Problem of the Week

Problem B and Solution

Equally Likely or A Sure Thing?

Problem

(a) Imagine you are drawing marbles one at a time from a bag which contains 1 red, 1 blue, 2 yellow, and 3 green marbles. You draw a marble without looking, record the colour, and return it to the bag. Suppose that each marble is equally likely to be drawn.

Which of the following events are equally likely to occur?

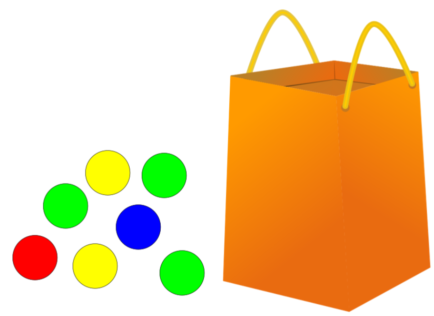
(i) You draw a red marble.

(ii) You draw a blue marble.

(iii) You draw a yellow marble.

(iv) You draw a green marble.

(v) You draw a red OR a blue marble.



Justify your answers by comparing the theoretical probabilities of the events.

(b) Suppose you have two unusual six-sided dice (number cubes), one with the even numbers 2, 4, 6, 8, 10, and 12 on its faces, and the other with the odd numbers 1, 3, 5, 7, 9, and 11 on its faces. When you roll the dice together, you find the sum of the two top faces. What is the probability of each of the following events?

(i) The sum is odd.

(ii) The sum is 7.

(iii) The sum is 25.

(c) Which of the events in part (b) can be called *certain*? Which of the events in part (b) can be called *impossible*?

Solution

(a) The theoretical probability in each case is equal to

$$\frac{\text{the number of marbles of the desired colour(s)}}{\text{the total number of marbles}}$$

Using this and the fact that there are 7 marbles in total, we calculate the probability of each event.

(i) The probability of drawing a red marble is $\frac{1}{7}$, since there is only one red marble.

(ii) The probability of drawing a blue marble is $\frac{1}{7}$, since there is only one blue marble.

(iii) The probability of drawing a yellow marble is $\frac{2}{7}$, since there are two yellow marbles and either of the two could be drawn.

(iv) The probability of drawing a green marble is $\frac{3}{7}$, since there are three green marbles and any of the three could be drawn.

(v) The probability of drawing a red or a blue marble is $\frac{2}{7}$, since there are two marbles that are red or blue.

Therefore, events (i) and (ii) are equally likely to occur. Also, events (iii) and (v) are equally likely to occur.

(b) For each roll of the die with even numbers on its faces, there are 6 possible rolls for the die with odd numbers on its faces. Thus, since the the die with even numbers on its faces has 6 faces, there are $6 \times 6 = 36$ possible rolls. Using this, we can calculate the probability of each event.

(i) An odd number plus an even number is always odd, so every roll will produce an odd sum. Thus, the probability that the sum is odd is equal to $\frac{36}{36} = 1$.

(ii) A sum of 7 could be produced by rolling a 1 and a 6, rolling a 2 and a 5, or rolling a 3 and a 4. Thus, the probability that the sum is 7 is equal to $\frac{3}{36} = \frac{1}{12}$.

(iii) The maximum possible sum is $11 + 12 = 23$, so there is no way to roll a sum of 25. Thus, the probability that the sum is 25 is equal to $\frac{0}{36} = 0$.

(c) Since the probability of event (i) is 1, then event (i) can be called *certain*. Since the probability of event (iii) is 0, then event (iii) can be called *impossible*.