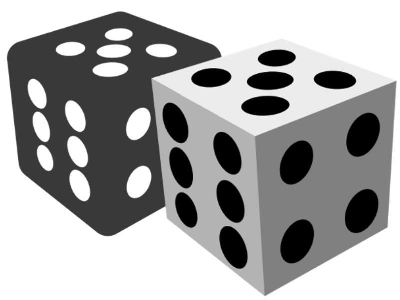
(c) If he rolls both dice and calculates the sum of the numbers on the two top faces, what sum(s) have the lowest theoretical probability of being rolled?

(d) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 7?

(e) What is the theoretical probability of rolling both dice and the sum of the numbers on the two top faces is 11?

(f) Roll two dice thirty-six times and keep track of the number of times the numbers on the two top faces sum to 11. What was your empirical probability of rolling a sum of 11?

(g) Share your results in part (f) with your classmates. How many had their empirical probability of rolling a sum of 11 equal the theoretical probability of rolling a sum of 11?



Solution

(a) Since the numbers on the faces of a standard six-sided die are 1, 2, 3, 4, 5, and 6, it is impossible to roll a 7. So the probability is 0.

(b) For each of the 6 possible numbers he could throw with the black die there are 6 possible numbers on the white die, so the total number of possible outcomes is $6 \times 6 = 36$. Thus, the theoretical probability that he throws a 1 on the black die and a 6 on the white die is 1 in 36, or $\frac{1}{36}$.

Alternatively, to solve this problem we can create a table where the columns show the possible numbers on the top face of the white die, the rows show the possible numbers on the top face of the black die, and each cell in the body of the table gives the sum of the corresponding pair of numbers.

		White Die					
		1	2	3	4	5	6
Black Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

From this table, we can conclude that there is 1 outcome out of 36 possible outcomes where the number on the top face of the black die is a 1 and the number on the top face of the white die is a 6. We will use this table in our answers to parts (c), (d), and (e).

(c) For each of the sums of 2 and 12, there is only one possible way to obtain that outcome (two ones or two sixes). Thus, each of these sums has the lowest theoretical probability, namely 1 in 36, or $\frac{1}{36}$.

(d) A sum of 7 can be obtained in 6 possible ways (as $1 + 6$ or $6 + 1$, $2 + 5$ or $5 + 2$, $3 + 4$ or $4 + 3$). So, there are six outcomes which give the desired sum. Thus, the theoretical probability that he rolls a 7 is 6 in 36, or $\frac{6}{36}$, which is equivalent to 1 in 6, or $\frac{1}{6}$.

(e) A sum of 11 can be obtained in 2 possible ways (as $5 + 6$ or $6 + 5$). Thus, the theoretical probability of rolling an 11 is 2 in 36, or $\frac{2}{36}$, which is equivalent to 1 in 18, or $\frac{1}{18}$.

(f) Answers will vary.

(g) Answers will vary.