Problem of the Week
Problem A and Solution
Patterned Savings

Problem

Rebecca begins saving money starting on January 1. She collects money in a jar in the following way. Every day she puts a quarter in the jar. Every second day, starting on January 2, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar.

So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar. In Canada, a quarter is worth 25 cents, a loonie is worth one dollar, and a toonie is worth two dollars. There are 100 cents in one dollar.

(a) How many coins does she add to the jar on January 12?

(b) How many coins does she add to the jar on January 23?

(c) How many coins does she add to the jar on January 30?

(d) How many coins in total does she have in the jar by the end of January?

(e) If she keeps saving this way, how much money will she have after 90 days?

Solution

One way to answer most of these questions is to keep track of how many coins are added each day and how much money is accumulated each day in a table.

Day	Number of Quarters Added	Number of Loonies Added	Number of Toonies Added	Money Added	Total Coins	Total Money
1	1	0	0	\$0.25	1	\$0.25
2	1	1	0	\$1.25	3	\$1.50
3	1	0	0	\$0.25	4	\$1.75
4	1	1	0	\$1.25	6	\$3.00
5	1	0	1	\$2.25	8	\$5.25
6	1	1	0	\$1.25	10	\$6.50
7	1	0	0	\$0.25	11	\$6.75
8	1	1	0	\$1.25	13	\$8.00
9	1	0	0	\$0.25	14	\$8.25
10	1	1	1	\$3.25	17	\$11.50

We could continue the table, and we would see that the amounts of coins and money added would repeat every 10 days. We observe that on every day that is a

multiple of 2 (except on day 10) Rebecca adds two coins to the jar, and on every day that is a multiple of 5 (except on day 10) Rebecca adds two coins to the jar. On day 10, which is a multiple of both 2 and 5, Rebecca adds three coins to the jar. On all other days, Rebecca adds just one coin to the jar.

(a) Since 12 is a multiple of 2 but not 5, on January 12, Rebecca adds 2 coins to the jar.

(b) Since 23 is neither a multiple of 2 nor a multiple of 5, Rebecca adds only 1 coin to the jar.

(c) Since 30 is a multiple of both 2 and 5, Rebecca adds 3 coins to the jar.

(d) From the table, we know that after 10 days, Rebecca will have saved 17 coins. This pattern will repeat every 10 days. So by January 30, the pattern will have repeated 3 times. On January 31, she will add one more quarter. So, by January 31 Rebecca will have $17 + 17 + 17 + 1 = 52$ coins in the jar.

(e) Every 10 days Rebecca will have saved a total of \$11.50. This pattern will repeat 9 times over a 90 day period. So after 90 days Rebecca will have \$11.50+\$11.50+\$11.50+\$11.50+\$11.50+\$11.50+\$11.50+\$11.50+\$11.50 = \$103.50.

Alternatively we could calculate the savings as $9 \times \$11.50 = \$103.50$.

Teacher's Notes

This problem shows a pattern that repeats every 10 days. In mathematics, we could refer to this kind of repetition as *periodic*. The length of the interval between repeated elements is known as the *period* of the function. In this case, the period of the savings function is 10, which is the *least common multiple* or *LCM* of the integers 1, 2, and 5. The LCM of a set of integers is the smallest positive integer that is a multiple of each integer in the set. In this case, we are looking for the smallest positive multiple of each of the individual periods of savings.

Periodic functions appear in mathematics and in the real world. Trigonometric functions such as *sin*, *cos*, and *tan* are periodic functions. Sound waves, phases of the moon, and your blood pressure, are all examples of periodic functions in nature.