The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

Problem of the Week
Problems and Solutions
2021 - 2022

Problem B (Grade 5/6)

Themes

(Click on a theme name to jump to that section.)

Number Sense (N)

Geometry & Measurement (G)

Algebra (A)

Data Management (D)

Computational Thinking (C)

The problems in this booklet are organized into themes.
A problem often appears in multiple themes.

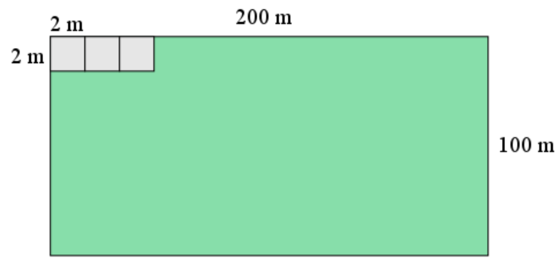
Number Sense (N)

TAKE ME TO THE COVER

Problem of the Week
Problem B
Work it Out

A gym is hosting an outdoor group exercise class. For many of the exercises, participants will need to make sure they are spaced well apart.

(a) A large grassy field has dimensions of 100 m by 200 m. The field was divided into squares that were each 2 m by 2 m, as shown.



If one person was in the middle of each square, how many people could be on the field?

(b) Imaginary Park is exactly 1 km by 1 km, or 1 km^2, which is equivalent to 100 hectares (ha) in size. If this park was divided into 2 m by 2 m squares for an exercise class like in part (a), and there is one person in the middle of each square, how many people would be in this park? How many people per hectare is that?

(c) Stanley Park is located in Vancouver, BC. While not a rectangle, it covers an area of 405 hectares. Suppose that $\frac{1}{5}$ of the park is not forested. If the number of people per hectare in the non-forested area of Stanley Park is the same as the number of people per hectare in Imaginary Park in part (b), how many people could do the exercise class in the non-forested area of Stanley Park?

Themes Geometry, Number Sense

Problem of the Week

Problem B and Solution

Work it Out

Problem

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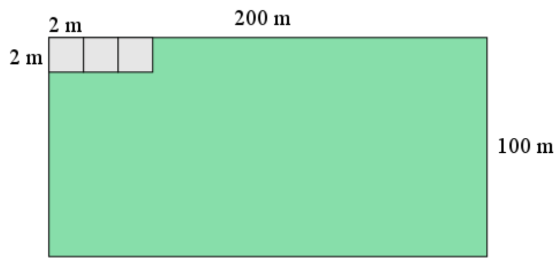

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Solution

(a) We need to figure out the number of 2 m by 2 m squares in the field. Since there are $200 \div 2 = 100$ squares along the long side of the park, and $100 \div 2 = 50$ squares along the short side, there are $100 \times 50 = 5000$ squares in total. That means the field could accommodate 5000 people.

(b) Since Imaginary Park is 1 km by 1 km (or 1000 m by 1000 m), there could be $1000 \div 2 = 500$ people in each row. Since there are $1000 \div 2 = 500$ such rows, there could be $500 \times 500 = 250\,000$ people in 100 ha of space. This works out to $250\,000 \div 100 = 2500$ people per ha.

(c) The non-forested area of Stanley Park is $\frac{1}{5}$ of 405 ha, or $\frac{1}{5} \times 405 = 81$ ha. This area will accommodate 2500 people per ha. This means a total of $2500 \times 81 = 202\,500$ people could do the exercise class in the non-forested area of Stanley Park at one time.

Problem of the Week
Problem B
The Puzzler

The Puzzler is the world's latest superhero. He uses his immense brain to win all battles by solving a series of math problems. He needs your help to solve the following problems.

Use a calculator to help when needed. You may also want to look up words like *consecutive* and *sum*.

(a) The numbers 3, 5, and 7 are three consecutive odd numbers that have a sum of $3 + 5 + 7 = 15$.
What are three consecutive odd numbers that have a sum of 399?

(b) What are three consecutive even numbers that have a sum of 5760?

(c) What are four consecutive whole numbers that have a sum of 2022?

Theme Number Sense

Problem of the Week
Problem B and Solution
The Puzzler

Problem

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What are three consecutive odd numbers that have a sum of 399?

(b) What are three consecutive even numbers that have a sum of 5760?

(c) What are four consecutive whole numbers that have a sum of 2022?

Solution

(a) The sum of the three consecutive odd numbers 3, 5, and 7 is $3 + 5 + 7 = 15$. We notice that $15 = 3 \times 5$ and 5 is the middle number. It seems that to find the middle of three consecutive odd numbers with a certain sum, we may divide that sum by 3.

Let's try using this to solve the problem. We note that $399 \div 3 = 133$. Therefore, the middle number could be 133. Then the first number would be 131 and the third number would be 135. The sum of these numbers is indeed $131 + 133 + 135 = 399$. Therefore, the three consecutive odd numbers are 131, 133, and 135.

(b) We will use a process like in (a). Noting that $5760 \div 3 = 1920$, we see that three consecutive even numbers could be 1918, 1920, and 1922. The sum of these numbers is indeed $1918 + 1920 + 1922 = 5760$. Therefore, the three consecutive even numbers are 1918, 1920, and 1922.

(c) Using a similar process, when we divide 2022 by 4 we get 505.5. Since 505 and 506 are the closest whole numbers to 505.5, they may be the two middle numbers. The four consecutive numbers may be 504, 505, 506, and 507. The sum of these numbers is indeed $504 + 505 + 506 + 507 = 2022$. Therefore, the four consecutive numbers are 504, 505, 506, and 507.

Problem of the Week

Problem B

Are We There Yet?

Planets travel around the sun in elliptical orbits (ovals). Mercury is the planet closest to the sun. The distance between Earth and Mercury ranges from 77 000 000 km at its closest distance to 222 000 000 km at its farthest distance.

Because distances are so great in the solar system, scientists measure them in **Astronomical Units**, or **AU**. One AU is equal to the average distance between the Earth and the Sun, or about 149 600 000 km.

Complete the missing information in the table below.

Planet	Distance in AU from Earth	Distance in km from Earth	Travel Time
Mars	0.52		
Venus			61 days
Saturn		1 275 000 000	
Neptune	29.09		

To calculate the travel time, assume you are travelling from Earth to the planet in a rocket at a speed of 28 000 km per hour throughout your flight. Pick the most reasonable unit of measure for time (for example, 15 000 hours doesn't mean much, but when divided by 24 to get 625 days, you know that it's almost 2 years).

Themes Geometry, Number Sense

Problem of the Week
Problem B and Solution
Are We There Yet?

Problem

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Complete the missing information in the table.

To calculate the travel time, assume you are travelling from Earth to the planet in a rocket at a speed of 28 000 km per hour throughout your flight. Pick the most reasonable unit of measure for time (for example, 15 000 hours doesn't mean much, but when divided by 24 to get 625 days, you know that it's almost 2 years).

Solution

The completed table is shown below.

Planet	**Distance in AU from Earth**	**Distance in km from Earth**	**Travel Time**
Mars	0.52	77 792 000	2 778 hr = 116 days
Venus	0.27	40 992 000	61 days
Saturn	8.52	1 275 000 000	45 536 hr = 1897 days = 5+ years
Neptune	29.09	4 351 864 000	155 424 hr = 6476 days = 17+ years

Since 1 AU = 149 600 000 km, to convert from the distance in AU to the distance in km (for Mars and Neptune), we multiply the distance in AU by 149 600 000. Similarly, to convert from the distance in km to the distance in AU (for Saturn), we divide the distance in km by 149 600 000. This allows us to fill in both distance columns for Mars, Saturn, and Neptune.

To calculate the travel time, we use the speed of the rocket, which is 28 000 km per hour. If we divide the distance in km by 28 000 km per hour, we will get the number of hours it takes to travel that distance, which is the travel time. We can then convert this to a more appropriate unit as we see fit.

To calculate the distance in km from the travel time (for Venus), note that 61 days is equal to $61 \times 24 = 1464$ hours. Thus, travelling at 28 000 km per hour, the distance covered would be $28\,000 \times 1464 = 40\,992\,000$ km. We can then convert the distance in km to the distance in AU as we did for Saturn.

Problem of the Week

Problem B

When is This Deal a Deal?

Danielle uses a battery-powered magnifying headlamp when creating silver jewellery. The headlamp requires one AA battery.

Instead of buying a 10-pack of non-rechargeable AA batteries for \$17.50, she decides to buy one rechargeable battery and a charger for \$40.

Suppose each non-rechargeable battery is used until it no longer works and the rechargeable battery is used until it needs to be recharged. Also suppose that the length of time until a non-rechargeable battery no longer works is the same as the length of time until a rechargeable battery needs to be recharged.

After how many rechargeable battery uses will Danielle's choice be a better deal than buying 10-packs?

You may find the table below to be useful. For example, after 5 uses of the rechargeable battery, the price per use for the rechargeable battery will be $\$40 \div 5 = \$8.00$.

Number of Rechargeable Battery Uses	Price Per Rechargeable Battery Use
5	\$8.00
10	
15	
20	
$\vdots$	

Theme Number Sense

Problem of the Week

Problem B and Solution

When is This Deal a Deal?

Problem

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After how many rechargeable battery uses will Danielle's choice be a better deal than buying 10-packs?

Solution

The cost of a single battery in a 10-pack is $\$17.50 \div 10 = \$1.75$. Therefore, the price per non-rechargeable battery use is \$1.75.

We will use the following completed table to answer the question.

Number of Rechargeable Battery Uses	Price Per Rechargeable Battery Use
5	\$8.00
10	\$4.00
15	\$2.67
20	\$2.00
25	\$1.60
22	\$1.82
23	\$1.74

Examining the completed table above, we see that when we look at increasing the number of rechargeable battery uses by 5, Danielle's purchase becomes a better deal after 25 uses. We then calculate the price per battery use for 22 and 23 uses. We notice that 23 uses is the smallest number of uses where the price per battery use is less than \$1.75.

Therefore, on the 23$^{\text{rd}}$ use the rechargeable battery is a better deal.

Problem of the Week

Problem B

Screen Size, Now and Then

Flat screen TVs usually have a screen ratio of $16:9$. This means that if the screen is 16 units wide, then it will be 9 units high. If the screen is 32 units wide, then since $32 = 16 \times 2$, it will be $9 \times 2 = 18$ units high, and so on.

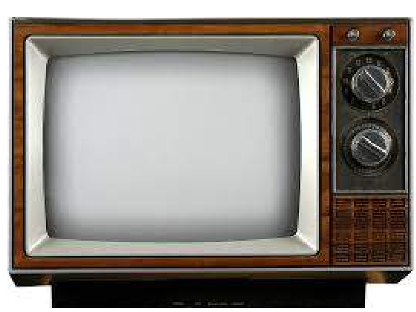
(a) Starting in the bottom-left corner of a grid that is 20 units wide and 10 units high, use a ruler to draw a flat screen TV screen that is 16 units wide and 9 units high.

(b) Older TVs had a screen ratio of $4:3$. If an older TV was 9 units high, how many units wide would it be?

(c) Draw the TV screen from part (b) on the same grid used in part (a), also starting in the bottom-left corner.

(d) How many more square units of area does the flat screen TV screen have compared to the older TV screen, if they both have a height of 9 units?

(e) A 4K flat screen TV has 3840×2160 pixels. If the screen is 122 cm wide by 69 cm high, how many pixels per cm^2 are there? Round to the nearest whole number.



Themes Geometry, Number Sense

Problem of the Week

Problem B and Solution

Screen Size, Now and Then

Problem

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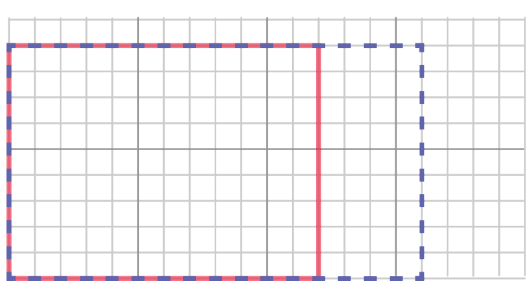
Solution

(a) The drawing of the flat screen TV screen on the grid is shown in part (c).

(b) The screen ratio of an older TV is $4:3$, so if the the height is 9 units, that means we have multiplied the 3 in our screen ratio by 3 to get 9. So the width would be $4 \times 3 = 12$ units.

(c) The grid below shows the flat screen TV with a dashed blue line and the older TV with a solid red line.



(d) We can count the squares on our grid that are part of the flat screen TV but not the older TV. We notice that the flat screen TV has 4 more squares of width, and since the height is 9 units for both TVs, there are $4 \times 9 = 36$ more square units of area in the flat screen TV.

(e) There are $3840 \times 2160 = 8\,294\,400$ pixels in total, and the area of the TV is $122 \times 69 = 8418$ cm^2. Thus there are $8\,294\,400 \div 8418 = 985$ pixels per cm^2.

Problem of the Week
Problem B
Money for Music

The CEMCers are a new up-and-coming band. They only play music that they have written. Over the past year, they had to cancel a few concerts. They will try to recover some of the money lost by using the online radio station RipRap.

(a) On the streaming radio station RipRap, musicians are paid on average \$0.0038 every time one of their songs is played. If The CEMCers usually make around \$10 000 per concert, how many times will RipRap have to play one of their songs for The CEMCers to make an income equivalent to the income made in one concert?

(b) Riprap doesn't play an artist's songs non-stop, all day, every day. Suppose that RipRap plays one of The CEMCers songs three times every day, starting on January 1, 2022. How long will it take until RipRap has paid \$10 000 to The CEMCers? (To take leap years into account, assume each year has 365.25 days.)

Theme Number Sense

Problem of the Week

Problem B and Solution

Money for Music

Problem

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Solution

(a) The desired relationship is

$$\$10\,000 = \text{the number of plays of their songs} \times \$0.0038$$

Thus, the required number of plays is

$$10\,000 \div 0.0038 \approx 2\,631\,578.95$$

Since a whole number of songs are played, The CEMCers will have made the income made in one concert once RipRap has played one of their songs 2 631 579 times.

(b) The number of days needed to play 2 631 579 of an artist's songs at 3 plays per day is

$$2\,631\,579 \div 3 = 877\,193 \text{ days}$$

Assuming each year averages 365.25 days, this is equivalent to

$$877\,193 \div 365.25 \approx 2401.6235 \text{ years}$$

That is, it would take 2401 years plus approximately $0.6235 \times 365.25 \approx 228$ days to play enough songs to pay the artist \$10 000.

This would be in the year 4423. It looks like The CEMCers will need to find another source of income to make up for their lost income.

Extension: How many plays a day would result in the band earning \$10 000 in 20 years?

Problem of the Week

Problem B

Running Low on Gas

Driving home from a meeting late one evening, Ming notices that her gas gauge is showing that a mere $\frac{1}{10}$ of a tank remains. Luckily, just then she spots a 24-hour gas station. She has just enough money to add 20 litres of gas to the tank, bringing her gas tank up to $\frac{1}{2}$ full.

(a) Given that the gas tank went from $\frac{1}{10}$ full to $\frac{1}{2}$ full, determine the fraction of the tank filled by the gas that Ming added. Hint: Use equivalent fractions.

(b) The fraction of the tank you found in part (a) holds 20 L. How many litres are there in $\frac{1}{10}$ of a full tank?

(c) Given what you discovered in part (b), what is the full capacity, in litres, of Ming's gas tank?

Theme Number Sense

Problem of the Week
Problem B and Solution
Running Low on Gas

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Solution

(a) Since $\frac{1}{2} = \frac{5}{10}$ and Ming started with $\frac{1}{10}$ of a tank, the gas Ming added filled

$$\frac{5}{10} \text{ of a tank} - \frac{1}{10} \text{ of a tank} = \frac{4}{10} \text{ of a tank.}$$

(b) Since $\frac{4}{10}$ of a tank holds 20 litres, $\frac{1}{10}$ of a tank holds $20 \div 4 = 5$ litres.

(c) Since $\frac{1}{10}$ of a tank holds 5 litres, the full capacity of Ming's tank is $10 \times 5 = 50$ litres.

Problem of the Week

Problem B

Orange You Glad?

Betsy is shopping for orange juice. She has discovered that it comes in a variety of containers at different prices.

- At one store, a 2.63 L container of orange juice costs \$4.00, and a pack of eight 200 mL orange juice boxes costs \$2.64.
- At another store, 2 L of orange juice costs \$3.59.
- At both stores, concentrated orange juice in a 295 mL can costs \$1.71. (This must be mixed with three cans of water to obtain $4 \times 295 = 1180$ mL of drinkable juice.)

Which purchase will give Betsy the best value for her money?

You may find calculating the price per 100 mL for each container size in the following table useful.

Amount of Orange Juice	**Price**	**Price per 100 mL**
2.63 L	\$4.00	
$8 \times 200 = 1600$ mL	\$2.64	
2 L	\$3.59	
1180 mL (mixed from concentrate)	\$1.71	

Themes Geometry, Number Sense

Problem of the Week

Problem B and Solution

Orange You Glad?

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Which purchase will give Betsy the best value for her money?

Solution

The 2.63 L container of orange juice costs $\$4.00 \div 2.63 \approx \$1.521$ per litre. Since 100 mL is $\frac{1}{10}$ of a litre, the cost is approximately $\$1.521 \div 10 = \$0.1521$ or 15.2¢ per 100 mL.

The 8-pack costs \$2.64 for 1600 mL, or $\$2.64 \div 1600 = \$0.00165$ per mL.
This is equal to $\$0.00165 \times 100 = \$0.165$ or 16.5¢ per 100 mL.

The 2 L container costs $\$3.59 \div 2 = \$1.795$ per litre. Since 100 mL is $\frac{1}{10}$ of a litre, the cost is $\$1.795 \div 10 = \$0.1795$ or about 18¢ per 100 mL.

The frozen concentrate costs $\$1.71 \div 1180 \approx \$0.00145$ per mL.
Therefore, the cost is approximately $\$0.00145 \times 100 = \$0.145$ or 14.5¢ per 100 mL.

The cost per 100 mL for each item is summarized in the completed table below.

Amount of Orange Juice	Price	Price per 100 mL
2.63 L	\$4.00	15.2¢
$8 \times 200 = 1600$ mL	\$2.64	16.5¢
2 L	\$3.59	18¢
1180 mL (mixed from concentrate)	\$1.71	14.5¢

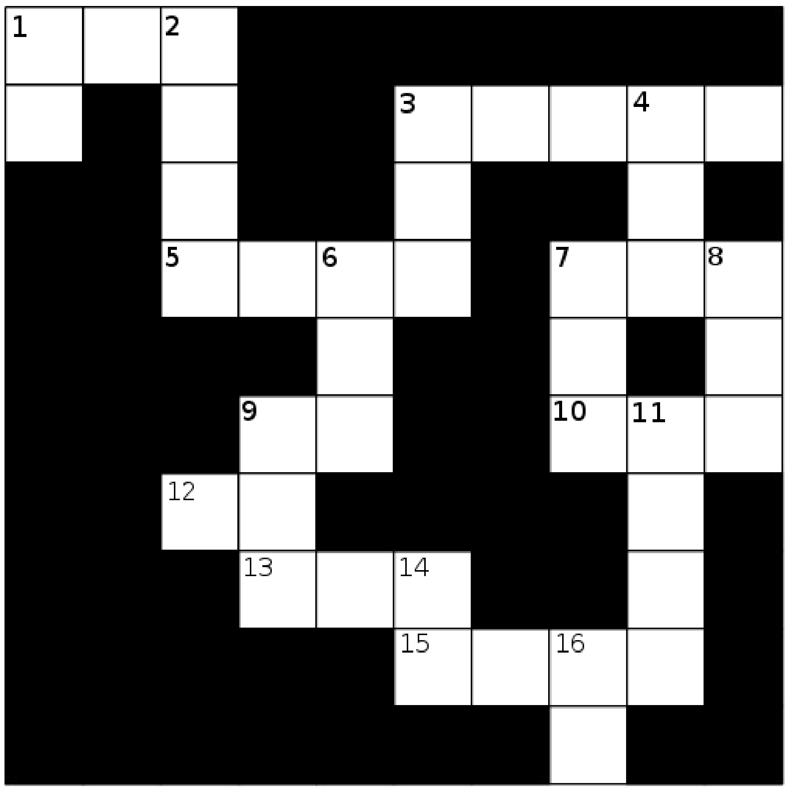
Since the concentrated orange juice has the lowest price of 14.5¢ per 100 mL, the best value for her money is the concentrated orange juice.

Problem of the Week

Problem B

This Puzzle Has Got Your Number!

Use the clues below the grid to complete the puzzle. The answer to each clue is a whole number. To put an answer in the grid, place one digit in each square.



Across Clues:

1. 1 hundred + 2 tens + 3 ones
3. $30\,000 + 2000 + 300 + 60 + 7$
5. $3444 + 3345$
7. 114×3
9. $2 \times 2 \times 2 \times 3 \times 3$
10. $369 - 234$
12. 4^3 or $4 \times 4 \times 4$
13. $3 \times 2 \times 2 \times 2 \times 2 \times 19$
15. 3841×2

Down Clues:

1. 1 ten + 7 ones
2. $10\,000 - 6544$
3. $999 - 630$
4. $1000 - 356$
6. 421×2
7. 3×107
8. 15^2 or 15×15
9. 7×107
11. $1500 + 1600 + 90 + 2$
14. 3^3 or $3 \times 3 \times 3$
16. 3^4 or $3 \times 3 \times 3 \times 3$

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THEME NUMBER SENSE

Problem of the Week

Problem B and Solution

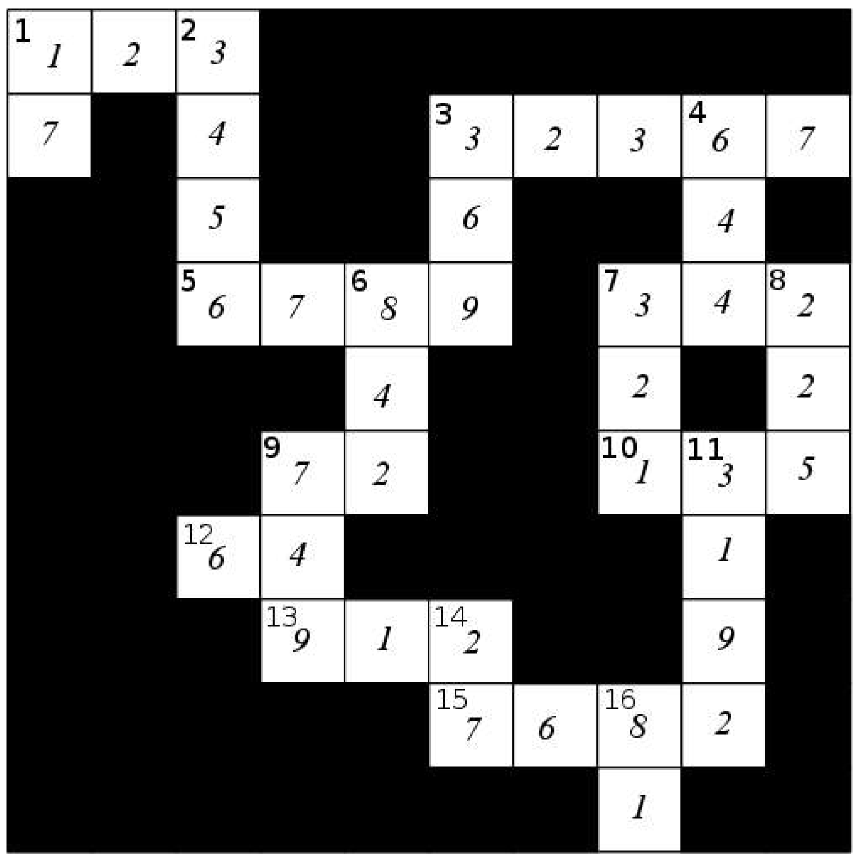
This Puzzle Has Got Your Number!

Problem

Use the clues below the grid to complete the puzzle. The answer to each clue is a whole number. To put an answer in the grid, place one digit in each square.

Solution

The completed grid is shown, as well as the answer to each clue.



Across Clues:

1. 1 hundred + 2 tens + 3 ones = 123
3. 30 000 + 2000 + 300 + 60 + 7 = 32 367
5. 3444 + 3345 = 6789
7. $114 \times 3 = 342$
9. $2 \times 2 \times 2 \times 3 \times 3 = 72$
10. $369 - 234 = 135$
12. 4^3 or $4 \times 4 \times 4 = 64$
13. $3 \times 2 \times 2 \times 2 \times 2 \times 19 = 912$
15. $3841 \times 2 = 7682$

Down Clues:

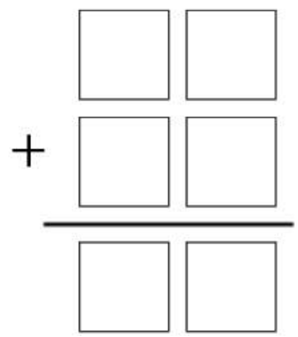
1. 1 ten + 7 ones = 17
2. $10\,000 - 6544 = 3456$
3. $999 - 630 = 369$
4. $1000 - 356 = 644$
6. $421 \times 2 = 842$
7. $3 \times 107 = 321$
8. 15^2 or $15 \times 15 = 225$
9. $7 \times 107 = 749$
11. $1500 + 1600 + 90 + 2 = 3192$
14. 3^3 or $3 \times 3 \times 3 = 27$
16. 3^4 or $3 \times 3 \times 3 \times 3 = 81$

Problem of the Week

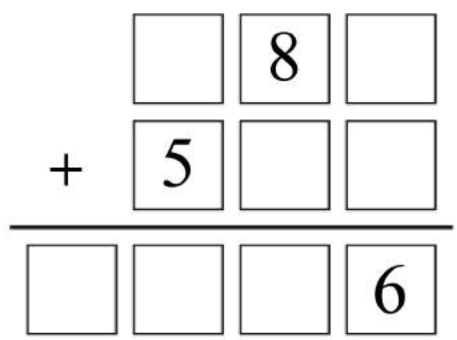
Problem B

That's About Right

(a) Place the digits 1, 3, 6, 7, 8, and 9 in the boxes shown so that each box contains a different digit, and the sum is as close as possible to 99.



(b) The digits 5, 6, and 8 have been placed in three of the boxes shown. Place the digits 0, 1, 2, 3, 4, 7, and 9 in the remaining boxes so that each box contains a different digit, and the sum is as close as possible to 1000.



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Problem of the Week

Problem B and Solution

That's About Right

Problem

(a) Place the digits 1, 3, 6, 7, 8, and 9 in the boxes shown so that each box contains a different digit, and the sum is as close as possible to 99.

$$\begin{array}{rcc} & \boxed{} & \boxed{} \\ + & \boxed{} & \boxed{} \\ \hline & \boxed{} & \boxed{} \end{array}$$

(b) The digits 5, 6, and 8 have been placed in three of the boxes shown. Place the digits 0, 1, 2, 3, 4, 7, and 9 in the remaining boxes so that each box contains a different digit, and the sum is as close as possible to 1000.

$$\begin{array}{cccc} & \boxed{} & \boxed{8} & \boxed{} \\ + & \boxed{5} & \boxed{} & \boxed{} \\ \hline \boxed{} & \boxed{} & \boxed{} & \boxed{6} \end{array}$$

Not printing this page? Try our interactive worksheet.

Solution

(a) Notice that 98 is the closest number to 99 that could possibly be formed using the digits 1, 3, 6, 7, 8, and 9. Let's see if we can arrange the remaining digits, 1, 3, 6, and 7, to get a sum of 98.

Using the digits 1, 3, 6, and 7, to get a sum with a ones digit of 8, we must place the 1 and the 7 in the two boxes in the ones column. If we place the remaining digits, 3 and 6, in the tens column, we will get a sum with a tens digit of 9. Therefore, it is possible to arrange the digits to get a sum of 98, which is the closest possible sum to 99. We also see that there are four possible ways to arrange the digits in the boxes to produce this sum.

$$\begin{array}{rcc} & \boxed{6} & \boxed{1} \\ + & \boxed{3} & \boxed{7} \\ \hline & \boxed{9} & \boxed{8} \end{array} \qquad \begin{array}{rcc} & \boxed{6} & \boxed{7} \\ + & \boxed{3} & \boxed{1} \\ \hline & \boxed{9} & \boxed{8} \end{array} \qquad \begin{array}{rcc} & \boxed{3} & \boxed{1} \\ + & \boxed{6} & \boxed{7} \\ \hline & \boxed{9} & \boxed{8} \end{array} \qquad \begin{array}{rcc} & \boxed{3} & \boxed{7} \\ + & \boxed{6} & \boxed{1} \\ \hline & \boxed{9} & \boxed{8} \end{array}$$

(b) Given the digits 0, 1, 2, 3, 4, 7, and 9, along with the placement of the 6, the closest number to 1000 that could be formed is 0976. The next closest number is 1026.

We will first see if we can place the digits to get a sum of 0976. If the sum is 0976, then the digits 0, 9, and 7 have been placed, and the remaining boxes will be filled with the digits 1, 2, 3, and 4. Of these digits, the only two that have a sum with ones digit 6 are 2 and 4. Therefore, the 2 and the 4 would need to go in the ones column. This leaves 1 and 3 to be placed. Looking at the tens column of the sum, we need to place one of these numbers in the tens column so that the sum of that number with 8 has a ones digit of 7. This is not possible. Therefore, we see that it is not possible to place the numbers so that the sum is 0976.

Next, we try to place the digits to get a sum of 1026. If the sum is 1026, then the digits 0, 1, and 2 have been placed, and the remaining boxes must be filled with the digits 3, 4, 7, and 9. Of these digits, the only two that have a sum with ones digit 6 are 7 and 9. Therefore, the 7 and the 9 would need to go in the ones column. This leaves 3 and 4 to be placed. Looking at the tens column of the sum, we need to place one of these numbers in the tens column so that the sum of that number with 8 and 1 (the carry from the ones column) has a ones digit of 2. This is possible if we place the 3 in this box. That leaves the 4 to go in the empty box in the hundreds column. Indeed, we see that the sum of 4 with 5 and 1 (the carry from the tens column) is 10, as required.

Thus, it is possible to arrange the digits to get a sum of 1026, and this is the closest we can get to a sum of 1000. We see that there are two possible ways to arrange the digits in the boxes to produce this sum.

$$\begin{array}{cccc} & \boxed{4} & \boxed{8} & \boxed{9} \\ + & \boxed{5} & \boxed{3} & \boxed{7} \\ \hline \boxed{1} & \boxed{0} & \boxed{2} & \boxed{6} \end{array} \qquad \begin{array}{cccc} & \boxed{4} & \boxed{8} & \boxed{7} \\ + & \boxed{5} & \boxed{3} & \boxed{9} \\ \hline \boxed{1} & \boxed{0} & \boxed{2} & \boxed{6} \end{array}$$

Extension: Can you find a better solution for (b) by placing the 5, 6, and 8 elsewhere?

Problem of the Week

Problem B

Where's the Audience?

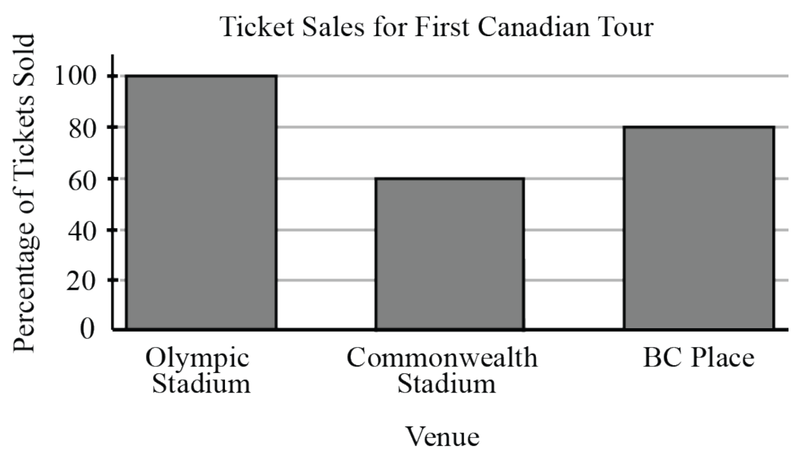
The Pythagorean Triples are a rock band who recently returned from their second Canadian tour.

(a) Information about ticket sales for three of the venues they played at is summarized in the following table.

Venue	Number of Tickets Available	Number of Tickets Sold
Olympic Stadium	60 000	45 000
Commonwealth Stadium	55 000	44 000
BC Place	54 000	48 600

For each venue, what percentage of available tickets were sold?

(b) Two years ago, the Pythagorean Triples played at the same three venues on their first Canadian tour. For each venue, the percentage of available tickets that were sold is shown in the bar graph below.



If the number of tickets available for each venue was the same for both tours, which tour sold more tickets for these three venues combined? Justify your answer.

Problem of the Week
Problem B and Solution
Where's the Audience?

Problem

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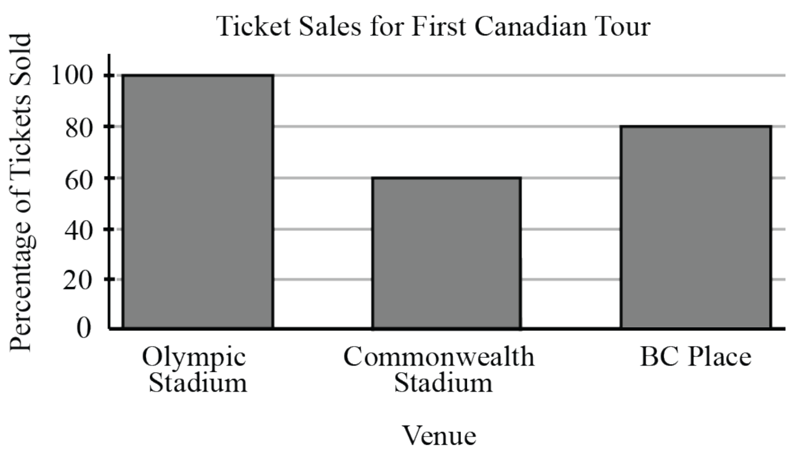


If the number of tickets available for each venue was the same for both tours, which tour sold more tickets for these three venues combined? Justify your answer.

Solution

(a) To calculate the percentage of available tickets that were sold, we divide the number of tickets sold by the number of tickets available, and then multiply by 100% to convert the decimal to a percentage.

- Olympic Stadium: $45\,000 \div 60\,000 = 0.75$, and $0.75 \times 100\% = 75\%$.
- Commonwealth Stadium: $44\,000 \div 55\,000 = 0.8$, and $0.8 \times 100\% = 80\%$.
- BC Place: $48\,600 \div 54\,000 = 0.9$, and $0.9 \times 100\% = 90\%$.

(b) We need to calculate the total number of tickets sold for the three venues for each of the tours.

- For the second Canadian tour, we can add up the number of tickets sold for each venue in the table from part (a).

$$45\,000 + 44\,000 + 48\,600 = 137\,600$$

- For the first Canadian tour, we first need to use the percentages in the bar graph to calculate the number of tickets sold at each venue. The bar graph shows that 100% of the available tickets at Olympic stadium were sold, 60% were sold at Commonwealth Stadium, and 80% were sold at BC Place.
  - Olympic Stadium: 100% of $60\,000$ is $60\,000$.
  - Commonwealth Stadium: 60% of $55\,000$ is equal to $\frac{60}{100} \times 55\,000$ or $\frac{3}{5} \times 55\,000$, which equals $33\,000$.
  - BC Place: 80% of $54\,000$ is equal to $\frac{80}{100} \times 54\,000$ or $\frac{4}{5} \times 54\,000$, which equals $43\,200$.

  Thus, the total number of tickets sold for the three venues for the first Canadian tour is

$$60\,000 + 33\,000 + 43\,200 = 136\,200$$

Since $137\,600 > 136\,200$, it follows that the second Canadian tour sold more tickets for the three venues combined.

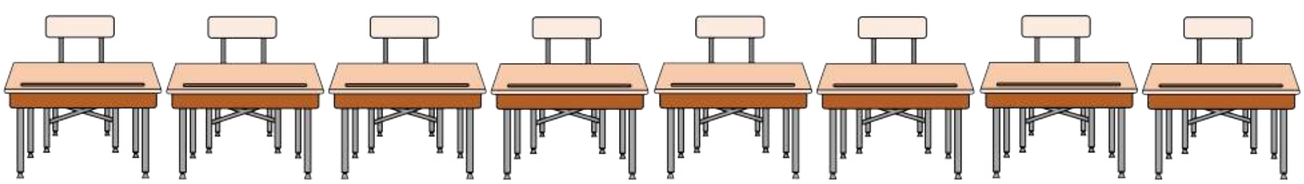
Problem of the Week
Problem B
Let's Divvy These Up

(a) At Yore Elementary School, each class has between 15 and 35 students. If there are 78 students in Grade 6, and all Grade 6 classes have the same number of students, how many Grade 6 classes are there?

(b) At Hyz Elementary School, there is the same number of Grade 6 classes as at Yore, but there are 84 Grade 6 students. If all Grade 6 classes have the same number of students, how many students are in each Grade 6 class?

(c) For each of the numbers in the first column of the table given below, first determine if it could be the total number of students in three very large classes of equal size, and then calculate the digit sum of the number. Do you notice any connection between these two things?
Note: The *digit sum* of a number is the sum of its digits. For example, the digit sum of 63 is $6 + 3 = 9$.

Number	Could be total?	Digit Sum
1008		
1023		
1741		
2238		
1759		
1902		



Theme Number Sense

Problem of the Week
Problem B and Solution
Let's Divvy These Up

Problem

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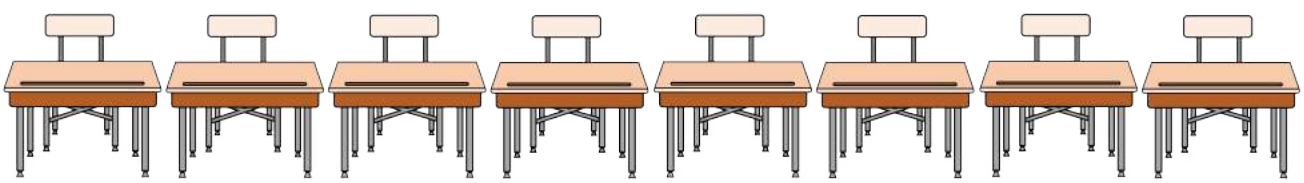


Solution

(a) Since the classes are of equal size, the number of classes must divide evenly into 78. The numbers which do so are $1, 2, 3, 6, 13, 26, 39$, and 78. The only number that is between 15 and 35 is 26. Since $3 \times 26 = 78$, that means there are 3 classes of 26 students.

(b) If there are 84 students in 3 classes, then there are $84 \div 3 = 28$ students in each class.

Alternatively, since $84 - 78 = 6$, that means there are $6 \div 3 = 2$ more students in each class at Hyz Elementary than at Yore. So there are $26 + 2 = 28$ students in each class.

(c) We can try dividing each number by 3.

$$1008 \div 3 = 336$$
$$1023 \div 3 = 341$$
$$1741 \div 3 = 580.\bar{3}$$
$$2238 \div 3 = 746$$
$$1759 \div 3 = 586.\bar{3}$$
$$1902 \div 3 = 634$$

Since 3 divides evenly into $1008, 1023, 2238$, and 1902, these four numbers could be the total number of students in 3 classes of equal size. Since 3 does not divide evenly into 1741 or 1759, these two numbers could not be the total number of students in 3 classes of equal size.

We add this information, along with the digit sum of each number, to the table.

Number	Could be total?	Digit Sum
1008	Yes	9
1023	Yes	6
1741	No	13
2238	Yes	15
1759	No	22
1902	Yes	12

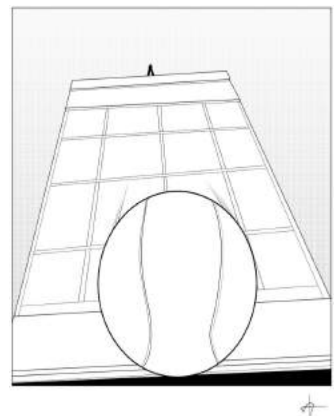
The digit sums for the numbers $1008, 1023, 2238$, and 1902 are all multiples of 3. These are the numbers that can be divided into 3 equal groups. The digit sums for the other two numbers are not multiples of 3. In fact, a number is a multiple of 3 exactly when its digit sum is also a multiple of 3.

Problem of the Week

Problem B

Boing! Boing! Boing!

A Superbball is a special kind of ball that will always bounce to half of the height from which it fell. Supposing that it is dropped from a building that is 128 m tall, answer the following questions.



(a) How high will it bounce after it hits the ground for the third time?

(b) How many times must the ball hit the ground so that the next bounce has a height of 2 m?

(c) How many times must the ball hit the ground so that the next bounce has a height of 25 cm?

(d) How tall would a building have to be if, after hitting the ground ten times, the ball bounces to 1 m? Is there a building this tall?

EXTENSION: Test different balls to see how high they bounce when dropped from 1 m.

THEMES ALGEBRA, NUMBER SENSE

Problem of the Week

Problem B and Solution

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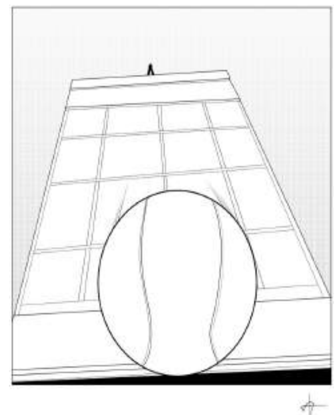

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Extension: Test different balls to see how high they bounce when dropped from 1 m.

Solution

(a) The building is 128 m high, so after the first bounce the ball will reach a height of $128 \div 2 = 64$ m. After the second bounce, the ball will reach a height of $64 \div 2 = 32$ m, and after the third bounce the ball will reach a height of $32 \div 2 = 16$ m. Thus, the ball will bounce to a height of 16 m after it hits the ground for the third time.

(b) Continuing the pattern from part (a), after the fourth bounce the ball will reach a height of 8 m, after the fifth bounce the ball will reach a height of 4 m, and after the sixth bounce the ball will reach a height of 2 m. So the ball must hit the ground six times to bounce to a height of 2 m.

(c) It's helpful to switch to centimetres at this point. From part (b), we know that after the sixth bounce, the ball will reach a height of 2 m or 200 cm. After the seventh, eighth, and ninth bounces, the ball will reach heights of 100 cm, 50 cm, and 25 cm, respectively. So the ball must hit the ground nine times to bounce to a height of 25 cm.

(d) To discover how tall the building would need to be so that after the tenth bounce the ball reaches a height of 1 m, we work backwards and double the height ten times. Doubling 1 m ten times gives the sequence 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 m. Thus, a building would have to be 1024 m tall in order for the ball to bounce to a height of 1 m after the tenth time it hits the ground. The tallest building in the world is currently the Burj Khalifa which is just under 830 m, so there isn't a building as tall as 1024 m.

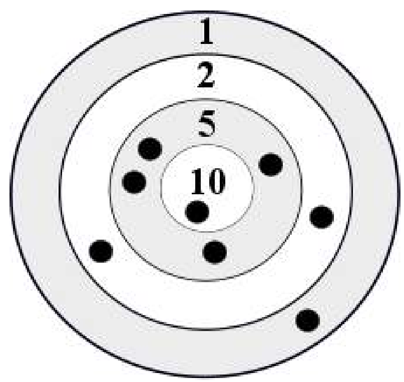
Problem of the Week

Problem B

Who Hit the Middle?

A dart board consists of four regions: an inner circle and three concentric circular bands. Any dart landing in the inner circle will receive 10 points. Any dart landing in the first band will receive 5 points. Any dart landing in the second band will receive 2 points. Any dart landing in the third band will receive 1 point.

Serena and Ebony each threw four darts. The locations where the eight darts landed are shown as black dots on the diagram below.



(a) What was the total number of points scored by the two players?

(b) If Ebony's total score was 1 more than Serena's, what was each person's score?

(c) What individual shots could each player have had to get their scores?

(d) Whose dart landed in the inner circle?

Problem of the Week

Problem B and Solution

Who Hit the Middle?

Problem

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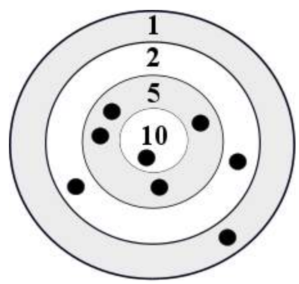

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(d) Whose dart landed in the inner circle?

Solution

(a) Since one dart landed in the band worth 1 point, two darts landed in the band worth 2 points, four darts landed in the band worth 5 points, and one dart landed in the inner circle worth 10 points, the total number of points scored by the two players was

$$1 + 2 + 2 + 5 + 5 + 5 + 5 + 10 = 35$$

(b) Since $35 = 17 + 18$, Ebony scored 18 points and Serena scored 17 points.

(c) Trying all combinations of four shots, we can see that the only way to get a score of 18 is as $1 + 2 + 5 + 10$.

Therefore, Ebony made shots worth 1, 2, 5, and 10 points. This means that Serena made shots worth 2, 5, 5, and 5 points.

(d) Since the inner circle is worth 10 points, then one of Ebony's darts landed in the inner circle.

Problem of the Week
Problem B
Melody's Posts

Melody is building a fence around her beautiful garden. She bought ten wooden posts from Thrifty Buys Used Lumber for the fence. Unfortunately, her bargain posts are all of different lengths.

Using a tape measure, she has found the lengths, in inches, to be:

$$57\tfrac{2}{3},\ 55\tfrac{7}{12},\ 55,\ 56\tfrac{3}{4},\ 57\tfrac{1}{2},\ 55\tfrac{3}{4},\ 56\tfrac{7}{12},\ 57\tfrac{1}{3},\ 56\tfrac{2}{3},\ \text{and}\ 56\tfrac{11}{12}.$$

Now she needs to decide how to build her fence using these posts.

(a) Write the ten lengths of the posts in order from shortest to longest.

(b) Melody decides to adjust the depth of the hole for each post so that all the posts will be the same height above the ground. If she wants all the posts to be 3 feet above the ground, what is the deepest hole she will need to dig? It may be helpful to note that 12 inches equals one foot.

(c) Suppose that Melody's garden is rectangular. The fence posts are needed in every corner and every 10 feet along the fence, as measured from the middle of one fence post to the middle of the next fence post. Draw a diagram for a possible fenced rectangular garden that uses all ten posts. What are the dimensions of your garden, in feet?

Themes Geometry, Number Sense

Problem of the Week

Problem B and Solution

Melody's Posts

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Solution

(a) The easiest way to compare fractions is to write them with a common denominator. All of the fractions in the post lengths can be converted to an equivalent fraction with a denominator of 12 as follows.

$$57\tfrac{2}{3} = 57\tfrac{8}{12} \qquad 56\tfrac{3}{4} = 56\tfrac{9}{12} \qquad 57\tfrac{1}{2} = 57\tfrac{6}{12}$$
$$55\tfrac{3}{4} = 55\tfrac{9}{12} \qquad 57\tfrac{1}{3} = 57\tfrac{4}{12} \qquad 56\tfrac{2}{3} = 56\tfrac{8}{12}$$

The lengths of Melody's posts are then:

$$57\tfrac{8}{12},\ 55\tfrac{7}{12},\ 55,\ 56\tfrac{9}{12},\ 57\tfrac{6}{12},\ 55\tfrac{9}{12},\ 56\tfrac{7}{12},\ 57\tfrac{4}{12},\ 56\tfrac{8}{12},\ 56\tfrac{11}{12}$$

Writing these lengths in order from shortest to longest gives:

$$55,\ 55\tfrac{7}{12},\ 55\tfrac{9}{12},\ 56\tfrac{7}{12},\ 56\tfrac{8}{12},\ 56\tfrac{9}{12},\ 56\tfrac{11}{12},\ 57\tfrac{4}{12},\ 57\tfrac{6}{12},\ 57\tfrac{8}{12}$$

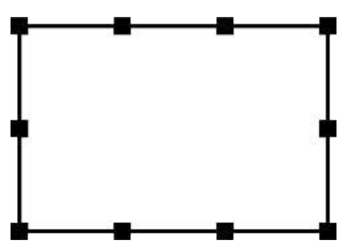
Then we can rewrite the lengths in order from shortest to longest with their original denominators.

$$55,\ 55\tfrac{7}{12},\ 55\tfrac{3}{4},\ 56\tfrac{7}{12},\ 56\tfrac{2}{3},\ 56\tfrac{3}{4},\ 56\tfrac{11}{12},\ 57\tfrac{1}{3},\ 57\tfrac{1}{2},\ 57\tfrac{2}{3}$$

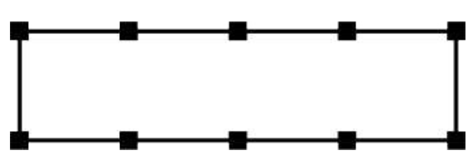
(b) The longest post will require the deepest hole. From part (a) we know the length of the longest post is $57\frac{2}{3}$ inches. Since 3 feet equals $3 \times 12 = 36$ inches, the depth of this hole will be $57\frac{2}{3} - 36 = 21\frac{2}{3}$ inches.

(c) There are two possibilities for a fenced rectangular garden that uses all ten posts. We need to place one post in each of the four corners of the rectangle, which leaves six posts to place on the sides. The dimensions are calculated by adding up the 10 foot spaces between each pair of fence posts.

The first option is a garden with dimensions 30 feet by 20 feet, as shown.



The second option is a garden with dimensions 40 feet by 10 feet, as shown.



Geometry & Measurement (G)

TAKE ME TO THE COVER

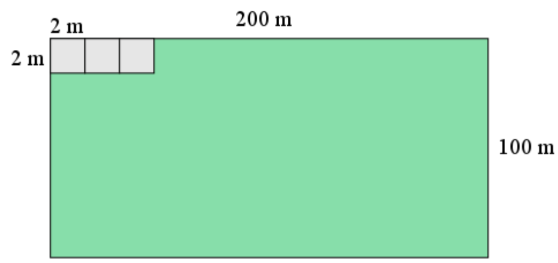
Problem of the Week

Problem B

Work it Out

A gym is hosting an outdoor group exercise class. For many of the exercises, participants will need to make sure they are spaced well apart.

(a) A large grassy field has dimensions of 100 m by 200 m. The field was divided into squares that were each 2 m by 2 m, as shown.



If one person was in the middle of each square, how many people could be on the field?

(b) Imaginary Park is exactly 1 km by 1 km, or 1 km^2, which is equivalent to 100 hectares (ha) in size. If this park was divided into 2 m by 2 m squares for an exercise class like in part (a), and there is one person in the middle of each square, how many people would be in this park? How many people per hectare is that?

(c) Stanley Park is located in Vancouver, BC. While not a rectangle, it covers an area of 405 hectares. Suppose that $\frac{1}{5}$ of the park is not forested. If the number of people per hectare in the non-forested area of Stanley Park is the same as the number of people per hectare in Imaginary Park in part (b), how many people could do the exercise class in the non-forested area of Stanley Park?

Themes Geometry, Number Sense

Problem of the Week
Problem B and Solution
Work it Out

Problem

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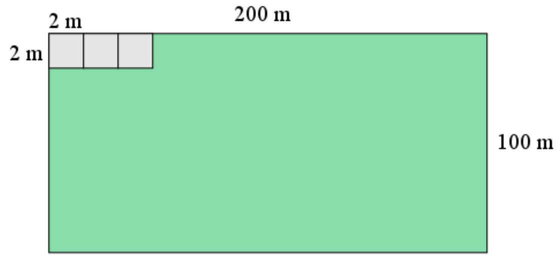

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Solution

(a) We need to figure out the number of 2 m by 2 m squares in the field. Since there are $200 \div 2 = 100$ squares along the long side of the park, and $100 \div 2 = 50$ squares along the short side, there are $100 \times 50 = 5000$ squares in total. That means the field could accommodate 5000 people.

(b) Since Imaginary Park is 1 km by 1 km (or 1000 m by 1000 m), there could be $1000 \div 2 = 500$ people in each row. Since there are $1000 \div 2 = 500$ such rows, there could be $500 \times 500 = 250\,000$ people in 100 ha of space. This works out to $250\,000 \div 100 = 2500$ people per ha.

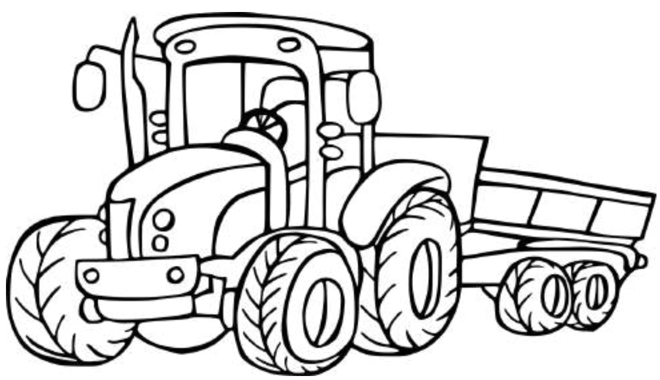
(c) The non-forested area of Stanley Park is $\frac{1}{5}$ of 405 ha, or $\frac{1}{5} \times 405 = 81$ ha. This area will accommodate 2500 people per ha. This means a total of $2500 \times 81 = 202\,500$ people could do the exercise class in the non-forested area of Stanley Park at one time.

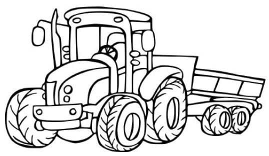
Problem of the Week
Problem B
Farmer Holly's Hay Wagon

Farmer Holly has a wagon with a wood deck and four tires. When empty, the wagon has a total mass of 770 kg. She uses her wagon to transport hay bales in the shape of rectangular prisms. Each hay bale is 1.4 m long by 1.5 m wide by 2 m high, and has a mass of 300 kg.

(a) Each of the four tires on the wagon has a maximum load of 1100 kg. That is, with each tire an additional 1100 kg can be added to the load on the wagon. How many hay bales could Farmer Holly put on her wagon without risking a tire blowout? Don't forget that the tires are also carrying the load of the wagon itself!

(b) Her wagon deck is 5.6 m long and 3 m wide. Farmer Holly wants to tightly pack the hay bales on the wagon with the 1.4 m by 1.5 m side face down on the wagon. How many hay bales can she fit on her wagon if she does not stack them?





Problem of the Week
Problem B and Solution
Farmer Holly's Hay Wagon

Problem

Farmer Holly has a wagon with a wood deck and four tires. When empty, the wagon has a total mass of 770 kg. She uses her wagon to transport hay bales in the shape of rectangular prisms. Each hay bale is 1.4 m long by 1.5 m wide by 2 m high, and has a mass of 300 kg.

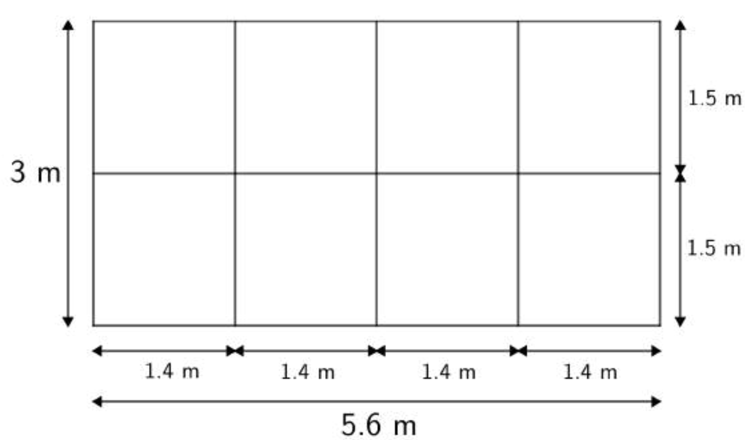
(a) Each of the four tires on the wagon has a maximum load of 1100 kg. That is, with each tire an additional 1100 kg can be added to the load on the wagon. How many hay bales could Farmer Holly put on her wagon without risking a tire blowout? Don't forget that the tires are also carrying the load of the wagon itself!

(b) Her wagon deck is 5.6 m long and 3 m wide. Farmer Holly wants to tightly pack the hay bales on the wagon with the 1.4 m by 1.5 m side face down on the wagon. How many hay bales can she fit on her wagon if she does not stack them?

Solution

(a) The total mass on the tires is equal to the mass of the hay plus the mass of the wagon. Since the wagon has four tires, the total mass the wagon can support is $4 \times 1100 = 4400$ kg. Subtracting the mass of the wagon, the total mass of hay the tires can support is $4400 - 770 = 3630$ kg. If each hay bale weighs 300 kg, then the number of hay bales that could be supported is $3630 \div 300 = 12.1$ hay bales. Since there must be a whole number of hay bales, she could put 12 hay bales on her wagon without going over the maximum load.

(b) Since $5.6 = 4 \times 1.4$, and $3 = 2 \times 1.5$, Farmer Holly could tightly pack $4 \times 2 = 8$ hay bales on her wagon in one layer.



Problem of the Week
Problem B
Are We There Yet?

Planets travel around the sun in elliptical orbits (ovals). Mercury is the planet closest to the sun. The distance between Earth and Mercury ranges from 77 000 000 km at its closest distance to 222 000 000 km at its farthest distance.

Because distances are so great in the solar system, scientists measure them in **Astronomical Units**, or **AU**. One AU is equal to the average distance between the Earth and the Sun, or about 149 600 000 km.

Complete the missing information in the table below.

Planet	Distance in AU from Earth	Distance in km from Earth	Travel Time
Mars	0.52		
Venus			61 days
Saturn		1 275 000 000	
Neptune	29.09		

To calculate the travel time, assume you are travelling from Earth to the planet in a rocket at a speed of 28 000 km per hour throughout your flight. Pick the most reasonable unit of measure for time (for example, 15 000 hours doesn't mean much, but when divided by 24 to get 625 days, you know that it's almost 2 years).

Themes Geometry, Number Sense

Problem of the Week

Problem B and Solution

Are We There Yet?

Problem

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Complete the missing information in the table.

To calculate the travel time, assume you are travelling from Earth to the planet in a rocket at a speed of 28 000 km per hour throughout your flight. Pick the most reasonable unit of measure for time (for example, 15 000 hours doesn't mean much, but when divided by 24 to get 625 days, you know that it's almost 2 years).

Solution

The completed table is shown below.

Planet	Distance in AU from Earth	Distance in km from Earth	Travel Time
Mars	0.52	77 792 000	2 778 hr = 116 days
Venus	0.27	40 992 000	61 days
Saturn	8.52	1 275 000 000	45 536 hr = 1897 days = 5+ years
Neptune	29.09	4 351 864 000	155 424 hr = 6476 days = 17+ years

Since 1 AU = 149 600 000 km, to convert from the distance in AU to the distance in km (for Mars and Neptune), we multiply the distance in AU by 149 600 000. Similarly, to convert from the distance in km to the distance in AU (for Saturn), we divide the distance in km by 149 600 000. This allows us to fill in both distance columns for Mars, Saturn, and Neptune.

To calculate the travel time, we use the speed of the rocket, which is 28 000 km per hour. If we divide the distance in km by 28 000 km per hour, we will get the number of hours it takes to travel that distance, which is the travel time. We can then convert this to a more appropriate unit as we see fit.

To calculate the distance in km from the travel time (for Venus), note that 61 days is equal to $61 \times 24 = 1464$ hours. Thus, travelling at 28 000 km per hour, the distance covered would be $28\,000 \times 1464 = 40\,992\,000$ km. We can then convert the distance in km to the distance in AU as we did for Saturn.

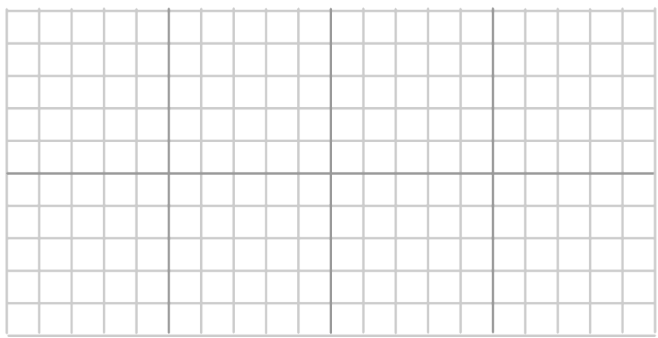
Problem of the Week

Problem B

Screen Size, Now and Then

Flat screen TVs usually have a screen ratio of $16:9$. This means that if the screen is 16 units wide, then it will be 9 units high. If the screen is 32 units wide, then since $32 = 16 \times 2$, it will be $9 \times 2 = 18$ units high, and so on.

(a) Starting in the bottom-left corner of a grid that is 20 units wide and 10 units high, use a ruler to draw a flat screen TV screen that is 16 units wide and 9 units high.

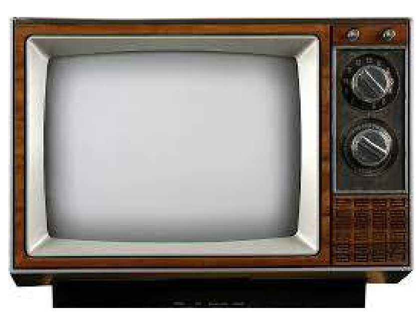


(b) Older TVs had a screen ratio of $4:3$. If an older TV was 9 units high, how many units wide would it be?

(c) Draw the TV screen from part (b) on the same grid used in part (a), also starting in the bottom-left corner.

(d) How many more square units of area does the flat screen TV screen have compared to the older TV screen, if they both have a height of 9 units?

(e) A 4K flat screen TV has 3840×2160 pixels. If the screen is 122 cm wide by 69 cm high, how many pixels per cm^2 are there? Round to the nearest whole number.



Themes Geometry, Number Sense

Problem of the Week
Problem B and Solution
Screen Size, Now and Then

Problem

Flat screen TVs usually have a screen ratio of $16:9$. This means that if the screen is 16 units wide, then it will be 9 units high. If the screen is 32 units wide, then since $32 = 16 \times 2$, it will be $9 \times 2 = 18$ units high, and so on.

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(c) Draw the TV screen from part (b) on the same grid used in part (a), also starting in the bottom-left corner.

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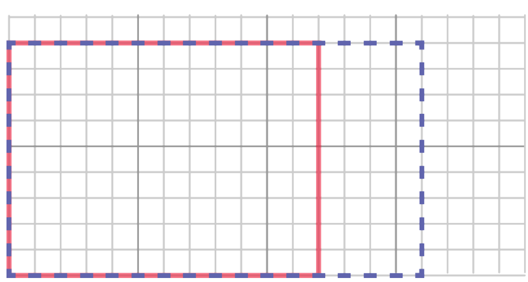
(e) A 4K flat screen TV has 3840×2160 pixels. If the screen is 122 cm wide by 69 cm high, how many pixels per cm^2 are there? Round to the nearest whole number.

Solution

(a) The drawing of the flat screen TV screen on the grid is shown in part (c).

(b) The screen ratio of an older TV is $4:3$, so if the the height is 9 units, that means we have multiplied the 3 in our screen ratio by 3 to get 9. So the width would be $4 \times 3 = 12$ units.

(c) The grid below shows the flat screen TV with a dashed blue line and the older TV with a solid red line.



(d) We can count the squares on our grid that are part of the flat screen TV but not the older TV. We notice that the flat screen TV has 4 more squares of width, and since the height is 9 units for both TVs, there are $4 \times 9 = 36$ more square units of area in the flat screen TV.

(e) There are $3840 \times 2160 = 8\,294\,400$ pixels in total, and the area of the TV is $122 \times 69 = 8418$ cm^2. Thus there are $8\,294\,400 \div 8418 = 985$ pixels per cm^2.

Problem of the Week
Problem B
Orange You Glad?

Betsy is shopping for orange juice. She has discovered that it comes in a variety of containers at different prices.

- At one store, a 2.63 L container of orange juice costs \$4.00, and a pack of eight 200 mL orange juice boxes costs \$2.64.
- At another store, 2 L of orange juice costs \$3.59.
- At both stores, concentrated orange juice in a 295 mL can costs \$1.71. (This must be mixed with three cans of water to obtain $4 \times 295 = 1180$ mL of drinkable juice.)

Which purchase will give Betsy the best value for her money?

You may find calculating the price per 100 mL for each container size in the following table useful.

Amount of Orange Juice	**Price**	**Price per 100 mL**
2.63 L	\$4.00	
$8 \times 200 = 1600$ mL	\$2.64	
2 L	\$3.59	
1180 mL (mixed from concentrate)	\$1.71	

Problem of the Week

Problem B and Solution

Orange You Glad?

Problem

Betsy is shopping for orange juice. She has discovered that it comes in a variety of containers at different prices.

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Which purchase will give Betsy the best value for her money?

Solution

The 2.63 L container of orange juice costs $\$4.00 \div 2.63 \approx \$1.521$ per litre. Since 100 mL is $\frac{1}{10}$ of a litre, the cost is approximately $\$1.521 \div 10 = \$0.1521$ or 15.2¢ per 100 mL.

The 8-pack costs \$2.64 for 1600 mL, or $\$2.64 \div 1600 = \$0.00165$ per mL.
This is equal to $\$0.00165 \times 100 = \$0.165$ or 16.5¢ per 100 mL.

The 2 L container costs $\$3.59 \div 2 = \$1.795$ per litre. Since 100 mL is $\frac{1}{10}$ of a litre, the cost is $\$1.795 \div 10 = \$0.1795$ or about 18¢ per 100 mL.

The frozen concentrate costs $\$1.71 \div 1180 \approx \$0.00145$ per mL.
Therefore, the cost is approximately $\$0.00145 \times 100 = \$0.145$ or 14.5¢ per 100 mL.

The cost per 100 mL for each item is summarized in the completed table below.

Amount of Orange Juice	**Price**	**Price per 100 mL**
2.63 L	\$4.00	15.2¢
$8 \times 200 = 1600$ mL	\$2.64	16.5¢
2 L	\$3.59	18¢
1180 mL (mixed from concentrate)	\$1.71	14.5¢

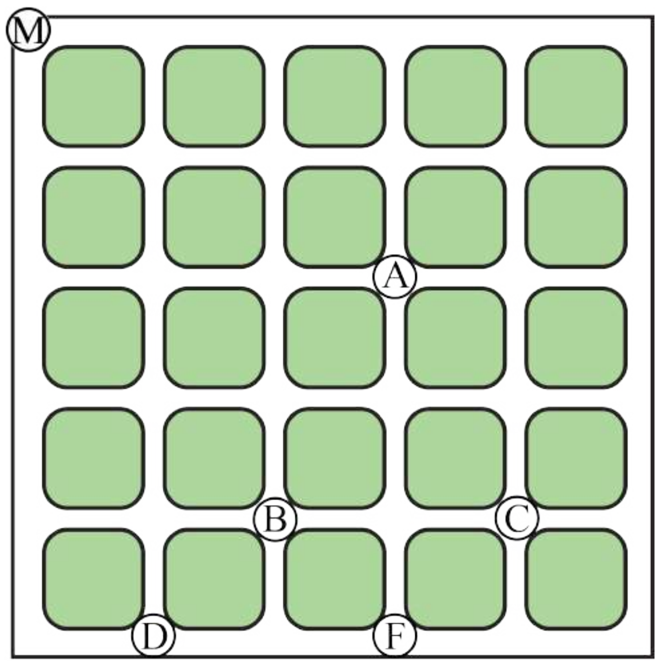
Since the concentrated orange juice has the lowest price of 14.5¢ per 100 mL, the best value for her money is the concentrated orange juice.

Problem of the Week

Problem B

For Your Amusement

The map of an amusement park looks like a grid with six horizontal paths and six vertical paths. The main gate and five rides are marked with letters, as shown.



M: Main Gate
A: Airplanes
B: Bumper Cars
C: Carousel
D: Drop Time
F: Ferris Wheel

All visitors must walk on the paths. It takes 1 minute for Anton to walk along a path from one intersection to the next, and 5 minutes to go on any ride.

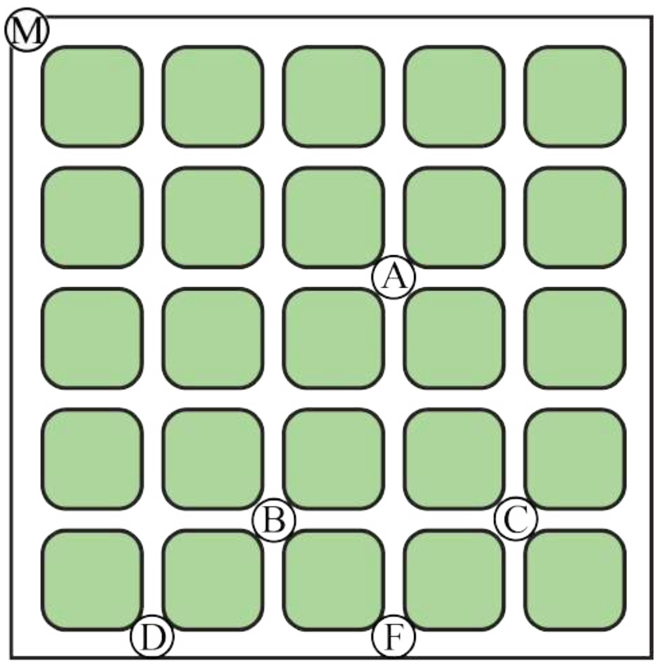
(a) Anton arrives at the main gate and wants to go on two rides before returning to the main gate for lunch in 25 minutes. Which two rides could he choose?

(b) Starting at the main gate, Anton wants to go on the Ferris Wheel, the Airplanes, and the Bumper Cars, and then back to the main gate to meet a friend. In which order should Anton go on the three rides if he wants to be back at the main gate as quickly as possible?

Problem of the Week
Problem B and Solution
For Your Amusement

Problem

The map of an amusement park looks like a grid with six horizontal paths and six vertical paths. The main gate and five rides are marked with letters, as shown.



M: Main Gate
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All visitors must walk on the paths. It takes 1 minute for Anton to walk along a path from one intersection to the next, and 5 minutes to go on any ride.

(a) Anton arrives at the main gate and wants to go on two rides before returning to the main gate for lunch in 25 minutes. Which two rides could he choose?

(b) Starting at the main gate, Anton wants to go on the Ferris Wheel, the Airplanes, and the Bumper Cars, and then back to the main gate to meet a friend. In which order should Anton go on the three rides if he wants to be back at the main gate as quickly as possible?

Solution

(a) Anton wants to go on two rides, and each ride takes 5 minutes, so he can walk for at most $25 - 5 - 5 = 15$ minutes.

Notice that it takes 8 minutes to walk from the main gate to the Carousel, and so takes 16 minutes to walk there and back. Thus, Anton cannot go on the Carousel. Similarly, it takes 8 minutes to walk from the main gate to the Ferris Wheel, so Anton cannot go on the Ferris Wheel.

That leaves the Airplanes, Bumper Cars, and Drop Time.

Can Anton go on the Airplanes and the Bumper Cars? It takes 5 minutes to walk between the main gate and the Airplanes, 3 minutes to walk between the Airplanes and the Bumper Cars, and 6 minutes to walk between the main gate and the Bumper Cars. Thus, this would take a total of $5 + 3 + 6 = 14$ minutes of walking. So one possibility is that Anton goes on the Airplanes and Bumper Cars.

Can Anton go on the Airplanes and Drop Time? It takes 5 minutes to walk between the main gate and the Airplanes, 5 minutes to walk between the Airplanes and Drop Time, and 6 minutes to walk between the main gate and Drop Time. Thus, this would take a total of $5 + 5 + 6 = 16$ minutes of walking. So it is not possible for Anton to go on the Airplanes and Drop Time.

Can Anton go on the Bumper Cars and the Drop Time? It takes 6 minutes to walk between the main gate and the Bumper Cars, 2 minutes to walk between the Bumper Cars and Drop Time, and 6 minutes to walk between the main gate and Drop Time. Thus, this would take a total of $6 + 2 + 6 = 14$ minutes of walking. So another possibility is that Anton goes on the Bumper Cars and Drop Time.

We have looked at all possibilities. Therefore, in 25 minutes, Anton could go on the Airplanes and Bumper Cars, or go on the Bumper Cars and Drop Time.

(b) There are six possible orderings of the three rides that Anton goes on.

- Suppose Anton goes from the main gate to the Ferris Wheel, then the Airplanes, then the Bumper Cars, then back to the main gate. This will take a total of $8 + 3 + 3 + 6 = 20$ minutes of walking.
- Suppose Anton goes from the main gate to the Ferris Wheel, then the Bumper Cars, then the Airplanes, then back to the main gate. This will take a total of $8 + 2 + 3 + 5 = 18$ minutes of walking.
- Suppose Anton goes from the main gate to the Airplanes, then the Ferris Wheel, then the Bumper Cars, then back to the main gate. This will take a total of $5 + 3 + 2 + 6 = 16$ minutes of walking.
- Suppose Anton goes from the main gate to the Airplanes, then the Bumper Cars, then the Ferris Wheel, then back to the main gate. This will take a total of $5 + 3 + 2 + 8 = 18$ minutes of walking.
- Suppose Anton goes from the main gate to the Bumper Cars, then the Ferris Wheel, then the Airplanes, then back to the main gate. This will take a total of $6 + 2 + 3 + 5 = 16$ minutes of walking.
- Suppose Anton goes from the main gate to the Bumper Cars, then the Airplanes, then the Ferris Wheel, then back to the main gate. This will take a total of $6 + 3 + 3 + 8 = 20$ minutes of walking.

So, it follows that Anton should go to the Airplanes, then the Ferris Wheel, then the Bumper Cars (or the reverse order) in order to get back to the main gate as quickly as possible.

Problem of the Week
Problem B
Melody's Posts

Melody is building a fence around her beautiful garden. She bought ten wooden posts from Thrifty Buys Used Lumber for the fence. Unfortunately, her bargain posts are all of different lengths.

Using a tape measure, she has found the lengths, in inches, to be:

$$57\tfrac{2}{3},\ 55\tfrac{7}{12},\ 55,\ 56\tfrac{3}{4},\ 57\tfrac{1}{2},\ 55\tfrac{3}{4},\ 56\tfrac{7}{12},\ 57\tfrac{1}{3},\ 56\tfrac{2}{3},\ \text{and } 56\tfrac{11}{12}.$$

Now she needs to decide how to build her fence using these posts.

(a) Write the ten lengths of the posts in order from shortest to longest.

(b) Melody decides to adjust the depth of the hole for each post so that all the posts will be the same height above the ground. If she wants all the posts to be 3 feet above the ground, what is the deepest hole she will need to dig? It may be helpful to note that 12 inches equals one foot.

(c) Suppose that Melody's garden is rectangular. The fence posts are needed in every corner and every 10 feet along the fence, as measured from the middle of one fence post to the middle of the next fence post. Draw a diagram for a possible fenced rectangular garden that uses all ten posts. What are the dimensions of your garden, in feet?

Themes Geometry, Number Sense

Problem of the Week
Problem B and Solution
Melody's Posts

Problem

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Now she needs to decide how to build her fence using these posts.

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(c) Suppose that Melody's garden is rectangular. The fence posts are needed in every corner and every 10 feet along the fence, as measured from the middle of one fence post to the middle of the next fence post. Draw a diagram for a possible fenced rectangular garden that uses all ten posts. What are the dimensions of your garden, in feet?

Solution

(a) The easiest way to compare fractions is to write them with a common denominator. All of the fractions in the post lengths can be converted to an equivalent fraction with a denominator of 12 as follows.

$$57\tfrac{2}{3} = 57\tfrac{8}{12} \qquad 56\tfrac{3}{4} = 56\tfrac{9}{12} \qquad 57\tfrac{1}{2} = 57\tfrac{6}{12}$$
$$55\tfrac{3}{4} = 55\tfrac{9}{12} \qquad 57\tfrac{1}{3} = 57\tfrac{4}{12} \qquad 56\tfrac{2}{3} = 56\tfrac{8}{12}$$

The lengths of Melody's posts are then:

$$57\tfrac{8}{12},\ 55\tfrac{7}{12},\ 55,\ 56\tfrac{9}{12},\ 57\tfrac{6}{12},\ 55\tfrac{9}{12},\ 56\tfrac{7}{12},\ 57\tfrac{4}{12},\ 56\tfrac{8}{12},\ 56\tfrac{11}{12}$$

Writing these lengths in order from shortest to longest gives:

$$55,\ 55\tfrac{7}{12},\ 55\tfrac{9}{12},\ 56\tfrac{7}{12},\ 56\tfrac{8}{12},\ 56\tfrac{9}{12},\ 56\tfrac{11}{12},\ 57\tfrac{4}{12},\ 57\tfrac{6}{12},\ 57\tfrac{8}{12}$$

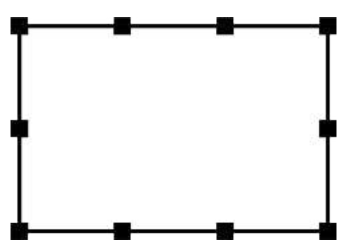
Then we can rewrite the lengths in order from shortest to longest with their original denominators.

$$55,\ 55\tfrac{7}{12},\ 55\tfrac{3}{4},\ 56\tfrac{7}{12},\ 56\tfrac{2}{3},\ 56\tfrac{3}{4},\ 56\tfrac{11}{12},\ 57\tfrac{1}{3},\ 57\tfrac{1}{2},\ 57\tfrac{2}{3}$$

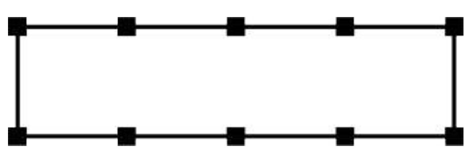
(b) The longest post will require the deepest hole. From part (a) we know the length of the longest post is $57\frac{2}{3}$ inches. Since 3 feet equals $3 \times 12 = 36$ inches, the depth of this hole will be $57\frac{2}{3} - 36 = 21\frac{2}{3}$ inches.

(c) There are two possibilities for a fenced rectangular garden that uses all ten posts. We need to place one post in each of the four corners of the rectangle, which leaves six posts to place on the sides. The dimensions are calculated by adding up the 10 foot spaces between each pair of fence posts.

The first option is a garden with dimensions 30 feet by 20 feet, as shown.



The second option is a garden with dimensions 40 feet by 10 feet, as shown.

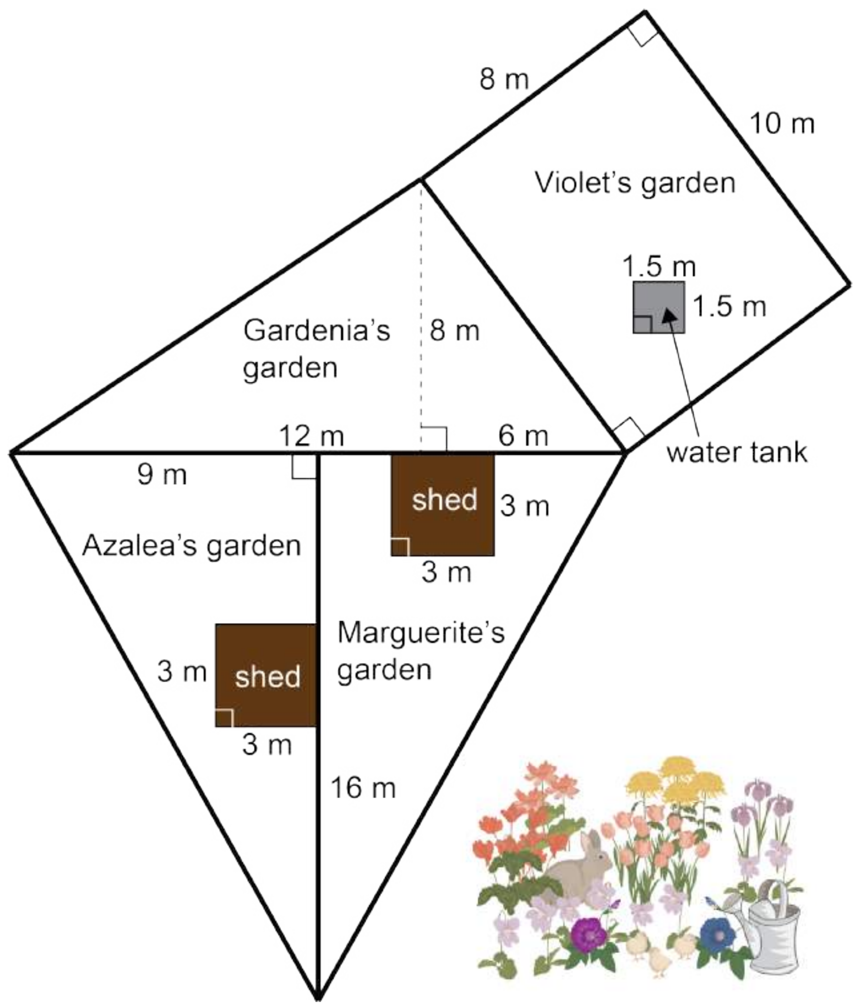


Problem of the Week

Problem B

Competing Gardens

Gardenia, Marguerite, Azalea, and Violet just moved into a new neighbourhood. Each says that she has the largest garden, but it is so hard to tell! The following diagram shows the shape and size of each garden and any other objects on the properties.



(a) Determine the area of each garden, excluding any objects on the property. Who has the largest garden?

(b) What is the difference in area between the largest garden and the smallest garden?

Theme Geometry

Problem of the Week
Problem B and Solution
Competing Gardens

Problem

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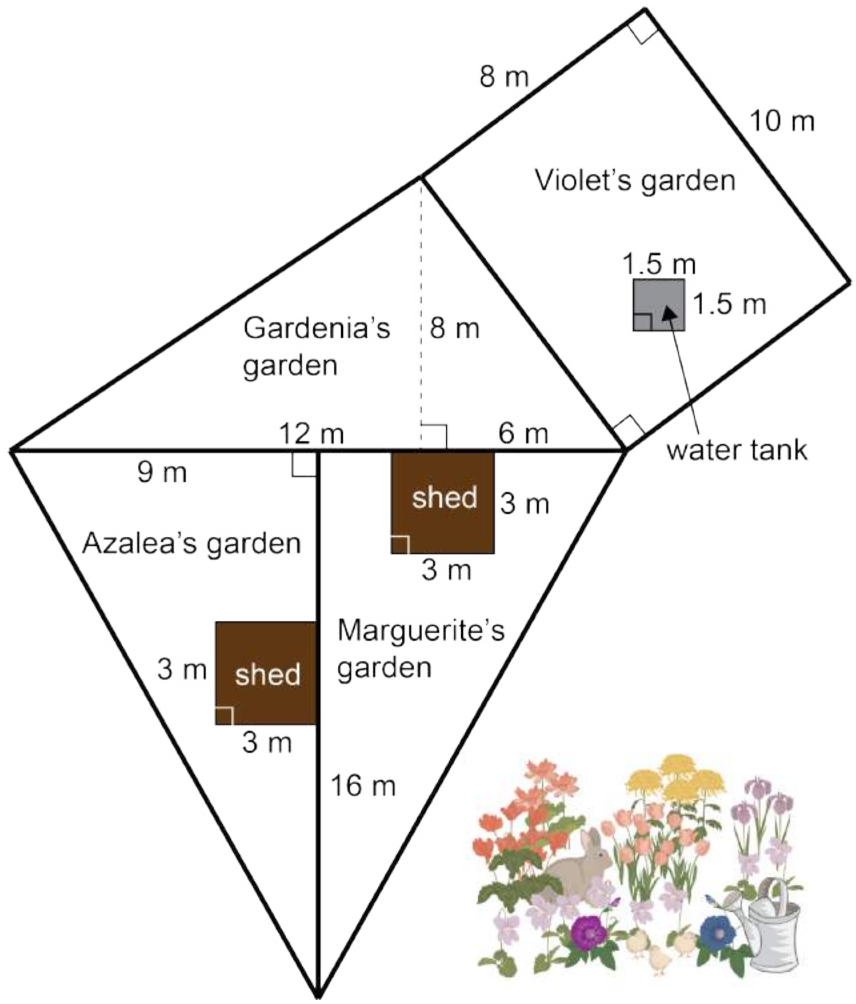


(a) Determine the area of each garden, excluding any objects on the property. Who has the largest garden?

(b) What is the difference in area between the largest garden and the smallest garden?

Solution

(a) Gardenia's garden is in the shape of a triangle with base $12 + 6 = 18$ m and height 8 m. Thus, the area of Gardenia's garden is $\frac{1}{2} \times 18 \times 8 = 72$ m^2.

Azalea's garden is in the shape of a triangle with base 9 m and height 16 m. It has a square shed on the property with side length 3 m. We need to subtract the area of the shed from the area of the garden. The area of the shed is $3 \times 3 = 9$ m^2. Thus, the area of Azalea's garden is $(\frac{1}{2} \times 9 \times 16) - 9 = 72 - 9 = 63$ m^2.

Marguerite's garden is in the shape of a triangle with base $12 + 6 - 9 = 9$ m and height 16 m. It also has a shed on the property with side length 3 m. We notice that Marguerite's garden and shed have the same dimensions as Azalea's garden and shed, so their gardens will have equal areas. Thus, the area of Marguerite's garden is also 63 m^2.

Violet's garden is in the shape of a rectangle with sides 8 m and 10 m. It has a square water tank on the property with side length 1.5 m. We need to subtract the area of the water tank from the area of the garden. The area of the water tank is $1.5 \times 1.5 = 2.25$ m^2. Thus, the area of Violet's garden is $(8 \times 10) - 2.25 = 80 - 2.25 = 77.75$ m^2.

Therefore the largest garden belongs to Violet, with an area of 77.75 m^2.

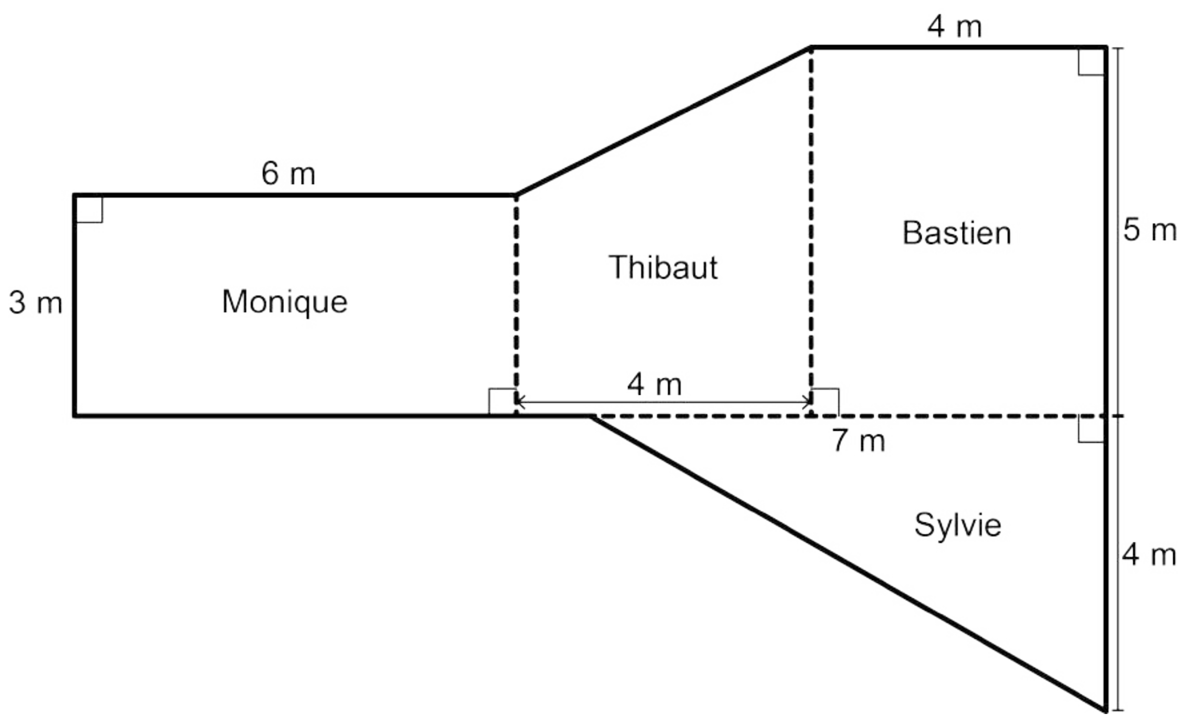
(b) The largest garden has an area of 77.75 m^2. The smallest garden has an area of 63 m^2. Thus, the difference is $77.75 - 63 = 14.75$ m^2.

Problem of the Week

Problem B

Share and Share Alike

Monique, Thibaut, Bastien, and Sylvie are siblings who share an irregularly-shaped room in their home. They have divided up the room so that each person has their own space. Each person's space is either a rectangle, a trapezoid, or a triangle. A floor plan, including some dimensions, is shown in the following diagram.



(a) Calculate the area of each person's space. Which person has the space with the largest area?

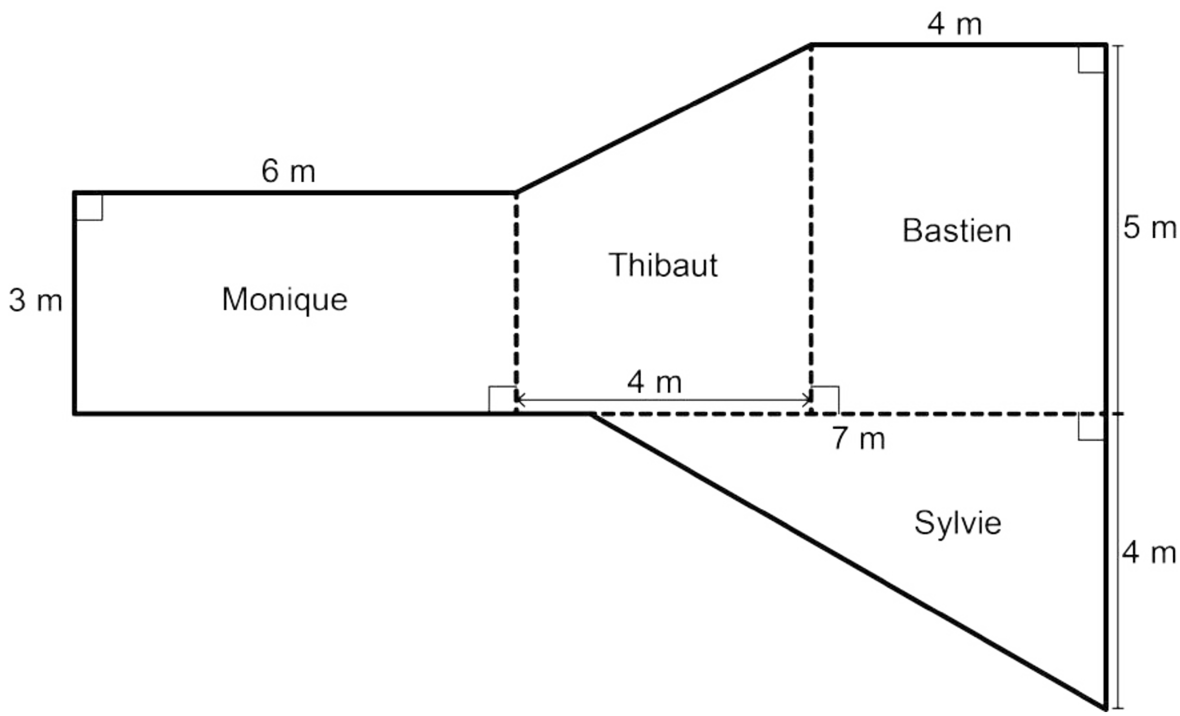
(b) The siblings have decided to divide up the room in a different way so that the area of each person's space is equal. After they do this, what is the area of each person's space?

(c) Redraw the inner dashed lines in the floor plan so that the area of each person's space is equal. Note that the shape of each person's space may no longer be a rectangle, trapezoid, or triangle.

Problem of the Week
Problem B and Solution
Share and Share Alike

Problem

Monique, Thibaut, Bastien, and Sylvie are siblings who share an irregularly-shaped room in their home. They have divided up the room so that each person has their own space. Each person's space is either a rectangle, a trapezoid, or a triangle. A floor plan, including some dimensions, is shown in the following diagram.



(a) Calculate the area of each person's space. Which person has the space with the largest area?

(b) The siblings have decided to divide up the room in a different way so that the area of each person's space is equal. After they do this, what is the area of each person's space?

(c) Redraw the inner dashed lines in the floor plan so that the area of each person's space is equal. Note that the shape of each person's space may no longer be a rectangle, trapezoid, or triangle.

Solution

(a) Monique's space is a rectangle with area $6 \times 3 = 18$ m^2.

Thibaut's space is a trapezoid with area $\frac{1}{2} \times (3 + 5) \times 4 = 16$ m^2.

Bastien's space is a rectangle with area $4 \times 5 = 20$ m^2.

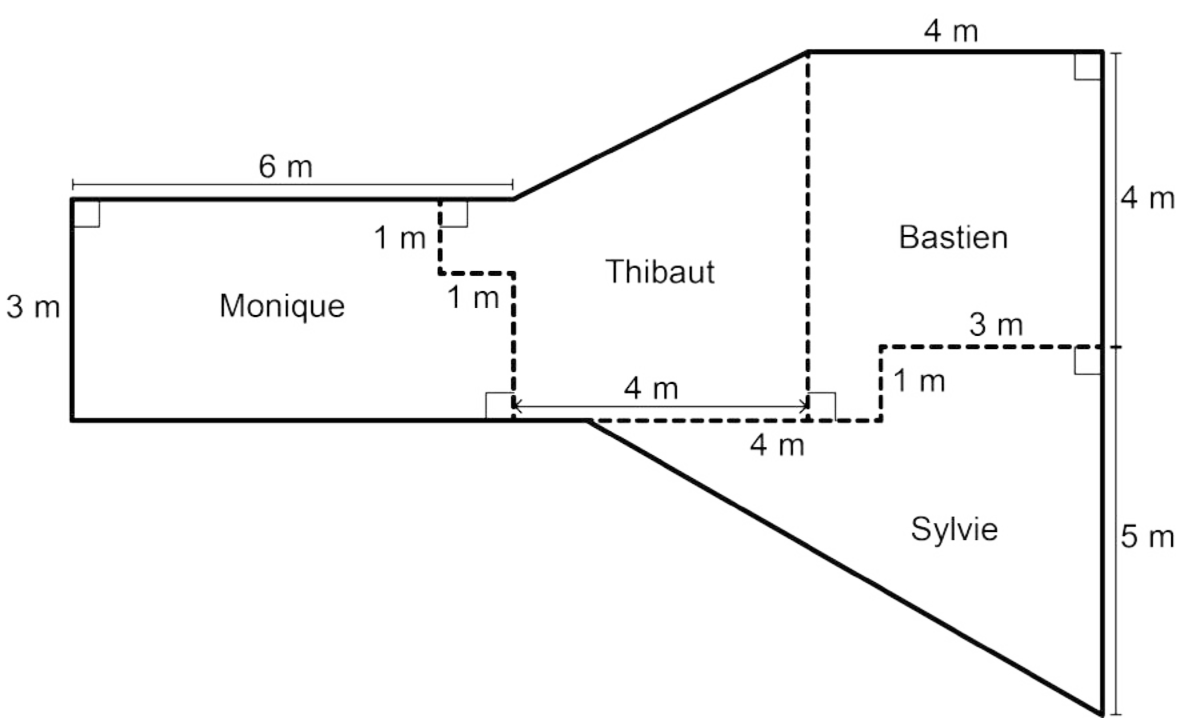
Sylvie's space is a triangle with area $\frac{1}{2} \times 7 \times 4 = 14$ m^2.

Thus, Bastien's space has the largest area.

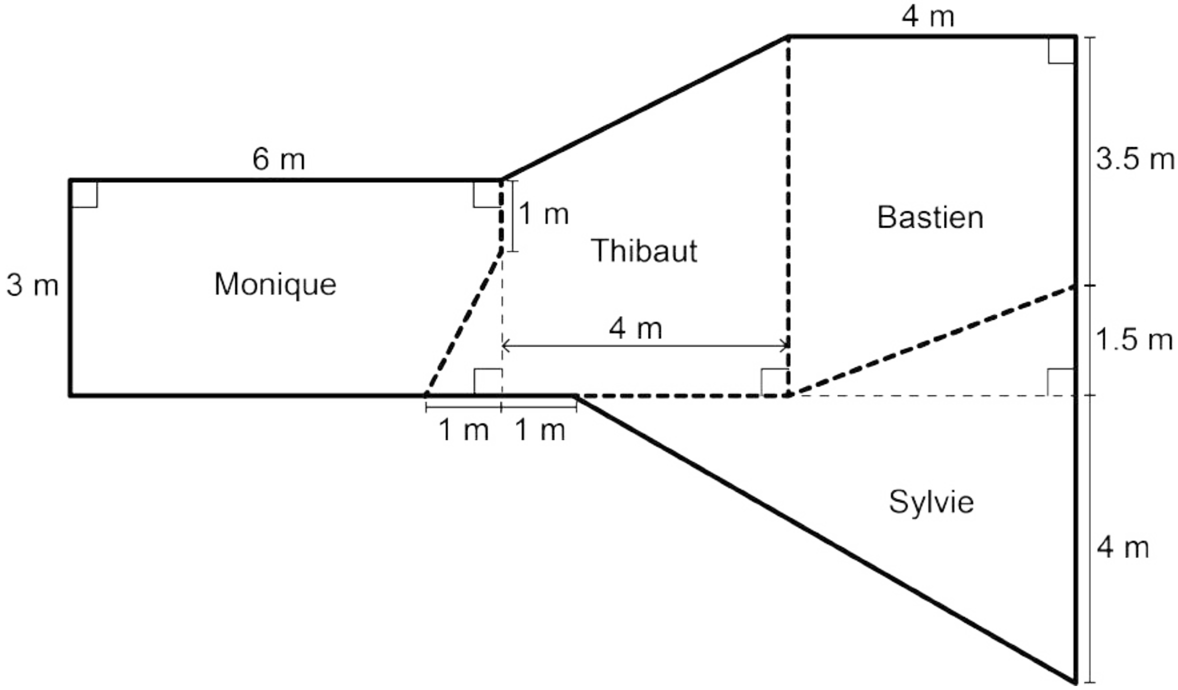
(b) If the area of each person's space is equal, then each person will have $\frac{1}{4}$ of the area of the room. The area of the room is $18 + 16 + 20 + 14 = 68$ m^2. Thus, the area of each person's space will be $\frac{1}{4} \times 68 = 17$ m^2.

(c) There are many ways to redraw the inner lines so that each person's space has an area of 17 m^2. It's easiest if we first think about how each person's space needs to change. Monique's space currently has an area of 18 m^2, so we need to remove an area of 1 m^2. Thibaut's space currently has an area of 16 m^2, so we need to add an area of 1 m^2. Bastien's space currently has an area of 20 m^2, so we need to remove an area of 3 m^2. Sylvie's space currently has an area of 14 m^2, so we need to add an area of 3 m^2.

One way to do this is to take a square 1 m^2 area from Monique's space and give it to Thibaut, and then take a rectangular 3 m^2 area from Bastien's space and give it to Sylvie. To achieve this, we can redraw two of the inner dashed lines as shown in the following floor plan.



Alternatively, we could reassign triangular areas instead of rectangular areas. One way to do this is shown in the following floor plan.

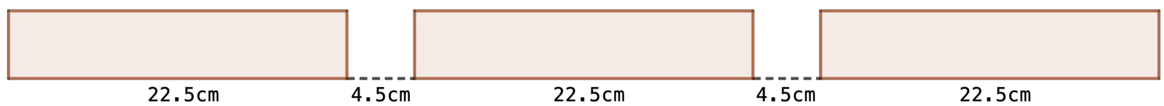


Problem of the Week

Problem B

Wall Construction

Moyo wants to build a wall in her backyard. She will build the wall out of bricks, and her measurements show that each brick is 22.5 cm long and 5.7 cm high. Moyo's bricks on the bottom layer of the wall are to be placed lengthwise and spaced 4.5 cm apart. For example, here is the bottom layer of a wall with three bricks.



(a) If she builds a wall with nine layers of bricks, what will the height of the wall be?

(b) If Moyo builds a wall with six bricks on the bottom layer, how long will this bottom layer be?

(c) If Moyo wanted the wall to be 130.5 cm long, how many bricks will Moyo use in the bottom layer of the wall?

Problem of the Week

Problem B and Solution

Wall Construction

Problem

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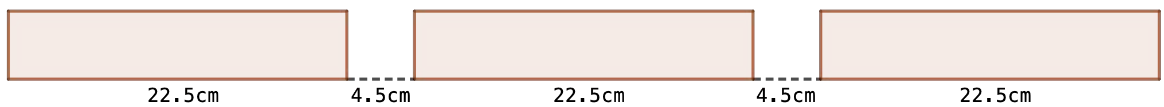


(a) If she builds a wall with nine layers of bricks, what will the height of the wall be?

(b) If Moyo builds a wall with six bricks on the bottom layer, how long will this bottom layer be?

(c) If Moyo wanted the wall to be 130.5 cm long, how many bricks will Moyo use in the bottom layer of the wall?

Solution

(a) The total height of the nine layers will be $9 \times 5.7 = 51.3$ cm.

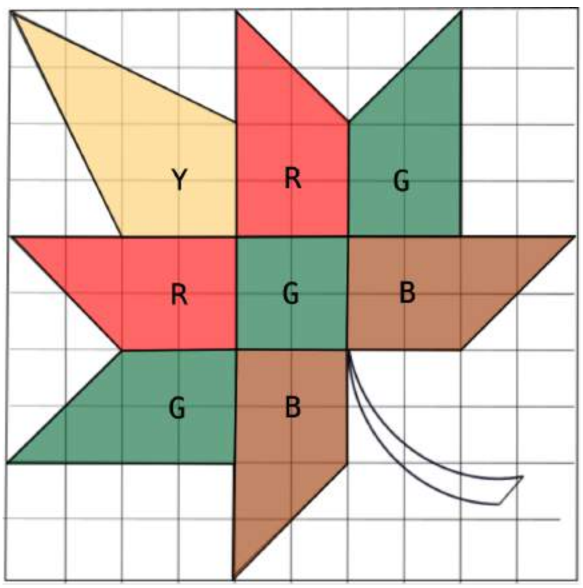
(b) The bottom layer, which consists of six bricks, will have six bricks and five spaces. Thus the total length will be $(6 \times 22.5) + (5 \times 4.5) = 135 + 22.5 = 157.5$ cm or 1.575 m.

(c) One brick plus one space is $22.5 + 4.5 = 27$ cm long. Since $27 \times 5 = 135$, and 135 is close to 130.5, we guess that Moyo would use about five bricks, with four spaces between them. Indeed, this gives $(5 \times 22.5) + (4 \times 4.5) = 112.5 + 18 = 130.5$ cm, the required length.

Note: Another possible way to solve this is to notice that 130.5 cm is 27 cm less than 157.5 cm. Now, 27 cm is the length of one brick plus one space. Therefore, the bottom layer has the 6 bricks in part (b) minus 1 brick. Therefore, Moyo will have 5 bricks on the bottom layer.

Problem of the Week
Problem B
Maple Leaf Quilting

The 10 cm by 10 cm square on the left is the pattern for a single square in the maple leaf quilt on the right. In the pattern, the interior of the maple leaf has been divided into eight shapes, which have each been coloured yellow (Y), red (R), green (G), or brown (B). The rest of the square, including the stem, is white.



(a) Draw dotted lines on the pattern which divide the interior of each coloured shape into pieces that are squares or triangles.

(b) For each of the four colours in the pattern, what is the area of the maple leaf that is covered by that colour?

(c) What is the area of the rest of the pattern, including the stem, that is white? That is, what is the area of the square that is not covered by the leaf?

Problem of the Week
Problem B and Solution
Maple Leaf Quilting

Problem

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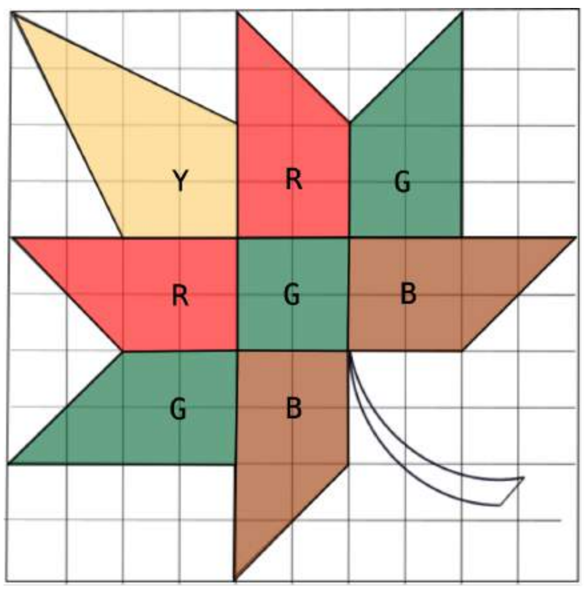

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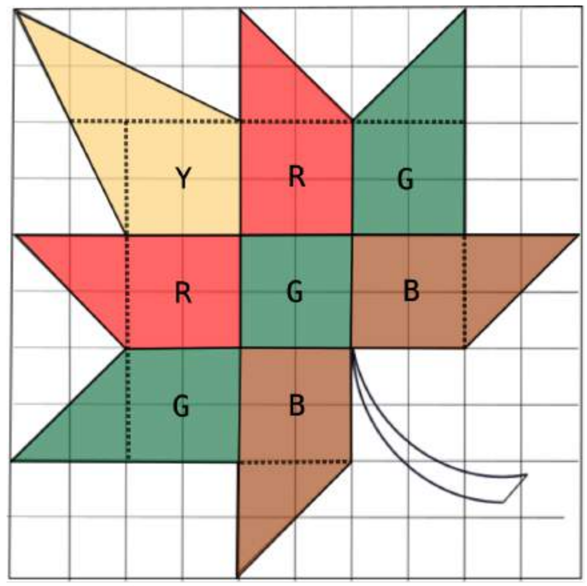
(b) For each of the four colours in the pattern, what is the area of the maple leaf that is covered by that colour?

(c) What is the area of the rest of the pattern, including the stem, that is white? That is, what is the area of the square that is not covered by the leaf?

Solution

(a) One way to divide the interior of each coloured shape into pieces that are squares or triangles has been shown below.



(b) The two red regions each consist of a square with area $2 \times 2 = 4\,\text{cm}^2$ and a triangle with area $2 \times 2 \div 2 = 2\,\text{cm}^2$. Therefore, the area of one red region is $4 + 2 = 6\,\text{cm}^2$, and the total area of the two red regions is $2 \times 6 = 12\,\text{cm}^2$.

The two brown regions also each consist of a square with area $2 \times 2 = 4\,\text{cm}^2$ and a triangle with area $2 \times 2 \div 2 = 2\,\text{cm}^2$. Therefore, the area of one brown region is $4 + 2 = 6\,\text{cm}^2$, and the total area of the two brown regions is $2 \times 6 = 12\,\text{cm}^2$.

The three green regions consist of two shapes that have the same dimensions as the red shapes, and the centre square. Since the area of the centre square is $2 \times 2 = 4\,\text{cm}^2$, the area of the three green regions is $12 + 4 = 16\,\text{cm}^2$.

The yellow region has an area equal to the area of the 4 cm by 4 cm square in the upper left corner minus the areas of the two white triangles in that square. Each of these triangles has a base of 2 cm and a height of 4 cm, and so has an area of $2 \times 4 \div 2 = 4\,\text{cm}^2$. Therefore, the total area of the white triangles is $4 + 4 = 8\,\text{cm}^2$. Thus, the area of the yellow region is $4 \times 4 - 8 = 16 - 8 = 8\,\text{cm}^2$.

(c) The total area of the maple leaf, excluding the stem, is the sum of the areas of the red, brown, green, and yellow coloured regions, or $12 + 12 + 16 + 8 = 48\,\text{cm}^2$.

Thus, the area of square that is not covered by the leaf is equal to the total area of the square minus the total coloured area or $10 \times 10 - 48 = 52\,\text{cm}^2$.

Algebra (A)

TAKE ME TO THE COVER

Problem of the Week
Problem B
Sven's Gym-Cans

Recently, Sven decided to start exercising at home instead of going to the gym. He does not have the nice weights that they have at the gym, but he does have different containers of food that he can use as weights. Sven has containers of soup, beans, breadcrumbs, jam, and peanuts. He also has one can whose label fell off that he calls the Mystery Can.

Sven used his balance scale and came up with the following discoveries:

- Two cans of soup have the same mass as one can of beans.
- Five containers of breadcrumbs have the same mass as one jar of jam.
- One can of beans and two containers of bread crumbs together have the same mass as the container of peanuts.
- The Mystery Can and one can of soup together have the same mass as the container of peanuts.

Sven's friend, Rob, who has perfect estimation skills, determined that the Mystery Can has a mass of 580 grams and the can of beans has a mass of 640 grams. Assuming that Rob is correct, determine the mass of each container, in grams. To do so, you may find it helpful to set up algebraic equations to show the relationships among the cans' masses.

Problem of the Week
Problem B and Solution
Sven's Gym-Cans

Problem

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Solution

First we will write algebraic equations to show the relationships given by Sven's four discoveries.

To avoid a lot of writing, we will use the following variables: S represents the mass, in grams, of one can of soup, P represents the mass, in grams, of one container of peanuts, B represents the mass, in grams, of one can of beans, C represents the mass, in grams, of one container of breadcrumbs, J represents the mass, in grams, of one jar of jam, and M represents the mass, in grams, of the Mystery Can.

Now we can write an equation for each of Sven's four discoveries.

- $2 \times S = B$
- $5 \times C = J$
- $B + 2 \times C = P$
- $M + S = P$

We are given a mass of 640 grams for one can of beans. Thus the first equation tells us that $2 \times S = 640$. Since $2 \times 320 = 640$, we can conclude that $S = 320$.

We are also given that the mass of the Mystery Can is 580 grams. Using the fourth equation we get $M + S = 580 + 320 = 900$. Therefore $P = 900$.

Using the third equation we get $640 + 2 \times C = 900$. Subtracting 640 from both sides of this equation gives us $2 \times C = 260$. Since $2 \times 130 = 260$, it follows that $C = 130$.

Finally, using the second equation we get $5 \times 130 = J$. Thus $J = 650$.

Therefore, the mass of each container is as follows.

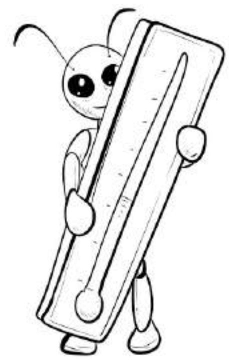
Container	**Mass (grams)**
can of soup	320
container of peanuts	900
container of breadcrumbs	130
jar of jam	650

Problem of the Week

Problem B

'Temp'ting Crickets

Crickets can help determine the temperature, in degrees Celsius. One possible way to make this calculation is to follow the steps below.



Step 1: Count the number of chirps in 25 seconds.

Step 2: Divide the number from Step 1 by 3.

Step 3: Add 4 to the number from Step 2.

(a) By filling in each ____ in the following equation with either a variable or a number, write an equation to show how to get the temperature, t, based on a certain number of chirps, c, in 25 seconds.

$$t = \_\_\_\_ \div \_\_\_\_ + \_\_\_\_$$

(b) Fill in the second column of the following table.

Chirps (c) in 25 seconds	Temperature (t) in degrees Celsius
60	
54	
66	

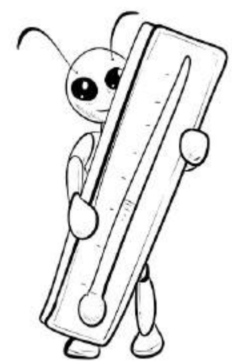
(c) Fill in the first column of the following table.

Chirps (c) in 25 seconds	Temperature (t) in degrees Celsius
	18
	20
	16

THEME ALGEBRA

Problem of the Week
Problem B and Solution
'Temp'ting Crickets

Problem



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54	
66	

(c) Fill in the first column of the following table.

Chirps (c) in 25 seconds	Temperature (t) in degrees Celsius
	18
	20
	16

Solution

(a) To determine the temperature, t, we take the number of chirps in 25 seconds, c, divide by 3, then add 4. That is, $t = \underline{c} \div \underline{3} + \underline{4}$.

(b) You may use the given steps or the equation from part (a) to fill in the table.

For example when there are 60 chirps, we divide by 3 to get 20, and then add 4 to get 24 degrees Celsius.

Or we may use the equation $t = 60 \div 3 + 4 = 20 + 4 = 24$.

Chirps (c) in 25 seconds	Temperature (t) in degrees Celsius
60	24
54	22
66	26

(c) To find the number of chirps for a given temperature, we work backwards, reversing the steps as we go. That is, we subtract 4 from the given temperature, and then multiply by 3.

For example when the temperature is 18 degrees Celsius, we subtract 4 to get 14, and then multiply 14 by 3 to get 42 chirps.

The equation to calculate chirps, c, given temperature, t, is $c = (t - 4) \times 3$.

Chirps (c) in 25 seconds	Temperature (t) in degrees Celsius
42	18
48	20
36	16

Problem of the Week
Problem B
Boing! Boing! Boing!

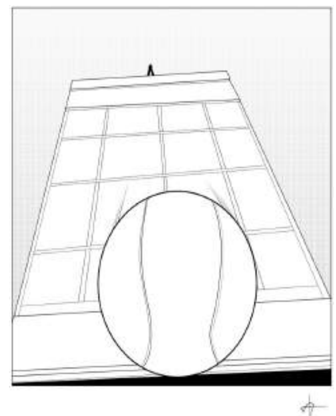
A Superbball is a special kind of ball that will always bounce to half of the height from which it fell. Supposing that it is dropped from a building that is 128 m tall, answer the following questions.

(a) How high will it bounce after it hits the ground for the third time?

(b) How many times must the ball hit the ground so that the next bounce has a height of 2 m?

(c) How many times must the ball hit the ground so that the next bounce has a height of 25 cm?

(d) How tall would a building have to be if, after hitting the ground ten times, the ball bounces to 1 m? Is there a building this tall?



EXTENSION: Test different balls to see how high they bounce when dropped from 1 m.

Problem of the Week

Problem B and Solution

Boing! Boing! Boing!

Problem

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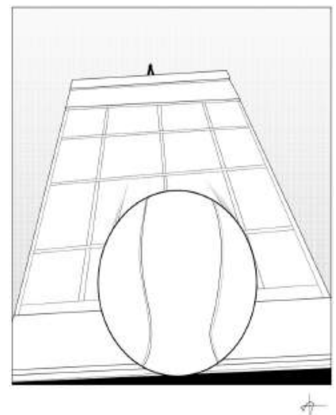

EXTENSION: Test different balls to see how high they bounce when dropped from 1 m.

Solution

(a) The building is 128 m high, so after the first bounce the ball will reach a height of $128 \div 2 = 64$ m. After the second bounce, the ball will reach a height of $64 \div 2 = 32$ m, and after the third bounce the ball will reach a height of $32 \div 2 = 16$ m. Thus, the ball will bounce to a height of 16 m after it hits the ground for the third time.

(b) Continuing the pattern from part (a), after the fourth bounce the ball will reach a height of 8 m, after the fifth bounce the ball will reach a height of 4 m, and after the sixth bounce the ball will reach a height of 2 m. So the ball must hit the ground six times to bounce to a height of 2 m.

(c) It's helpful to switch to centimetres at this point. From part (b), we know that after the sixth bounce, the ball will reach a height of 2 m or 200 cm. After the seventh, eighth, and ninth bounces, the ball will reach heights of 100 cm, 50 cm, and 25 cm, respectively. So the ball must hit the ground nine times to bounce to a height of 25 cm.

(d) To discover how tall the building would need to be so that after the tenth bounce the ball reaches a height of 1 m, we work backwards and double the height ten times. Doubling 1 m ten times gives the sequence 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 m. Thus, a building would have to be 1024 m tall in order for the ball to bounce to a height of 1 m after the tenth time it hits the ground. The tallest building in the world is currently the Burj Khalifa which is just under 830 m, so there isn't a building as tall as 1024 m.

Data Management (D)

TAKE ME TO THE COVER

Problem of the Week

Problem B

Jumping Jacks with Jym

Jym's class has been working on jumping jacks and charting their progress. The mean (average) for the class was determined using each student's information, giving a value of 40 jumping jacks (JJs) per minute.

Over 10 days, Jym measured how many jumping jacks he can do in one session, and how long it took him to do them. The tables below show his results.

Day	Number of JJs	Time taken	JJs per minute
1	32	58 s	
2	32	57 s	
3	34	59 s	
4	35	57 s	
5	40	1 min 2 s	

Day	Number of JJs	Time taken	JJs per minute
6	42	1 min 5 s	
7	40	58 s	
8	45	1 min 2 s	
9	46	1 min	
10	48	1 min	

(a) Complete the tables by finding Jym's JJs per minute for each day, rounding to the nearest decimal.

(b) Calculate the mean of the values you found in (a). How does this mean for Jym compare to the rest of his class?

(c) Which type of graph would be most appropriate to show Jym's improvement over time? Create a suitable graph with appropriate labels.

Theme Data Management

Problem of the Week
Problem B and Solution
Jumping Jacks with Jym

Problem

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(b) Calculate the mean of the values you found in (a). How does this mean for Jym compare to the rest of his class?

(c) Which type of graph would be most appropriate to show Jym's improvement over time? Create a suitable graph with appropriate labels.

Solution

(a) The completed tables are as follows.

Day	Number of JJs	Time taken	JJs per minute
1	32	58 s	33.1
2	32	57 s	33.7
3	34	59 s	34.6
4	35	57 s	36.8
5	40	1 min 2 s	38.7

Day	Number of JJs	Time taken	JJs per minute
6	42	1 min 5 s	38.8
7	40	58 s	41.4
8	45	1 min 2 s	43.5
9	46	1 min	46
10	48	1 min	48

Jym's JJs per minute each day were found by taking his total number of JJs and dividing by the time taken, in seconds, to get the number of JJs per second, and then multiplying this number by 60 to get the number of JJs per minute.

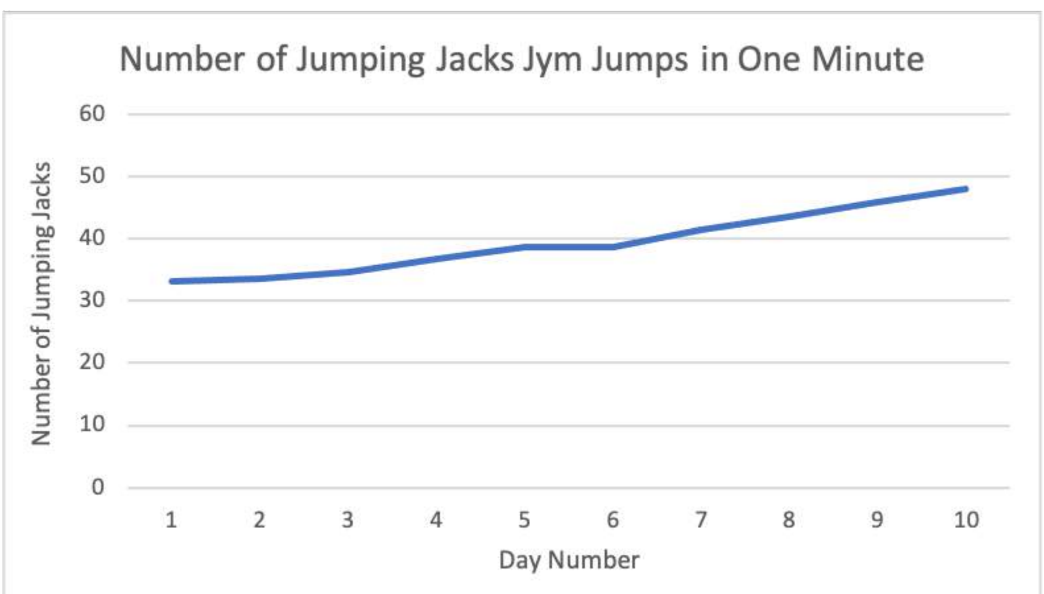
For example, on Day 1 Jym jumped $\frac{32}{58}$ JJs per second, which is equivalent to $\frac{32}{58} \times 60 \approx 33.1$ JJs per minute.

(b) Jym's mean JJs per minute is

$$\frac{33.1 + 33.7 + 34.6 + 36.8 + 38.7 + 38.8 + 41.4 + 43.5 + 46 + 48}{10} = \frac{394.6}{10} = 39.46 \approx 39.5$$

Jym's mean of 39.5 JJs per minute is slightly less than the mean for the class.

(c) The most meaningful graph would be one showing Jym's average JJs per minute versus time, as a broken line graph (or a bar graph). A broken line graph is shown below.



Problem of the Week
Problem B
Trucks, Bikes, and Cars

Mohammed wants to graph a record of the traffic that goes by his house between 4:30 p.m. and 4:40 p.m. He creates a data collection sheet and collects the following data while standing on the sidewalk beside his home during that period.

Time	Vehicle
4:30.12	Car
4:30.43	Car
4:31.24	Truck
4:31.58	Bicycle
4:32.34	Car
4:33.08	Car

Time	Vehicle
4:33.37	Truck
4:34.21	Car
4:34.52	Car
4:35.23	Car
4:36.14	Bicycle
4:36.45	Truck

Time	Vehicle
4:37.29	Car
4:38.36	Bicycle
4:39.16	Truck
4:39.48	Car
4:40.10	Car
4:40.38	Car

(a) Is the data Mohammed collected *primary* or *secondary* data?

(b) What type of graph or plot would be appropriate to display this data?

(c) Create this graph or plot using proper titles and labels.

Problem of the Week
Problem B and Solution
Trucks, Bikes, and Cars

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(a) Is the data Mohammed collected *primary* or *secondary* data?

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(c) Create this graph or plot using proper titles and labels.

Solution

(a) The data Mohammed collected is primary since he collected it. (If he gave the data to city planners to use, then it would be secondary data to them, as they did not collect it themselves.)

(b) The appropriate type of graph or plot depends on its purpose. Some ideas include a stem-and-leaf plot for minutes and seconds which displays when each vehicle passed, or a bar graph or circle graph to contrast the numbers or ratios of cars, trucks, and bicycles.

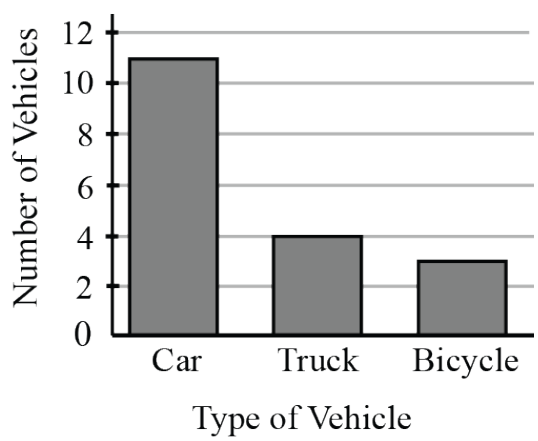
(c) Examples of a stem-and-leaf plot, a bar graph, and a circle graph are shown below.

Traffic In Front of My House

Stem	Leaf
4:30	12 43
4:31	24 58
4:32	34
4:33	08 37
4:34	21 52
4:35	23
4:36	14 45
4:37	29
4:38	36
4:39	16 48
4:40	10 38

Key: 4:40|10 = 4:40.10 p.m.

Traffic In Front of My House



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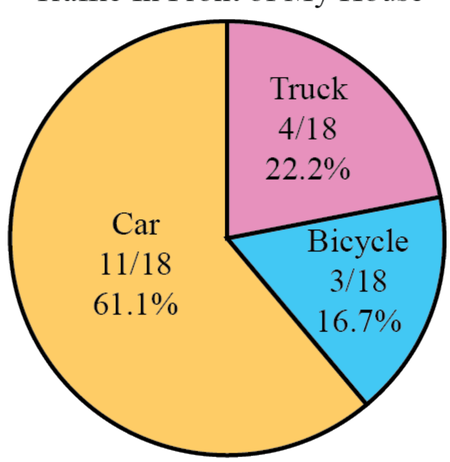


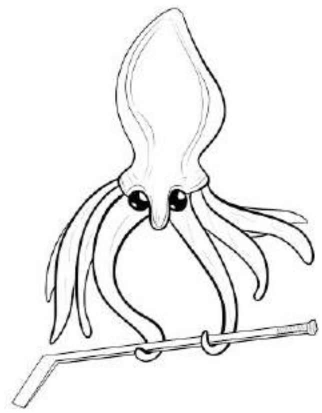
Problem of the Week

Problem B

Get Kraken!

The Seattle Kraken are the newest team to join the National Hockey League. Let's say they determined the colours for their uniforms by picking two choices from the table below. One colour must come from the first column and one colour must come from the second column.

Colour 1	**Colour 2**
Green	White
Blue	Black
Teal	Silver
	Gold



(a) List all the possible colour combinations. How many different options do they have?

(b) What pair of colours would you choose? If the team chooses the colour combination randomly from the colour combination options in part (a), what is the probability that the pair of colours you want gets picked?

(c) What is the probability that the uniform colours do not include teal, black, or gold?

Problem of the Week
Problem B and Solution
Get Kraken!

Problem

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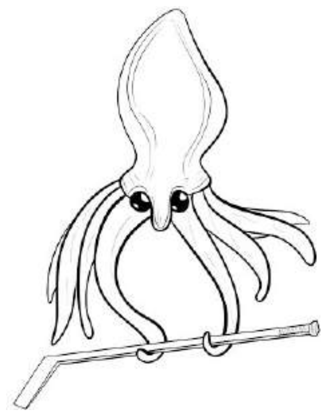

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(c) What is the probability that the uniform colours do not include teal, black, or gold?

Solution

(a) Each of the three colours from the Colour 1 column can be combined with each of the four colours from the Colour 2 column, giving the following possible combinations:

- Green and White, Green and Black, Green and Silver, and Green and Gold.
- Blue and White, Blue and Black, Blue and Silver, and Blue and Gold.
- Teal and White, Teal and Black, Teal and Silver, and Teal and Gold.

Thus, there are $3 \times 4 = 12$ possible combinations of colours.

(b) Answers will vary due to individual choices. If colours are picked randomly, the probability that the pair of colours you want gets picked is 1 in 12 or $\frac{1}{12}$.

(c) The probability that the uniform colours do not contain teal, black, or gold is determined by first noting that there are only 4 such combinations: Green and White, Green and Silver, Blue and White, and Blue and Silver.

Thus, there are 4 possible combinations of the remaining colours.

Hence, the probability is $\frac{4}{12}$ or $\frac{1}{3}$.

Problem of the Week

Problem B

Where's the Audience?

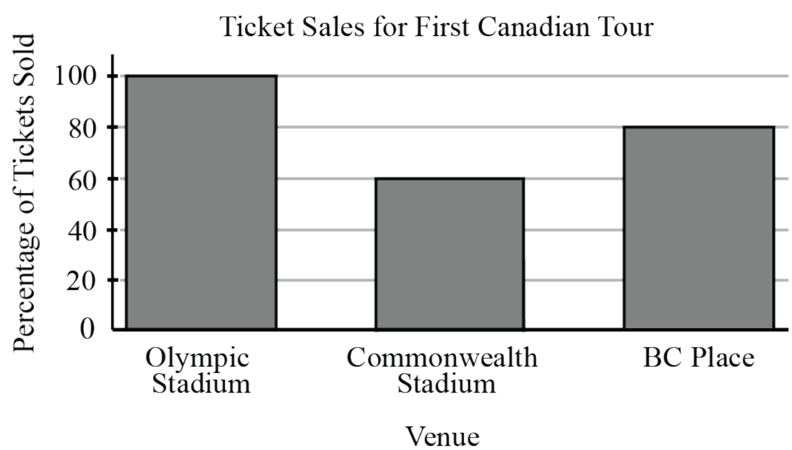
The Pythagorean Triples are a rock band who recently returned from their second Canadian tour.

(a) Information about ticket sales for three of the venues they played at is summarized in the following table.

Venue	Number of Tickets Available	Number of Tickets Sold
Olympic Stadium	60 000	45 000
Commonwealth Stadium	55 000	44 000
BC Place	54 000	48 600

For each venue, what percentage of available tickets were sold?

(b) Two years ago, the Pythagorean Triples played at the same three venues on their first Canadian tour. For each venue, the percentage of available tickets that were sold is shown in the bar graph below.



If the number of tickets available for each venue was the same for both tours, which tour sold more tickets for these three venues combined? Justify your answer.

Problem of the Week

Problem B and Solution

Where's the Audience?

Problem

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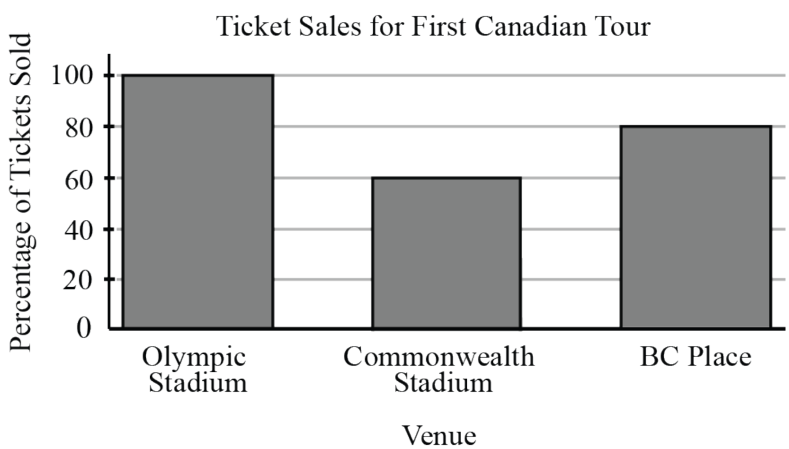

If the number of tickets available for each venue was the same for both tours, which tour sold more tickets for these three venues combined? Justify your answer.

Solution

(a) To calculate the percentage of available tickets that were sold, we divide the number of tickets sold by the number of tickets available, and then multiply by 100% to convert the decimal to a percentage.

- Olympic Stadium: $45\,000 \div 60\,000 = 0.75$, and $0.75 \times 100\% = 75\%$.
- Commonwealth Stadium: $44\,000 \div 55\,000 = 0.8$, and $0.8 \times 100\% = 80\%$.
- BC Place: $48\,600 \div 54\,000 = 0.9$, and $0.9 \times 100\% = 90\%$.

(b) We need to calculate the total number of tickets sold for the three venues for each of the tours.

- For the second Canadian tour, we can add up the number of tickets sold for each venue in the table from part (a).

$$45\,000 + 44\,000 + 48\,600 = 137\,600$$

- For the first Canadian tour, we first need to use the percentages in the bar graph to calculate the number of tickets sold at each venue. The bar graph shows that 100% of the available tickets at Olympic stadium were sold, 60% were sold at Commonwealth Stadium, and 80% were sold at BC Place.
  - Olympic Stadium: 100% of $60\,000$ is $60\,000$.
  - Commonwealth Stadium: 60% of $55\,000$ is equal to $\frac{60}{100} \times 55\,000$ or $\frac{3}{5} \times 55\,000$, which equals $33\,000$.
  - BC Place: 80% of $54\,000$ is equal to $\frac{80}{100} \times 54\,000$ or $\frac{4}{5} \times 54\,000$, which equals $43\,200$.

  Thus, the total number of tickets sold for the three venues for the first Canadian tour is

$$60\,000 + 33\,000 + 43\,200 = 136\,200$$

Since $137\,600 > 136\,200$, it follows that the second Canadian tour sold more tickets for the three venues combined.

Problem of the Week

Problem B

How Tall are They, Really?

There were 12 players on the Canadian 2020 Olympic Women's Basketball Team. The table below shows the height of each player.



Name	Height
Natalie Achonwa	6 feet, 3 inches
Kayla Alexander	6 feet, 4 inches
Laeticia Amihere	6 feet, 2 inches
Miranda Ayim	6 feet, 3 inches
Bridget Carleton	6 feet, 1 inch
Shay Colley	5 feet, 8 inches
Aaliyah Edwards	6 feet, 3 inches
Nirra Fields	5 feet, 7 inches
Kim Gaucher	6 feet, 1 inch
Kia Nurse	6 feet, 0 inches
Shaina Pellington	5 feet, 8 inches
Nayo Raincock-Ekunwe	6 feet, 2 inches

Source: https://www.basketball.ca/team-canada-en/tokyo-2020

(a) Create a stem-and-leaf plot to represent the heights in feet and inches of the players on the team. Use the number of feet as the stems, and the number of inches as the leaves.

(b) Using your stem-and-leaf plot, find the median height of the players on the team.

(c) Convert all the heights to inches, and then calculate the mean height of the players on the team. Recall that 1 foot $=$ 12 inches.

For example, 6 feet is equal to $6 \times 12 = 72$ inches, so a height of 6 feet, 3 inches is equal to $72 + 3 = 75$ inches.

(d) Assume that the mean height of a Canadian woman is 5 feet, 4 inches. How much taller, on average, are the players on this team?

Theme Data Management

Problem of the Week

Problem B and Solution

How Tall are They, Really?

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For example, 6 feet is equal to $6 \times 12 = 72$ inches, so a height of 6 feet, 3 inches is equal to $72 + 3 = 75$ inches.

(d) Assume that the mean height of a Canadian woman is 5 feet, 4 inches. How much taller, on average, are the players on this team?

Solution

(a) The stem-and-leaf plot is shown below.

stem	leaf
5	7 8 8
6	0 1 1 2 2 3 3 3 4

key: 6 | 3 = 6 feet, 3 inches

(b) The median is halfway between 6 feet, 1 inch and 6 feet, 2 inches, which is 6 feet, 1.5 inches.

(c) The table below shows all the heights converted to inches.

Name	**Height (feet, inches)**	**Height (inches)**
Natalie Achonwa	6 feet, 3 inches	$72 + 3 = 75$
Kayla Alexander	6 feet, 4 inches	$72 + 4 = 76$
Laeticia Amihere	6 feet, 2 inches	$72 + 2 = 74$
Miranda Ayim	6 feet, 3 inches	$72 + 3 = 75$
Bridget Carleton	6 feet, 1 inch	$72 + 1 = 73$
Shay Colley	5 feet, 8 inches	$60 + 8 = 68$
Aaliyah Edwards	6 feet, 3 inches	$72 + 3 = 75$
Nirra Fields	5 feet, 7 inches	$60 + 7 = 67$
Kim Gaucher	6 feet, 1 inch	$72 + 1 = 73$
Kia Nurse	6 feet, 0 inches	72
Shaina Pellington	5 feet, 8 inches	$60 + 8 = 68$
Nayo Raincock-Ekunwe	6 feet, 2 inches	$72 + 2 = 74$

The mean is found by adding up all the heights in inches, and dividing by 12.

$$75 + 76 + 74 + 75 + 73 + 68 + 75 + 67 + 73 + 72 + 68 + 74 = 870$$

Since $870 \div 12 = 72.5$, the mean height is 72.5 inches, or 6 feet, 0.5 inches.

(d) We can first convert 5 feet, 4 inches to inches. 5 feet is equal to $5 \times 12 = 60$ inches, so a height of 5 feet, 4 inches is equal to $60 + 4 = 64$ inches.

The mean height of the players on the team is 72.5 inches. Since $72.5 - 64 = 8.5$, that means on average, the players on the team are 8.5 inches taller than the average Canadian woman.

Problem of the Week

Problem B

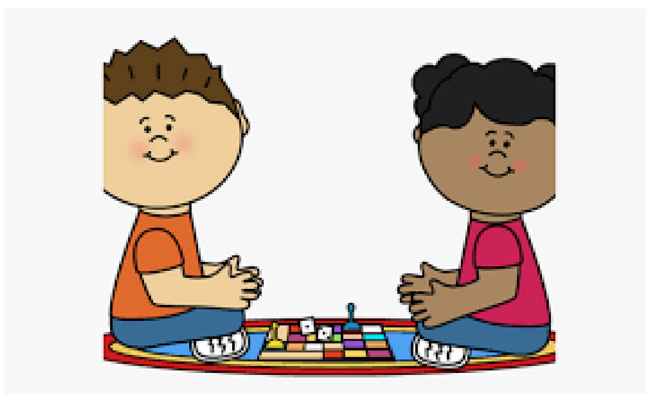
Let the Game Begin!

Parvinder and Carlos are each rolling a standard six-sided die, with numbers 1 to 6 on its sides, to see who gets to go first in their game. The person who rolls the lower number gets to go first. If they tie, they roll again.

(a) What is the theoretical probability that Parvinder will roll a number lower than Carlos on their first roll? HINT: What are the possibilities for Carlos' roll if Parvinder rolls a 1? a 2? a 3?

(b) What is the theoretical probability that Parvinder will roll a number greater than Carlos on their first roll?

(c) Is this a fair way to determine who goes first?



Problem of the Week
Problem B and Solution
Let the Game Begin!

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(b) What is the theoretical probability that Parvinder will roll a number greater than Carlos on their first roll?

(c) Is this a fair way to determine who goes first?

Solution

The following table shows all the possible rolls. For each possibility, there is a T if Parvinder and Carlos roll the same number, a P if Parvinder rolls a number lower than Carlos, and a C if Carlos rolls a number lower than Parvinder.

		Carlos' Roll					
		1	2	3	4	5	6
Parvinder's Roll	1	T	P	P	P	P	P
	2	C	T	P	P	P	P
	3	C	C	T	P	P	P
	4	C	C	C	T	P	P
	5	C	C	C	C	T	P
	6	C	C	C	C	C	T

The table shows that the total number of possible rolls is $6 \times 6 = 36$.

(a) There are 15 Ps in the table. These occur when Parvinder rolls a number lower than Carlos. Thus, the theoretical probability that Parvinder rolls a number lower than Carlos on their first roll is $\frac{15}{36} = \frac{5}{12}$.

(b) There are 15 Cs in the table. These occur when Parvinder rolls a number greater than Carlos. Thus, the theoretical probability that Parvinder rolls a number greater than Carlos on their first roll is $\frac{15}{36} = \frac{5}{12}$.

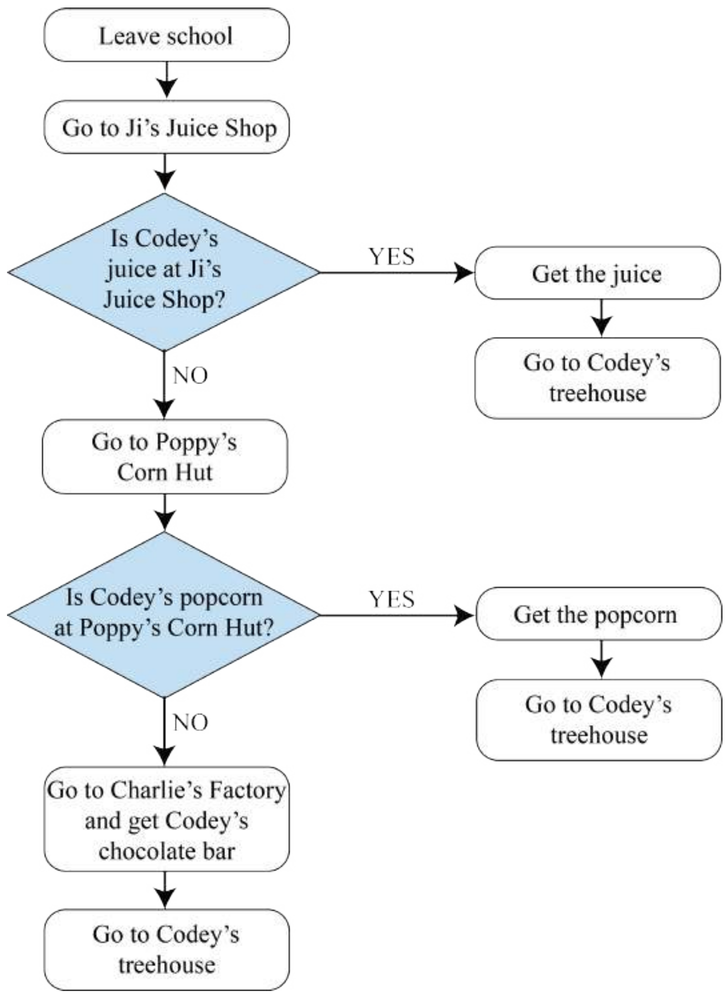
(c) From (a) and (b), we see that the probabilities of each person rolling the lower number are the same. Therefore, this is a fair way to determine who goes first.

Computational Thinking (C)

TAKE ME TO THE COVER

Problem of the Week
Problem B
Go With the Flow

Codey has asked three friends to pick up some food on their way from school to his treehouse. At three different stores he has paid for and set aside one item, and he has given his friends instructions to pick these items up using the following flow chart.



(a) Raj leaves school first, then Bharti, then Kylie. If they each take the same route and walk at the same speed, which item will each friend bring to Codey's treehouse?

(b) How could you modify the flow chart so that each friend has to go to only one store?

Problem of the Week

Problem B and Solution

Go With the Flow

Problem

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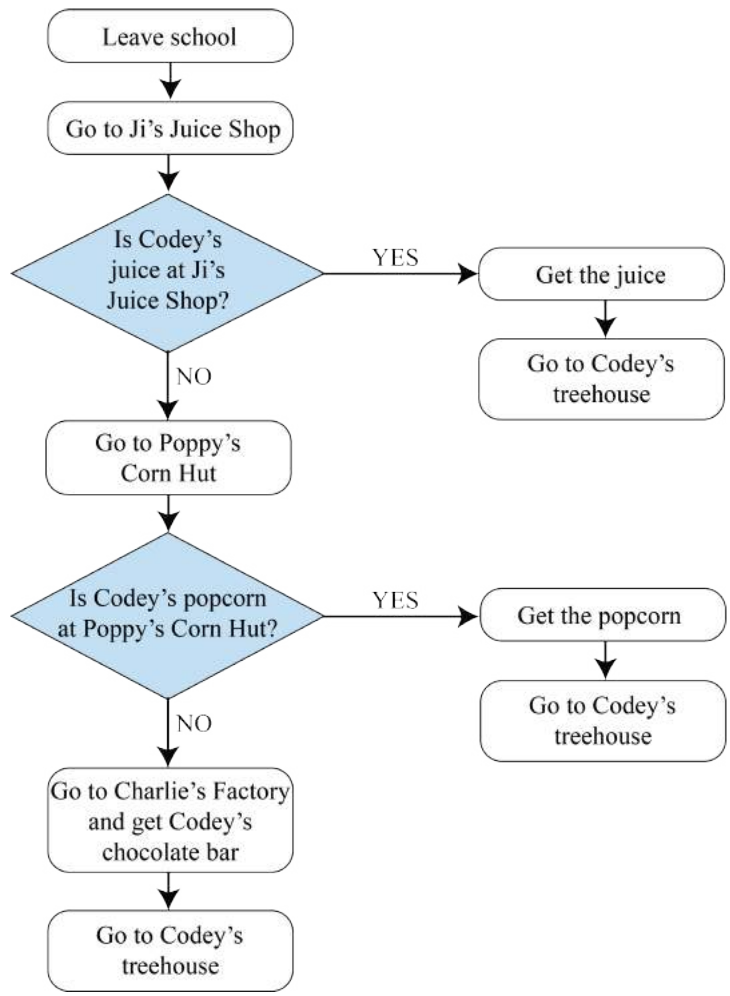


(a) Raj leaves school first, then Bharti, then Kylie. If they each take the same route and walk at the same speed, which item will each friend bring to Codey's treehouse?

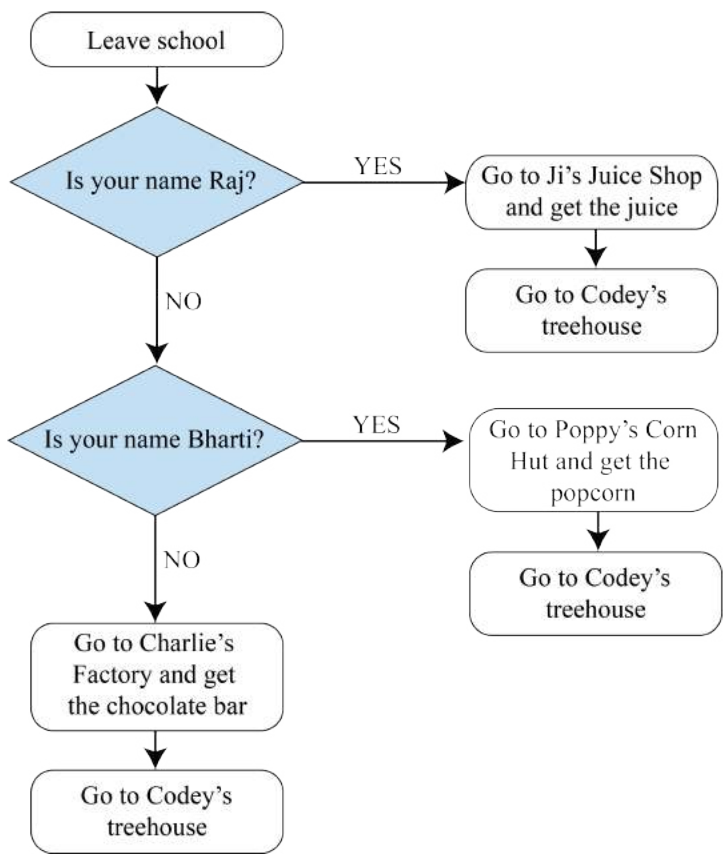
(b) How could you modify the flow chart so that each friend has to go to only one store?

Solution

(a) Since Raj leaves school first, he will arrive at Ji's Juice Shop first so will pick up the juice. Bharti leaves school next, so she will first go to Ji's Juice shop, but since the juice was already picked up, Bharti will then go to Poppy's Corn Hut and pick up the popcorn. Kylie leaves school last so she will first go to Ji's Juice Shop. Since the juice was already picked up, she will then go to Poppy's Corn Hut. Since the popcorn was already picked up, she will then go to Charlie's Factory and pick up the chocolate bar.

So, Raj will bring the juice, Bharti will bring the popcorn, and Kylie will bring the chocolate bar to Codey's treehouse.

(b) To modify the flow chart, we could assign each friend to a particular item. There are many ways to do this, but one way is shown in the flow chart below.

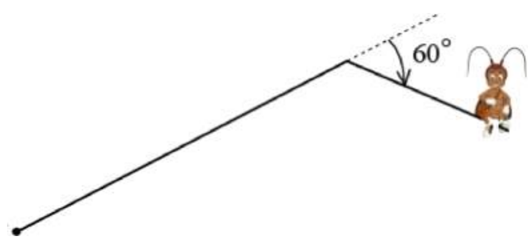


Problem of the Week
Problem B
A Spritely Shape

Peyton used a block coding program to get a sprite character to draw a shape. His sprite followed these steps:

1. Put pen down to write
2. Move 10 steps forward
3. Turn clockwise 60°
4. Repeat steps 2 and 3 five more times

Here is the sprite's drawing partway through the program:



(a) What type of polygon did the sprite draw?

(b) What type of pattern did Peyton use in this code?

(c) If the code were changed so that step 3 reads "Turn clockwise 45°", how would Peyton need to change step 4 in order to create a closed polygon?

Problem of the Week
Problem B and Solution
A Spritely Shape

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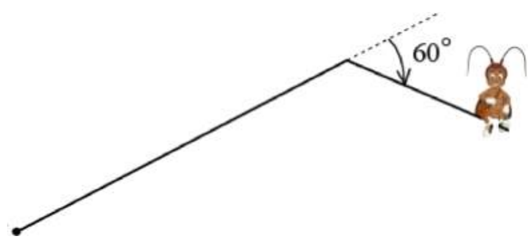


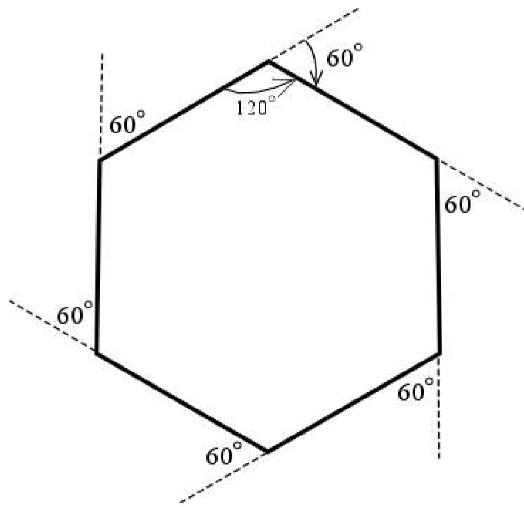
(a) What type of polygon did the sprite draw?

(b) What type of pattern did Peyton use in this code?

(c) If the code were changed so that step 3 reads "Turn clockwise 45°", how would Peyton need to change step 4 in order to create a closed polygon?

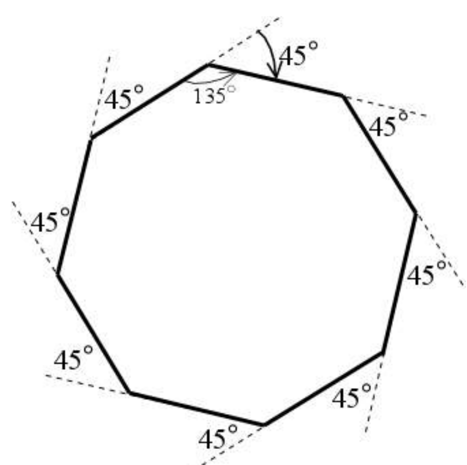
Solution

(a) The sprite moved 10 steps forward and then and turned 60° clockwise a total of six times. By doing this, the sprite created a regular hexagon (with interior angles of 120°, which sum to 720°). The completed hexagon is shown below.



(b) Peyton used a *repeating* pattern in this code; continuing will retrace the hexagon.

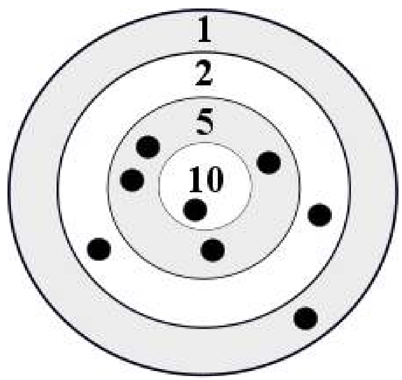
(c) If the code were changed so that step 3 reads "Turn clockwise 45° ", Peyton would have to revise step 4 to "Repeat steps 2 and 3 seven more times.", thus creating a regular octagon (with interior angles of 135°, which sum to 1080°). The completed octagon is shown below.



Problem of the Week
Problem B
Who Hit the Middle?

A dart board consists of four regions: an inner circle and three concentric circular bands. Any dart landing in the inner circle will receive 10 points. Any dart landing in the first band will receive 5 points. Any dart landing in the second band will receive 2 points. Any dart landing in the third band will receive 1 point.

Serena and Ebony each threw four darts. The locations where the eight darts landed are shown as black dots on the diagram below.



(a) What was the total number of points scored by the two players?

(b) If Ebony's total score was 1 more than Serena's, what was each person's score?

(c) What individual shots could each player have had to get their scores?

(d) Whose dart landed in the inner circle?

Problem of the Week
Problem B and Solution
Who Hit the Middle?

Problem

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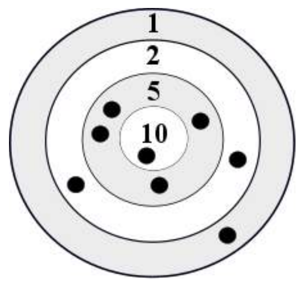

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(b) If Ebony's total score was 1 more than Serena's, what was each person's score?

(c) What individual shots could each player have had to get their scores?

(d) Whose dart landed in the inner circle?

Solution

(a) Since one dart landed in the band worth 1 point, two darts landed in the band worth 2 points, four darts landed in the band worth 5 points, and one dart landed in the inner circle worth 10 points, the total number of points scored by the two players was

$$1 + 2 + 2 + 5 + 5 + 5 + 5 + 10 = 35$$

(b) Since $35 = 17 + 18$, Ebony scored 18 points and Serena scored 17 points.

(c) Trying all combinations of four shots, we can see that the only way to get a score of 18 is as $1 + 2 + 5 + 10$.

Therefore, Ebony made shots worth 1, 2, 5, and 10 points. This means that Serena made shots worth 2, 5, 5, and 5 points.

(d) Since the inner circle is worth 10 points, then one of Ebony's darts landed in the inner circle.