Problem of the Week Problem A and Solution Scrabble Words

Problem

ScrabbleTM is a game where players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and a player's score for placing a word on the board is the sum of the point values for the letters in the word. The point values for each letter are in the following table.

Letter	А	В	С	D	E	F	G	Н	Ι	J	Κ	L	М
Value	1	3	3	2	1	4	2	4	1	8	5	1	3
								-					
Letter	Ν	0	Р	Q	R	S	Т	U	V	W	Х	Y	Z

In the Scrabble board shown, the coloured squares have special values.

Blue squares, marked \mathbf{B} on the board, are double letter squares, which means if you put a tile on that square, then the letter is worth twice as many points.

Grey squares, marked \mathbf{G} on the board, are triple letter squares, which means if you put a tile on that square, then the letter is worth three times as many points.

Red squares, marked \mathbf{R} on the board, are double word squares, which means if you put a tile on that square, then the whole word is worth twice as many points.

Orange squares, marked O on the board, are triple word squares, which means if you put a tile on that square, then the whole word is worth three times as many points.

Double and triple letter scores are calculated before the double and triple word scores.

- G
 I
 G
 G

 R
 I
 I
 I

 I
 R
 I
 I

 G
 I
 R
 I
 B

 G
 I
 R
 R
 I

 G
 I
 R
 I
 I

 G
 I
 B
 I
 O
- (a) How many points would the word EXACT be worth if you put the first letter in the first row and second column, and place the word horizontally across the board?
- (b) How many points would the word DEBUT be worth if you put the first letter in the second row and second column, and place the word vertically on the board?
- (c) How many points would the word CHEF be worth if you put the first letter of the word in the last column and third row, and place the word vertically on the board?

Solution

- (a) If we place the letters for EXACT horizontally on the board starting with the letter E in the first row and second column, then the letter C will be on a grey square. This means that letter is worth three times its normal value. That is, the letter C is worth $3 \times 3 = 9$ points. So the score for this word is 1 + 8 + 1 + 9 + 1 = 20 points.
- (b) If we place the letters for DEBUT vertically down the board starting with the D in the second row and second column, then the word covers a red square. This means the word is worth twice its normal value. The normal score for the word would be 2+1+3+1+1=8. The red square means that the word would be worth twice as much, so the score for the word at this location on the board is $2 \times 8 = 16$ points.
- (c) If we place the letters for CHEF vertically down the board starting with the letter C in the last column and third row, then the letter C will be on a blue square. This means that letter is worth twice its normal value. That is, in this placement of the word, the letter C is worth $2 \times 3 = 6$ points. The word also covers an orange square, which means the word is worth three times its normal value. We calculate the score including the double value of the letter C before applying the triple word score. The score for the word without considering the triple word square, but including its double letter score, is 6 + 4 + 1 + 4 = 15. When we consider the triple word square, the result is $3 \times 15 = 45$ points.

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Teacher's Notes

When we apply the rule that double and triple letter scores are calculated before the double and triple word scores, we are enforcing an *order of operations* to our calculations.

We often use the mnemonic **BEDMAS** to describe the standard order of operations for mathematical operators. This stands for **B**rackets, **E**xponents, **D**ivision and **M**ultiplication in the order that they appear, and **A**ddition and **S**ubtraction in the order that they appear.

We use brackets in places where we want some operation to take precedence over another operation that would otherwise happen first according to BEDMAS. For example, in part (b) of this problem, we want to add the letter values together before we apply the double word score. We could calculate the result like this:

$$(2+1+3+1+1) \times 2 = (8) \times 2 = 16$$

Without the brackets the calculation

$$2 + 1 + 3 + 1 + 1 \times 2$$

would equal

$$2 + 1 + 3 + 1 + 2 = 9$$

since according to BEDMAS the multiplication is done before the addition.

In part (c), we need to multiply the point value of the letter C by 2 since it is on a double letter square. Then we add the rest of the letter values together before multiplying that sum by 3. So we could calculate the result for part (c) like this:

$$(2 \times 3 + 4 + 1 + 4) \times 3 = (6 + 4 + 1 + 4) \times 3 = (15) \times 3 = 45$$

Note that we do not need brackets around 2×3 since, according to BEDMAS, the multiplication will be done before the addition in the calculation. However, sometimes we use brackets (even when they are not required) to make calculations clearer. To emphasize that the double letter score must be calculated before the sum of the letters is calculated, it may be easier to understand the calculation for part (c) if we used this expression:

$$((2 \times 3) + 4 + 1 + 4) \times 3$$