

# Problem of the Week Problems and Solutions 2021 - 2022

Problem A (Grade 3/4)

### **Themes**

(Click on a theme name to jump to that section.)

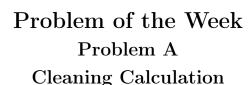
Number Sense (N)
Geometry & Measurement (G)
Algebra (A)
Data Management (D)
Computational Thinking (C)

The problems in this booklet are organized into themes.

A problem often appears in multiple themes.

### Number Sense (N)





A housekeeper is responsible for cleaning all the rooms on one floor of a hotel. The floor has 16 regular rooms and 5 suites. Regular rooms take 20 minutes each to clean. Suites take 30 minutes each to clean.

- (a) How long does it take to clean all the rooms on the floor of the hotel?
- (b) If the housekeeper starts cleaning at 10:00 a.m. and does not take a break, at what time is the job finished?







#### Problem of the Week Problem A and Solution Cleaning Calculation

#### Problem

A housekeeper is responsible for cleaning all the rooms on one floor of a hotel. The floor has 16 regular rooms and 5 suites. Regular rooms take 20 minutes each to clean. Suites take 30 minutes each to clean.

- (a) How long does it take to clean all the rooms on the floor of the hotel?
- (b) If the housekeeper starts cleaning at 10:00 a.m. and does not take a break, at what time is the job finished?

#### Solution

(a) We can calculate the time it takes to clean the regular rooms by skip counting by 20:

 $20,\ 40,\ 60,\ 80,\ 100,\ 120,\ 140,\ 160,\ 180,\ 200,\ 220,\ 240,\ 260,\ 280,\ 300,\ 320$ 

We see that it takes a total of 320 minutes to clean the regular rooms.

We can calculate the time it takes to clean the suites by skip counting by 30:

We see that it takes a total of 150 minutes to clean the suites. We add these two numbers together to see that it takes a total of 320 + 150 = 470 minutes to clean the entire floor.

From this total number of minutes, we could calculate the result in hours and minutes. However, we could also recognize that when skip counting by 20, we get to 60 minutes after three 20-minute intervals. This means it takes an hour to clean three regular rooms. We could count a bit differently:

20 min, 40 min, 1 hour, 80 min, 100 min, 2 hours, 140 min, 160 min, 3 hours, 200 min, 220 min, 4 hours, 260 min, 280 mins, 5 hours, 320 mins

We can see that 320 minutes is equal to 5 hours and 20 minutes.

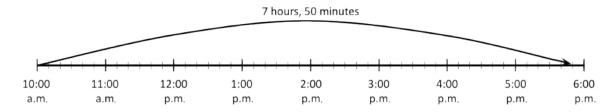
Similarly, we get to 60 minutes after two 30-minute intervals. With similar counting we see that 150 minutes is equal to 2 hours and 30 minutes.

Now we can find the total time taken as:

5 hours + 2 hours + 20 minutes + 30 minutes

which is equal to 7 hours and 50 minutes.

(b) We can use a timeline to determine the time that is 7 hours and 50 minutes after 10:00 a.m.



Therefore, the housekeeper will finish the job at 5:50 p.m.

#### Teacher's Notes

Although we often think of *rates* as being related to distance and time, we can describe this problem in terms of rates. The units we use when describing rates is:

For example, a car's speed may be measured in km/hr or a computer's download speed may be measured in MBits/sec.

When working with rates involving distance and time, we can consider this formula that shows the relationship between *rate*, *distance*, and *time*:

$$rate = \frac{distance}{time}$$

Given any two of the values, we can use the formula to calculate the third. For example, if we know the *rate* and *time*, we can calculate the distance. Often we rearrange the formula to make it easier to calculate the missing value, where the missing value is isolated on one side of the equals sign.

$$rate \times time = distance$$
 and  $time = \frac{distance}{rate}$ 

For this problem, we can consider the following formula for the rate of cleaning:

$$rate = \frac{rooms}{time}$$

For regular rooms, we know the rate is  $\frac{1 \text{ room}}{20 \text{ min}}$  and the number of rooms is 16.

We can rearrange the formula to isolate the time:

$$time = \frac{rooms}{rate}$$

Now we can substitute the actual numbers into this formula:

$$time = \frac{16 \text{ rooms}}{\frac{1 \text{ room}}{20 \text{ mins}}}$$

To calculate this result we must divide by a fraction, which we can do by inverting the fraction and multiplying like this:

$$time = 16 \text{ rooms} \times \frac{20 \text{ mins}}{1 \text{ room}}$$
 
$$time = 320 \text{ mins}$$



### Problem of the Week Problem A **Patio Planning**

Jody and Jillian are building a patio beside their house. They have square patio stones that have sides with a length of 50 cm. They have a total 75 patio stones available.

To build the patio, they place the patio stones so that two stones next to each other are touching and their sides are lined up exactly. They will only use whole patio stones and they want the entire patio covered in stones.

- (a) If they want to have their patio in the shape of a square, what are the dimensions of the largest possible patio they can build?
- (b) If they want to build a patio in the shape of a rectangle where the length of one side is twice as long as the length of the other side, what are the dimensions of the largest patio they can build?







### Problem of the Week Problem A and Solution Patio Planning

#### **Problem**

Jody and Jillian are building a patio beside their house. They have square patio stones that have sides with a length of 50 cm. They have a total 75 patio stones available.

To build the patio, they place the patio stones so that two stones next to each other are touching and their sides are lined up exactly. They will only use whole patio stones and they want the entire patio covered in stones.

- (a) If they want to have their patio in the shape of a square, what are the dimensions of the largest possible patio they can build?
- (b) If they want to build a patio in the shape of a rectangle where the length of one side is twice as long as the length of the other side, what are the dimensions of the largest patio they can build?

#### Solution

(a) The following table shows the possible sizes of the patio if the patio is in the shape of a square.

Length	Width	Number of Stones Used
1	1	$1 \times 1 = 1$
2	2	$2 \times 2 = 4$
3	3	$3 \times 3 = 9$
4	4	$4 \times 4 = 16$
5	5	$5 \times 5 = 25$
6	6	$6 \times 6 = 36$
7	7	$7 \times 7 = 49$
8	8	$8 \times 8 = 64$
9	9	$9 \times 9 = 81$

Since there are only 75 patio stones available, the biggest square patio that Jody and Jillian can build is 8 patio stones long by 8 patio stones wide. Since each square patio stone has sides with length 50 cm, then 8 stones in a line have a length of  $8 \times 50 = 400$  cm or 4 metres. So the largest square patio made out of 75 or fewer patio stones has dimensions 4 meters long by 4 metres wide.

(b) To investigate a patio that has one side that is twice as long as the other side, we can name the shorter side Length and the longer side Width. The following table shows the possible sizes of the patio.

Length	Width	Number of Stones Used
1	2	$1 \times 2 = 2$
2	4	$2 \times 4 = 8$
3	6	$3 \times 6 = 18$
4	8	$4 \times 8 = 32$
5	10	$5 \times 10 = 50$
6	12	$6 \times 12 = 72$
7	14	$7 \times 14 = 98$

Since we only have 75 patio stones, the biggest patio Jody and Jillian can build that has one side twice as long as the other side is a rectangle that is 6 patio stones long by 12 patio stones wide. Since each square patio stone has sides with length 50 cm, then 6 stones in a line have a length of  $6 \times 50 = 300$  cm or 3 metres, and 12 stones in a line have a length of  $12 \times 50 = 600$  cm or 6 metres. So the largest patio with one side twice as long as the other side and made out of 75 or fewer patio stones, has dimensions 3 metres by 6 metres.



# Problem of the Week Problem A Non-cents

Canadian prices are normally in dollars and cents, with the dollar amount appearing before the decimal point and the cents amount appearing after the decimal point.

Zahra has a store that has decided to get rid of cents in their pricing. This means that all new prices will be rounded to the nearest dollar amount so that each price ends with .00.

These are some of the items in her store as well as their original prices:

Item	Price
Rollerblades	\$54.59
Helmet	\$25.25
Elbow Pads	\$15.88
Skateboard	\$75.75
Bicycle	\$105.32
Bike Lock	\$25.44
Wrist Guards	\$20.99

- (a) What will the new price of each item be?
- (b) If she sells one of each type of item, will Zahra be making more money or less money than before? Justify your answer.



THEME NUMBER SENSE





# Problem of the Week Problem A and Solution Non-cents

#### **Problem**

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Skateboard	\$75.75
Bicycle	\$105.32
Bike Lock	\$25.44
Wrist Guards	\$20.99

- (a) What will the new price of each item be?
- (b) If she sells one of each type of item, will Zahra be making more money or less money than before? Justify your answer.

#### Solution

(a) Here are the new prices rounded to the nearest dollar:

Item	New Price
Rollerblades	\$55.00 (rounded up)
Helmet	\$25.00 (rounded down)
Elbow Pads	\$16.00 (rounded up)
Skateboard	\$76.00 (rounded up)
Bicycle	\$105.00 (rounded down)
Bike Lock	\$25.00 (rounded down)
Wrist Guards	\$21.00 (rounded up)

(b) There are many ways to determine the impact of the new prices. One option is to find the totals of the original prices and the new prices, and then calculate the difference of those totals. To keep things a bit simpler, since the tens and hundreds place values have not changed, then we can ignore the tens and hundreds place values of the prices.

Original total (ignoring the tens and hundreds place values):

$$4.59 + 5.25 + 5.88 + 5.75 + 5.32 + 5.44 + 0.99 = $33.22$$

New total (ignoring the tens and hundreds place values):

$$5+5+6+6+5+5+1=$33$$

We see that the new total is 22 cents less than the original total. Therefore, Zahra will be making less money with the new pricing.

Another way to determine the difference is to find out how much more or less the new individual prices will be compared to the old prices. These amounts are all in cents. For example:

- The new rollerblades price is 41 cents more than the original price since we rounded up and 59 + 41 = 100.
- The new helmet price is 25 cents less than the original price since we rounded down.

Zahra will make more money on the items where the prices were rounded up, and less on the items that were rounded down.

More money: 41 + 12 + 25 + 1 = 79 cents

Less money: 25 + 32 + 44 = 101 cents

The "less money" amount is greater than the "more money" amount. Overall, Zahra will make 101 - 79 = 22 cents less than before with the new pricing.



# Problem of the Week Problem A What Number Am I?

I am a five-digit number made of the digits 0, 3, 4, 6, and 9. In my number, the following are true.

- The digit **4** is in a position whose place value is 10 times the place value of the position of the digit **9** and 100 times the place value of the position of the digit **0**.
- The digit **3** is in a position whose place value is 100 times the place value of the position of the digit **9**.
- The digit **0** is in a position whose place value is 10 times the place value of the position of the digit **6**.

What number am I?





#### Problem of the Week Problem A and Solution What Number Am I?

#### Problem

I am a five-digit number made of the digits 0, 3, 4, 6, and 9. In my number, the following are true.

- The digit 4 is in a position whose place value is 10 times the place value of the position of the digit 9 and 100 times the place value of the position of the digit 0.
- The digit **3** is in a position whose place value is 100 times the place value of the position of the digit **9**.
- The digit **0** is in a position whose place value is 10 times the place value of the position of the digit **6**.

What number am I?

#### Solution

The positions in a five-digit number are: ones (units), tens, hundreds, thousands, and ten thousands.

Since the 4 is in a position whose place value is 10 and 100 times the place value of other positions, the 4 cannot be in the ones or tens position.

Since the 3 is in a position whose place value is 100 times the place value of another position, the 3 cannot be in the ones or tens position.

Since the 0 is in a position whose place value is 10 times the place value of another position, the 0 cannot be in the ones position.

So the ones position must contain either the 9 or the 6.

Let's assume that the 9 is in the ones position. Based on the first clue, the 4 must be in the tens position since  $10 \times 1 = 10$ . However, we have already stated that the 4 cannot be in the ones or tens position. So we cannot put the 9 in the ones position.

Therefore, we can conclude that the 6 is in the ones position. Based on the third clue, the 0 must be in the tens position since  $10 \times 1 = 10$ . Knowing that the 0 is in the tens position, we can use the first clue to conclude that the 4 must be in the thousands position, since  $100 \times 10 = 1000$ . Also, the 9 must be in the hundreds position, since  $10 \times 100 = 1000$ . Now the only position that is left, the ten thousands position, must contain the only digit left which is the 3. This is consistent with the second clue since  $100 \times 100 = 10000$ .

Therefore, the number must be 34 906.

# Problem of the Week Problem A Perfect Punch

Su is going to make punch for her friends. She wants to mix 3 L of orange juice, 1 L of pop,  $\frac{1}{2}$  L of grape juice, and 300 mL of cranberry juice in a punch bowl.

- (a) To avoid spilling, Su plans to use a punch bowl with a capacity of at least 200 mL more than the liquid it holds. What is the smallest capacity that her punch bowl should have?
- (b) Su has cups that can each hold 300 mL of punch. How many of these cups can she fill with the punch she makes?







## Problem of the Week Problem A and Solution Perfect Punch

#### Problem

Su is going to make punch for her friends. She wants to mix 3 L of orange juice, 1 L of pop,  $\frac{1}{2}$  L of grape juice, and 300 mL of cranberry juice in a punch bowl.

- (a) To avoid spilling, Su plans to use a punch bowl with a capacity of at least 200 mL more than the liquid it holds. What is the smallest capacity that her punch bowl should have?
- (b) Su has cups that can each hold 300 mL of punch. How many of these cups can she fill with the punch she makes?

#### Solution

(a) The smallest capacity of the punch bowl is the sum of the volumes of each liquid, plus the 200 mL of extra space to avoid spilling.

One way to calculate this would be to convert all the volumes to millilitres.

- 3 L = 3000 mL
- 1 L = 1000 mL
- $\frac{1}{2}$  L = 500 mL

So the minimum capacity is 3000 + 1000 + 500 + 300 + 200 = 5000 mL.

Alternatively, we might notice that the sum of the volume of cranberry juice and the extra room for spillage is: 300 + 200 = 500 mL or  $\frac{1}{2}$  L.

So the minimum capacity is  $3 + 1 + \frac{1}{2} + \frac{1}{2} = 5$  L.

(b) The volume of punch is 3000 + 1000 + 500 + 300 = 4800 mL.

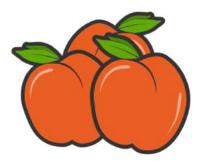
We can use skip counting to figure out how many cups of punch Su can fill: 300, 600, 900, 1200, 1500, 1800, 2100, 2400, 2700, 3000, 3300, 3600, 3900, 4200, 4500, 4800

We can see from this that Su can fill 16 cups with punch.

Alternatively, we can calculate the number of cups of punch by dividing  $4800 \div 300 = 16$  cups.

# Problem of the Week Problem A Tasty Tarts

Genevieve and Dima decided to make peach tarts. According to their recipe, each tart requires three-quarters of a peach. If they bought a dozen peaches, how many tarts can they make?







# Problem of the Week Problem A and Solution Tasty Tarts

#### **Problem**

Genevieve and Dima decided to make peach tarts. According to their recipe, each tart requires three-quarters of a peach. If they bought a dozen peaches, how many tarts can they make?

#### Solution

Here is one way to solve the problem. We can draw a diagram of a dozen peaches where each peach is divided into four quarters. Then we group three quarters together with a common label, corresponding to the tart that those quarters are used in. We continue to identify groups of three quarters over the dozen peaches. Finally, we count the number of groups we have identified.

1	1	2	2	3	4
1	2	3	3	4	4
5	5	6	6	7	8
5	6	7	7	8	8
9	9	10	10	11	12
9	9	10 11	10 11		12

From this diagram, we can see that Genevieve and Dima can make 16 tarts with a dozen peaches.



### Problem of the Week Problem A **Puppy Plans**

Robbie wants to get a puppy, so he decides to help pay the costs. He wants to donate \$60 to the animal shelter he gets the puppy from, and he needs to contribute \$10 every month to help buy the puppy's food.

- (a) How much money does Robbie need to help pay the cost of getting the puppy and feeding the puppy in the first year?
- (b) Robbie needs to save the money calculated in part (a) before getting the puppy. He has \$87 in his piggy bank. He earns a \$7 allowance every week for doing his chores. How long will it take Robbie to earn the rest of the money he needs?







#### Problem of the Week Problem A and Solution Puppy Plans

#### Problem

Robbie wants to get a puppy, so he decides to help pay the costs. He wants to donate \$60 to the animal shelter he gets the puppy from, and he needs to contribute \$10 every month to help buy the puppy's food.

- (a) How much money does Robbie need to help pay the cost of getting the puppy and feeding the puppy in the first year?
- (b) Robbie needs to save the money calculated in part (a) before getting the puppy. He has \$87 in his piggy bank. He earns a \$7 allowance every week for doing his chores. How long will it take Robbie to earn the rest of the money he needs?

#### Solution

- (a) Since there are 12 months in a year and Robbie wants to contribute \$10 each month, he needs  $12 \times 10 = $120$  for puppy food in the first year. Counting the shelter donation, Robbie needs \$60 + \$120 = \$180 in total.
- (b) We can make a table to calculate how long it will take for Robbie to save enough money to have at least \$180. He starts with \$87 and will increase his total by \$7 each week.

Week	Total Saved (in \$)
0	87
1	94
2	101
3	108
4	115
5	122
6	129
7	136
8	143
9	150
10	157
11	164
12	171
13	178
14	185

So it will take Robbie 14 weeks to earn enough money to help cover the shelter donation and the costs of feeding the puppy in the first year.

Alternatively, since Robbie has already saved \$87, he needs to earn \$180 - \$87 = \$93 more. We calculate how long it will take to earn the remaining amount using division. Since  $93 \div 7 = 13$  with a remainder of 2, we round up to determine that it will take 14 weeks to earn the additional \$93 Robbie needs.



### Problem of the Week Problem A Cookies!

Benoit, Jin, Manuel, and Sarah shared a package of 30 cookies. After several days, all the cookies in the package were eaten by these four people and each person only ate whole cookies. Benoit recorded the following observations:

- Sarah ate twice as many cookies as Jin.
- Manuel ate two-thirds as many cookies as Sarah.
- Benoit ate half as many cookies as Manuel.
- (a) Who ate the most cookies? Justify your answer.
- (b) What fraction of the number of cookies eaten by Sarah is the number of cookies eaten by Benoit? Justify your answer.
- (c) Who ate the fewest cookies? Justify your answer.
- (d) Someone ate 6 cookies. This was not the person who ate the most or fewest cookies. How many cookies did each person eat? Justify your answer.





# Problem of the Week Problem A and Solution Cookies!

#### Problem

Benoit, Jin, Manuel, and Sarah shared a package of 30 cookies. After several days, all the cookies in the package were eaten by these four people and each person only ate whole cookies. Benoit recorded the following observations:

- Sarah ate twice as many cookies as Jin.
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- Benoit ate half as many cookies as Manuel.
- (a) Who ate the most cookies? Justify your answer.
- (b) What fraction of the number of cookies eaten by Sarah is the number of cookies eaten by Benoit? Justify your answer.
- (c) Who ate the fewest cookies? Justify your answer.
- (d) Someone at 6 cookies. This was not the person who at the most or fewest cookies. How many cookies did each person eat? Justify your answer.

#### Solution

- (a) Since Sarah ate more cookies than Jin, we know that Jin did not eat the most cookies. Since Manuel ate fewer cookies than Sarah, we know that Manuel did not eat the most cookies. Since Benoit ate fewer cookies than Manuel, we know that Benoit did not eat the most cookies. Therefore, Sarah ate the most cookies.
- (b) Since Benoit ate half as many cookies as Manuel, and Manuel ate two-thirds as many cookies as Sarah, we need to calculate one-half of two-thirds, which is one-third. So Benoit ate one-third as many cookies as Sarah.
- (c) Since Sarah ate twice as many cookies as Jin, we can say that Jin ate one-half as many cookies as Sarah. From part (b) we know Benoit ate one-third as many cookies as Sarah. We also know that Manuel ate two-thirds as many cookies as Sarah. Since one-third is less than one-half, which is less than two-thirds, it follows that Benoit ate the fewest cookies.

(d) Since we know that Sarah ate the most cookies, and Benoit ate the fewest cookies, then either Jin or Manuel ate 6 cookies. Let's guess that Jin ate 6 cookies.

In this case, from the first observation then Sarah ate  $2 \times 6 = 12$  cookies. We know that  $\frac{1}{3}$  of 12 is 4, so  $\frac{2}{3}$  of 12 is 8. From the second observation Manuel ate 8 cookies. From the third observation, Benoit ate  $\frac{1}{2}$  of 8, which is 4 cookies.

Since 4+6+8+12=30, then it is possible that Benoit ate 4 cookies, Jin ate 6 cookies, Manuel ate 8 cookies, and Sarah ate 12 cookies.

We should also check to see if it is possible that Manuel was the person who ate 6 cookies. In this case, Benoit would have eaten  $\frac{1}{2}$  of 6, which is 3 cookies. From part (b) we noticed that Benoit ate  $\frac{1}{3}$  as many cookies as Sarah. This means we can say that Sarah ate 3 times as many cookies as Benoit, so Sarah would have eaten  $3 \times 3 = 9$  cookies. However, since Jin ate half as many cookies as Sarah, and one-half of 9 is not a whole number, that means Sarah could not have eaten 9 cookies. So Manuel could not be the person who ate 6 cookies.

We see that it must be the case that Benoit ate 4 cookies, Jin ate 6 cookies, Manuel ate 8 cookies, and Sarah ate 12 cookies.

# Problem of the Week Problem A Gym Schedules

At Spruce Glen Public School, each day is divided into nine blocks, which are each 30 minutes long. There are six classrooms that share the gym, according to the weekly schedule shown.

	Monday	Tuesday	Wednesday	Thursday	Friday
Block A	Room 1	Room 3	Room 2	Room 3	Room 6
Block B	Room 5	Room 5	Room 2	Room 3	Room 1
Block C	Room 3	Room 5	Room 2	Room 2	Room 3
Block D	Room 3	Room 5	Room 5	Room 2	Room 3
Block E	Room 6	Room 1	Room 5	Room 6	Room 3
Block F	Room 4	Room 2	Room 6	Room 1	Room 5
Block G	Room 4	Room 6	Room 1	Room 1	Room 2
Block H	Room 2	Room 4	Room 3	Room 4	Room 2
Block I	Room 2	Room 4	Room 4	Room 5	Room 4

- (a) Make a bar chart showing the total gym time per week for each room.
- (b) List the rooms in order from least to greatest total gym time per week.





# Problem of the Week Problem A and Solution Gym Schedules

#### Problem

At Spruce Glen Public School, each day is divided into nine blocks, which are each 30 minutes long. There are six classrooms that share the gym, according to the weekly schedule shown.

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Block B	Room 5	Room 5	Room 2	Room 3	Room 1
Block C	Room 3	Room 5	Room 2	Room 2	Room 3
Block D	Room 3	Room 5	Room 5	Room 2	Room 3
Block E	Room 6	Room 1	Room 5	Room 6	Room 3
Block F	Room 4	Room 2	Room 6	Room 1	Room 5
Block G	Room 4	Room 6	Room 1	Room 1	Room 2
Block H	Room 2	Room 4	Room 3	Room 4	Room 2
Block I	Room 2	Room 4	Room 4	Room 5	Room 4

(a) Make a bar chart showing the total gym time per week for each room.

(b) List the rooms in order from least to greatest total gym time per week.

#### Solution

(a) One way to draw the bar chart is to calculate the total gym time for each room. Then we can use the total times to determine the height of each bar. Remember that each block is 30 minutes long.

For example, if we add up the gym times for Room 1 in minutes we get:

• Monday time: 30 minutes

• Tuesday time: 30 minutes

• Wednesday time: 30 minutes

• Thursday time: 30 + 30 = 60 minutes

• Friday time: 30 minutes

Total time for Room 1 is 30 + 30 + 30 + 60 + 30 = 180 minutes



We could do these calculations for each room to get:

Total time for Room 2 is 60 + 30 + 90 + 60 + 60 = 300 minutes

Total time for Room 3 is 60 + 30 + 30 + 60 + 90 = 270 minutes

Total time for Room 4 is 60 + 60 + 30 + 30 + 30 = 210 minutes

Total time for Room 5 is 30 + 90 + 60 + 30 + 30 = 240 minutes

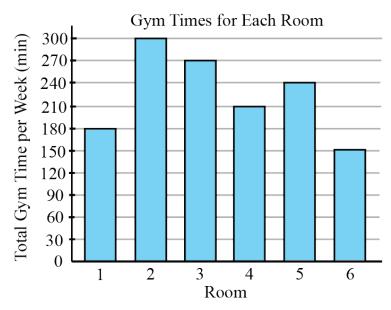
Total time for Room 6 is 30 + 30 + 30 + 30 + 30 = 150 minutes

Another way to build the bar chart is to count the total number of blocks that each class is in the gym. We can keep track of the number of blocks by using a tally chart.

Room	Number of Blocks
1	#
2	## ##
3	##
4	##
5	##
6	##

Then we can set our vertical axis to go up by 30 minute increments so that each tick mark in the bar chart represents one block in the gym.

The completed bar chart is shown.



(b) Looking at the size of the bars in the chart, the rooms in order from least to greatest total gym time per week is:

Room 6, Room 1, Room 4, Room 5, Room 3, Room 2

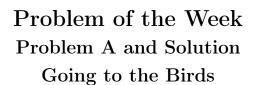


### Problem of the Week Problem A Going to the Birds

The following table shows some information about some of the world's tallest birds. However, the table is incomplete.

Bird	Average Height (in cm)
Mute Swan	
Ostrich	
King Penguin	92
Andean Condor	
Emu	
Southern Cassowary	155
Wandering Albatross	

- (a) Use the following information about the average heights of these birds to fill in the rest of the table:
  - The Andean Condor is 18 cm taller than the King Penguin.
  - The Mute Swan is 55 cm shorter than the Southern Cassowary.
  - The Wandering Albatross is 7 cm taller than the Mute Swan.
  - The Ostrich is 1 m taller than the Andean Condor.
  - The Southern Cassowary is 20 mm taller than the Emu.
- (b) List the birds in order from shortest to tallest.



#### Problem

The following table shows some information about some of the world's tallest birds. However, the table is incomplete.

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  - The Wandering Albatross is 7 cm taller than the Mute Swan.
  - The Ostrich is 1 m taller than the Andean Condor.
  - The Southern Cassowary is 20 mm taller than the Emu.
- (b) List the birds in order from shortest to tallest.

#### Solution

(a) Since the Andean Condor is 18 cm taller than the King Penguin, the Andean Condor is 92 + 18 = 110 cm tall.

Since the Mute Swan is 55 cm shorter than the Southern Cassowary, the Mute Swan is 155 - 55 = 100 cm tall.

Since the Wandering Albatross is 7 cm taller than the Mute Swan, the Wandering Albatross is 100 + 7 = 107 cm tall.

Since the Ostrich is 1 m taller than the Andean Condor, and 1 m = 100 cm, the Ostrich is 110 + 100 = 210 cm tall.

Since the Southern Cassowary is 20 mm taller than the Emu, and 20 mm = 2 cm, the Emu is 155 - 2 = 153 cm tall.

The completed version of the table is:

Bird	Average Height (in cm)
Mute Swan	100
Ostrich	210
King Penguin	92
Andean Condor	110
Emu	153
Southern Cassowary	155
Wandering Albatross	107

(b) We can rank the birds from shortest to tallest using the completed table from part (a). From shortest to tallest, the birds are:

King Penguin, Mute Swan, Wandering Albatross, Andean Condor, Emu, Southern Cassowary, Ostrich



### Problem of the Week Problem A Adding and Subtracting

Follow the steps below.

**Step 1:** Pick two different three-digit numbers. Label the larger number A and the smaller number B.

**Step 2:** Subtract B from 999 and label the difference C.

**Step 3:** Add A and C and label the sum D.

**Step 4:** Subtract 1000 from D and label the difference E.

**Step 5:** Add 1 to E and label the sum F.

- (a) What is the connection between the number F and the numbers A and B?
- (b) Try the same steps with different numbers. Do you think you will always get the same result? Why or why not?















# Problem of the Week Problem A and Solution Adding and Subtracting

#### Problem

Follow the steps below.

**Step 1:** Pick two different three-digit numbers. Label the larger number A and the smaller number B.

**Step 2:** Subtract B from 999 and label the difference C.

**Step 3:** Add A and C and label the sum D.

**Step 4:** Subtract 1000 from D and label the difference E.

**Step 5:** Add 1 to E and label the sum F.

- (a) What is the connection between the number F and the numbers A and B?
- (b) Try the same steps with different numbers. Do you think you will always get the same result? Why or why not?

#### Solution

- (a) We can work through this procedure with any two, random, three-digit numbers. Let's try starting with 814 and 275.
  - **Step 1:** The larger number 814 is labelled A, and the smaller number 275 is labelled B.
  - Step 2: The difference 999 B is 999 275 = 724, and is labelled C.
  - **Step 3:** The sum A + C is 814 + 724 = 1538, and is labelled D.
  - Step 4: The difference D 1000 is 1538 1000 = 538, and is labelled E.
  - **Step 5:** The sum E + 1 is 538 + 1 = 539, and is labelled F.

Notice that 814 - 275 = 539. So A - B = F.

- (b) If you try this procedure with any two three-digit numbers, it will always work out that A B = F. We will show this using algebra.
  - **Step 1:** We choose three-digit numbers A and B so that  $A \geq B$ .
  - **Step 2:** We calculate 999 B, and label this C. That is, C = 999 B.
  - **Step 3:** We add A + C. So, we calculate A + 999 B, and label this D. That is, D = A + 999 B.
  - **Step 4:** We subtract 1000 from D. So, we calculate A + 999 B 1000, and label this E. That is, E = A + 999 B 1000.
  - **Step 5:** We add 1 to *E*. So, we calculate A + 999 B 1000 + 1, and label this *F*. That is, F = A + 999 B 1000 + 1.

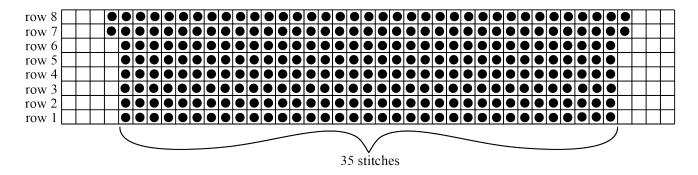
Now, we can simplify this expression to get A - B + 999 - 1000 + 1 = A - B, since 999 - 1000 + 1 = 0. So, F = A - B.



### Problem of the Week Problem A Knitting Patterns

Knitting is a process of making rows of stitches out of yarn. To knit a rectangular shape, the number of stitches in each row is always the same. However, to knit different shapes, the number of stitches in each row can be increased or decreased as you work through the pattern.

A particular pattern starts with 35 stitches in the first row. After every 6 rows, 2 more stitches are added to the row; one at the beginning and one at the end. So, the 7<sup>th</sup> row would have 37 stitches, and the 13<sup>th</sup> row would have 39 stitches. The first 8 rows of the knitting pattern are shown below, with dots representing the stitches. Note that knitting patterns are read from the bottom to the top.



If we follow this pattern, we will create an item that resembles a trapezoid.

- (a) How many stitches will be in the 20<sup>th</sup> row of the pattern?
- (b) The length of our finished item is the distance from the first row to the last row. Our finished item will have a length of 30 cm. If every 6 rows of knitting add 3 cm of length, how many rows will there be in total?
- (c) How many stitches will be in the last row found in part (b)?



ALGEBRA, NUMBER SENSE THEMES

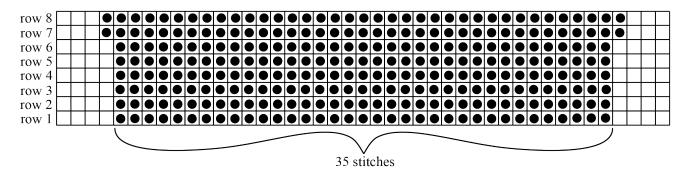


### Problem of the Week Problem A and Solution **Knitting Patterns**

#### Problem

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A particular pattern starts with 35 stitches in the first row. After every 6 rows, 2 more stitches are added to the row; one at the beginning and one at the end. So, the 7<sup>th</sup> row would have 37 stitches, and the 13<sup>th</sup> row would have 39 stitches. The first 8 rows of the knitting pattern are shown below, with dots representing the stitches. Note that knitting patterns are read from the bottom to the top.



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- (a) How many stitches will be in the 20<sup>th</sup> row of the pattern?
- (b) The length of our finished item is the distance from the first row to the last row. Our finished item will have a length of 30 cm. If every 6 rows of knitting add 3 cm of length, how many rows will there be in total?
- (c) How many stitches will be in the last row found in part (b)?





#### Solution

(a) We can make a table showing the number of stitches in each row as we follow the pattern.

Rows	Number of Stitches
1 - 6	35
7 - 12	37
13 - 18	39
19 - 24	41

Since 20 is between 19 and 24, there will be 41 stitches in the 20<sup>th</sup> row.

(b) We can make a table showing the row number and the length of the item after that row has been knitted. Here we will use skip counting and look at the length after every  $6^{\rm th}$  row.

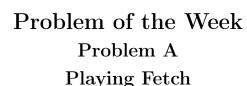
Row	Length (in cm)	
6	3	
12	6	
18	9	
24	12	
30	15	
36	18	
42	21	
48	24	
54	27	
60	30	

After 60 rows, our finished item will have a length of 30 cm.

(c) We can make another table showing the number of stitches in each row, again using skip counting and looking at every 6<sup>th</sup> row up to row 60.

Row	Number of Stitches
6	35
12	37
18	39
24	41
30	43
36	45
42	47
48	49
54	51
60	53

The 60<sup>th</sup> row will have 53 stitches.



Leslie loves playing fetch with her dog Spencer. Spencer always starts by sitting beside Leslie before Leslie throws the ball. When Leslie throws the ball, Spencer runs to it and brings the ball back to the same spot. Leslie throws the ball three times.

- The first time she throws it 8 metres.
- The second time she throws it twice as far as the first time.
- The third time she throws it 5 metres less than the second time.

How far does Spencer run in total?







# Problem of the Week Problem A and Solution Playing Fetch

#### Problem

Leslie loves playing fetch with her dog Spencer. Spencer always starts by sitting beside Leslie before Leslie throws the ball. When Leslie throws the ball, Spencer runs to it and brings the ball back to the same spot. Leslie throws the ball three times.

- The first time she throws it 8 metres.
- The second time she throws it twice as far as the first time.
- The third time she throws it 5 metres less than the second time.

How far does Spencer run in total?

#### Solution

On the first throw, Spencer runs  $2 \times 8 = 16$  metres.

The distance of the second throw is  $8 \times 2 = 16$  metres.

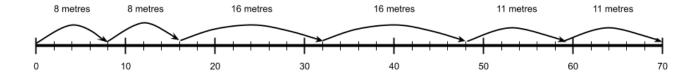
On the second throw, Spencer runs  $2 \times 16 = 32$  metres.

The distance of the third throw is 16 - 5 = 11 metres.

On the third throw, Spencer runs  $2 \times 11 = 22$  metres.

The total distance Spencer runs is: 16 + 32 + 22 = 70 metres.

We can also show the distance Spencer runs on a number line.





### Problem of the Week Problem A Carroll Diagram

Carroll diagrams are tables that you use to sort objects based on two properties. The table has two rows and two columns. The rows are for sorting based on whether or not the one property is present and the columns are for sorting based on whether or not the second property is present. At the intersection of a row and column is a cell that contains objects that have both properties identified by the row and column labels.

Arrange the following numbers into the Carroll diagram below:

12, 63, 15, 150, 56, 30, 55, 21, 88, 72

	Even Number	Odd Number
Multiple of 3	12	
Not a Multiple of 3		

The first number has been placed in the diagram already. The number 12 has been added to the cell that is at the intersection of the 'Even Number' column and the 'Multiple of 3' row.

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### Problem of the Week Problem A and Solution Carroll Diagram

#### Problem

Carroll diagrams are tables that you use to sort objects based on two properties. The table has two rows and two columns. The rows are for sorting based on whether or not the one property is present and the columns are for sorting based on whether or not the second property is present. At the intersection of a row and column is a cell that contains objects that have both properties identified by the row and column labels.

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12, 63, 15, 150, 56, 30, 55, 21, 88, 72

	Even Number	Odd Number
Multiple of 3	12	
Not a Multiple of 3		

The first number has been placed in the diagram already. The number 12 has been added to the cell that is at the intersection of the 'Even Number' column and the 'Multiple of 3' row.

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#### Solution

Here is the completed Carroll diagram:

	Even Number	Odd Number
Multiple of 3	12, 150, 30, 72	63, 15, 21
Not a Multiple of 3	56, 88	55

A quick trick to determine if a number is a multiple of 3 is to calculate the sum of its digits. If the sum of the digits is a multiple of 3, then the original number is also a multiple of 3. For example, for the number 72, the sum of the digits is 7+2=9. Since the sum of 9 is divisible by 3, then 72 is also divisible by 3. Also, for the number 88, the sum of the digits is 8 + 8 = 16. Since 16 is not divisible by 3, then 88 is not divisible by 3.

# Problem of the Week Problem A Deciding About Data

The Webb family is trying to decide on a new monthly internet plan. There are three choices:

Plan A: \$10 for the first 10 GB of data, and each additional 2 GB costs \$5.

Plan B: \$40 for the first 20 GB of data, and each additional 10 GB costs \$10.

Plan C: \$80 for unlimited GB of data.

Note that for Plan A, additional data has to be purchased in 2 GB increments. Similarly, for Plan B, additional data has to be purchased in 10 GB increments.

After keeping track of data used, the family decides they will use between 25 GB and 40 GB of data each month. Which plan should the Webb family choose? Justify your answer.





#### Problem of the Week Problem A and Solution Deciding About Data

#### Problem

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Note that for Plan A, additional data has to be purchased in 2 GB increments. Similarly, for Plan B, additional data has to be purchased in 10 GB increments.

After keeping track of data used, the family decides they will use between 25 GB and 40 GB of data each month. Which plan should the Webb family choose? Justify your answer.

#### Solution

One way to make a decision is to make a table showing how much each plan would cost for various amounts of data use.

Data Used	Total Cost (in \$)	Total Cost (in \$)	Total Cost (in \$)
(in GB)	for Plan A	for Plan B	for Plan C
10	10	40	80
12	15	40	80
14	20	40	80
16	25	40	80
18	30	40	80
20	35	40	80
22	40	50	80
24	45	50	80
26	50	50	80
28	55	50	80
30	60	50	80
32	65	60	80
34	70	60	80
36	75	60	80
38	80	60	80
40	85	60	80

Notice that at 25 GB, with Plan A they would need to get 26 GB of data. This would cost \$50. So, comparing the cost of the plans, we notice that Plan B is the lowest price from 25 GB to 40 GB. Therefore, the family should choose Plan B.

(Note that if the family uses less than 25 GB per month, then Plan A is better. Also, if they use more than 40 GB, then it looks like Plan B is better at first, but Plan C will eventually be better.)

# Problem of the Week Problem A Theatre Trip

The Grade 3 and Grade 4 classes are going on a field trip to the theatre. There are 143 students, 6 teachers, and 16 parent volunteers that will be travelling by school bus to the theatre. If each bus is limited to 45 passengers, what is the fewest number of buses needed for the trip?







### Problem of the Week Problem A and Solution Theatre Trip

#### Problem

The Grade 3 and Grade 4 classes are going on a field trip to the theatre. There are 143 students, 6 teachers, and 16 parent volunteers that will be travelling by school bus to the theatre. If each bus is limited to 45 passengers, what is the fewest number of buses needed for the trip?

#### Solution

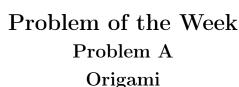
First we need to calculate how many people will be travelling in total.

There is a total of 143 + 6 + 16 = 165 people on the field trip.

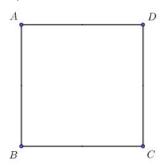
Now we can make a table that shows how many people can be transported on buses that hold 45 people each:

Number of Buses	Number of People
1	45
2	90
3	135
4	180

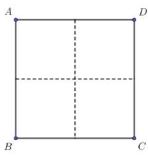
From this table, we see that 3 buses will not hold enough people for the trip, but 4 buses will have enough room for everybody. Therefore, the fewest number of buses needed is 4.



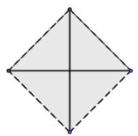
Laila starts with a square piece of paper. Starting at one corner and moving around the square, she labels the corners A, B, C, and D.



Laila folds the paper in half, by folding side AB onto side DC, to form a rectangle. She opens up the paper and folds it again to form another rectangle by folding side AD onto side BC. When she opens up the paper this time, she sees two creases in the paper as shown below.

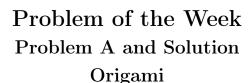


The centre of the square is the point where the two creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another smaller square made up of four triangular regions as shown below.



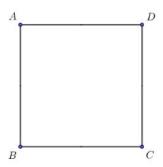
What fraction of the area of the original square is the area of this smaller square? Justify your answer.

THEMES GEOMETRY, NUMBER SENSE

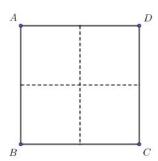


#### Problem

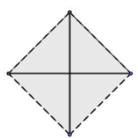
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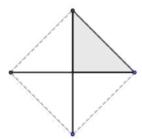


What fraction of the area of the original square is the area of this smaller square? Justify your answer.

#### Solution

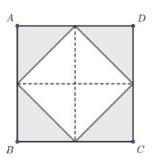
The smaller square has an area that is  $\frac{1}{2}$  the area of the original square. A justification of this is given below.

Consider the following image of the smaller square with only one of the four triangular regions shaded.



Underneath the shaded triangle is a region of the original square that is exactly the same size. That is true for all four of the triangles that were formed by having the corners meet at the centre of the original square.

Let's now shade all four of the triangular regions in the smaller square, and then open up the paper again.



We can make the following observations:

- The shaded triangles each have a matching unshaded triangle.
- The shaded triangles make up the area of the smaller square.
- The area of the original square is equal to the area of the shaded triangles plus the area of the smaller square. This means the area of the original square is 2 times the area of the smaller square.

It follows that the area of the smaller square is  $\frac{1}{2}$  the area of the original square.

# Problem of the Week Problem A Candy Store Counting

Ricardo and Nadia went to the candy store. The clues below describe the types and amount of candies they bought.

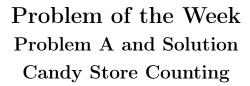
- They bought 8 packages of Sunbursts.
- They bought 4 more packages of Twinglers than Sunbursts.
- They bought 6 fewer packages of Nerts than Twinglers.
- They bought twice as many packages of W&Ws than Nerts.
- The number of packages of Sweet Patch Kids is equal to the total number of packages of Sunbursts and Nerts.
- The number of packages of Jolly Farmers is equal to the number of packages of Sweet Patch Kids minus the number of packages of Twinglers.

Complete the pictograph to show how many packages of each type of candy they purchased. Use one to represent two packages of candy. The first row has already been completed for you.

Type of Candy	Number of Packages	
Sunbursts		
Twinglers		
Nerts		
W&Ws		
Sweet Patch Kids		
Jolly Farmers		

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THEMES DATA MANAGEMENT, NUMBER SENSE



#### Problem

Ricardo and Nadia went to the candy store. The clues below describe the types and amount of candies they bought.

- They bought 8 packages of Sunbursts.
- They bought 4 more packages of Twinglers than Sunbursts.
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Complete the pictograph to show how many packages of each type of candy they purchased. Use one to represent two packages of candy. The first row has already been completed for you.

Type of Candy	Number of Packages
Sunbursts	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
Twinglers	
Nerts	
W&Ws	
Sweet Patch Kids	
Jolly Farmers	

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#### Solution

First we need to calculate how many packages of each kind of candy they bought. We know they bought 8 packages of Sunbursts.

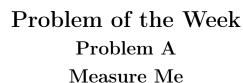
- Buying 4 more packages of Twinglers than Sunbursts means they bought 8+4=12 packages of Twinglers.
- Buying 6 fewer packages of Nerts than Twinglers means they bought 12-6=6 packages of Nerts.
- Buying twice as many packages of W&Ws than Nerts means they bought  $6 \times 2 = 12$  packages of W&Ws.
- The number of packages of Sweet Patch Kids they bought is 8 + 6 = 14.
- The number of packages of Jolly Farmers they bought is 14 12 = 2.

To complete the pictograph, we use to represent two packages of candy. So we divide the number of packages for each type of candy by 2, in order to calculate the number of needed for that candy in the pictograph.

The completed pictograph is shown below.

Type of Candy	Nun	nber o	of Pac	kages			
Sunbursts	65	65	65	65			
Twinglers	65	65	65	65	60	65	
Nerts	65	66	66				
W&Ws	60	66	66	60	65	60	
Sweet Patch Kids	65	65	65	65	65	65	65
Jolly Farmers	65						

**Key:** one represents 2 packages of candy.



Arezoo, Femke, Jackson, Nya, and Elisabeth made a table that recorded their heights at different stages of their lives.

Name	Height at Birth	Height at Age 6	Adult Height
Arezoo	44 cm	106 cm	1.51 m
Femke	47 cm	104 cm	153 cm
Jackson	49 cm	108 cm	2 m
Nya	48 cm	1.11 m	189 cm
Elisabeth	52 cm	1 m and 19 cm	1.84 m

- (a) Who grew the most between birth and age 6? Who grew the least between birth and age 6? Justify your answers.
- (b) Using estimation, arrange the list of friends from least growth to most growth from age 6 to adulthood.







# Problem of the Week Problem A and Solution Measure Me

#### Problem

Arezoo, Femke, Jackson, Nya, and Elisabeth made a table that recorded their heights at different stages of their lives.

Name	Height at Birth	Height at Age 6	Adult Height
Arezoo	44 cm	$106 \mathrm{\ cm}$	1.51 m
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Nya	48 cm	1.11 m	189 cm
Elisabeth	52 cm	1 m and 19 cm	1.84 m

- (a) Who grew the most between birth and age 6? Who grew the least between birth and age 6? Justify your answers.
- (b) Using estimation, arrange the list of friends from least growth to most growth from age 6 to adulthood.

#### Solution

(a) One way to solve this is to convert all of the measurements into centimetres and then find the difference between each person's height at age 6 and at birth.

Name	Height at Birth	Height at Age 6	Height Difference
Arezoo	44 cm	106 cm	106 - 44 = 62  cm
Femke	47 cm	104 cm	104 - 47 = 57  cm
Jackson	49 cm	108 cm	108 - 49 = 59  cm
Nya	48 cm	111 cm	111 - 48 = 63  cm
Elisabeth	52 cm	119 cm	119 - 52 = 67  cm

Elisabeth grew the most between birth and age 6.

Femke grew the least between birth and age 6.

(b) We will estimate the growth of each person by rounding numbers to the nearest 10 cm.

Name	Approximate Height at Age 6	Approximate Adult Height	Approximate Difference
Arezoo	110 cm	150 cm	150 - 110 = 40  cm
Femke	100 cm	150 cm	150 - 100 = 50  cm
Jackson	110 cm	200 cm	200 - 110 = 90  cm
Nya	110 cm	190 cm	190 - 110 = 80  cm
Elisabeth	120 cm	180 cm	180 - 120 = 60  cm

When we list the friends in order by approximate growth between age 6 and adulthood, from least growth to most growth, we get:

Arezoo, Femke, Elisabeth, Nya, and Jackson

It turns out that if we determine the actual difference between each person's adult height and height at age 6, and then list the friends in order from least growth to most growth, we would get the same ordering. However, this will not always be the case. For example, if Arezoo's adult height had been 1.54 m and Femke's adult height had been 151 cm, then the order when approximating would have been the same as above, but the actual order would have been Femke, Arezoo, Elisabeth, Nya, Jackson.

# Problem of the Week Problem A Aquarium Issues

Jacques got an aquarium for his birthday and wants to fill it with some guppies and tetras. Guppies are sold in groups of four and tetras are sold in groups of three. He can buy four guppies for \$12, and three tetras for \$7.

If Jacques spent exactly \$50 on fish, how many of each fish did he buy?







# Problem of the Week Problem A and Solution Aquarium Issues

#### Problem

Jacques got an aquarium for his birthday and wants to fill it with some guppies and tetras. Guppies are sold in groups of four and tetras are sold in groups of three. He can buy four guppies for \$12, and three tetras for \$7.

If Jacques spent exactly \$50 on fish, how many of each fish did he buy?

#### Solution

Since 50 is not a multiple of 12 or a multiple of 7, then Jacques must have bought some of each fish in order to spend exactly \$50. One way to solve this problem is to make a table to keep track of how much money it costs for multiples of four guppies, and how much of the \$50 would be left to buy tetras.

Number of	Cost for Guppies	Money Leftover
Guppies	(in \$)	for Tetras (in \$)
4	$1 \times 12 = 12$	50 - 12 = 48
8	$2 \times 12 = 24$	50 - 24 = 26
12	$3 \times 12 = 36$	50 - 36 = 14
16	$4 \times 12 = 48$	50 - 48 = 2

Now we look for a multiple of seven in the leftover money, because each group of tetras costs \$7. The only multiple of seven in the leftover money is 14. Since  $14 = 2 \times 7$ , Jacques must have bought two groups of tetras.

This means Jacques bought 12 guppies and  $2 \times 3 = 6$  tetras.



### Problem of the Week Problem A Crafty Construction

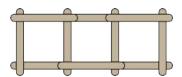
Sam is making square picture frames using popsicle sticks. He has one box of 50 popsicle sticks and wants to make 13 frames.

(a) Sam plans to make individual frames using four popsicle sticks as shown.



Will he have enough popsicle sticks to make 13 frames? Justify your answer.

(b) Sam changes his mind and instead of individual frames, he decides to connect the frames in a row. He starts by making a frame using four popsicle sticks, and then uses three popsicle sticks to create another frame attached to this frame. He then uses three more popsicle sticks to create another frame attached to the first two frames, so he has three connected frames, as shown.



Sam plans to continue this process, using three more populate sticks for each frame, until he has 13 frames connected in a row. Will he have enough popsicle sticks to make 13 frames? Justify your answer.

- (c) Can you draw another layout of the 13 frames that Sam could have built using at most 50 popsicle sticks? How many popsicle sticks does your layout use?
- (d) Can you draw a layout of the 13 frames that uses fewer than 35 popsicle sticks?



### Problem of the Week Problem A and Solution Crafty Construction

#### Problem

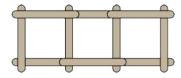
Sam is making square picture frames using popsicle sticks. He has one box of 50 popsicle sticks and wants to make 13 frames.

(a) Sam plans to make individual frames using four popsicle sticks as shown.



Will he have enough popsicle sticks to make 13 frames? Justify your answer.

(b) Sam changes his mind and instead of individual frames, he decides to connect the frames in a row. He starts by making a frame using four popsicle sticks, and then uses three popsicle sticks to create another frame attached to this frame. He then uses three more popsicle sticks to create another frame attached to the first two frames, so he has three connected frames, as shown.



Sam plans to continue this process, using three more popsicle sticks for each frame, until he has 13 frames connected in a row. Will he have enough popsicle sticks to make 13 frames? Justify your answer.

- (c) Can you draw another layout of the 13 frames that Sam could have built using at most 50 popsicle sticks? How many popsicle sticks does your layout use?
- (d) Can you draw a layout of the 13 frames that uses fewer than 35 popsicle sticks?

#### Solution

(a) Since each frame requires 4 popsicle sticks, the total number of popsicle sticks required for 13 frames is equal to  $13 \times 4 = 52$ .

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

Number of Frames	Number of Popsicle Sticks
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to make 13 individual frames, Sam will not have enough popsicle sticks to make all the frames.

(b) The first frame uses 4 popsicle sticks but the other 12 frames use only 3 popsicle sticks each. So the total number of popsicle sticks required can be found by adding 4 to  $12 \times 3$ . Since  $12 \times 3 = 36$ , this gives a total of 4 + 36 = 40 popsicle sticks.

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

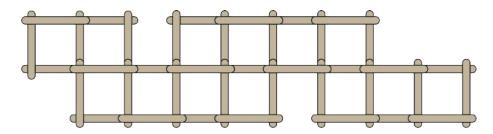
Number of Frames	Number of Popsicle Sticks
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

Either way, since it will take 40 popsicle sticks to build a row of 13 connected frames, Sam will have enough popsicle sticks to make all the frames.

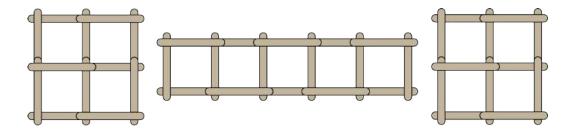


(c) There are many layouts of 13 frames you could make using at most 50 popsicle sticks. Two examples are shown.

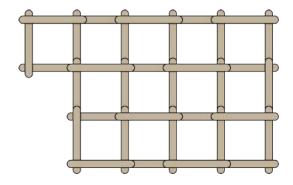
The following layout uses 39 popsicle sticks.



The following layout uses 40 popsicle sticks.



(d) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks, as shown.



### Geometry & Measurement (G)





### Problem of the Week Problem A **Patio Planning**

Jody and Jillian are building a patio beside their house. They have square patio stones that have sides with a length of 50 cm. They have a total 75 patio stones available.

To build the patio, they place the patio stones so that two stones next to each other are touching and their sides are lined up exactly. They will only use whole patio stones and they want the entire patio covered in stones.

- (a) If they want to have their patio in the shape of a square, what are the dimensions of the largest possible patio they can build?
- (b) If they want to build a patio in the shape of a rectangle where the length of one side is twice as long as the length of the other side, what are the dimensions of the largest patio they can build?







### Problem of the Week Problem A and Solution Patio Planning

#### **Problem**

Jody and Jillian are building a patio beside their house. They have square patio stones that have sides with a length of 50 cm. They have a total 75 patio stones available.

To build the patio, they place the patio stones so that two stones next to each other are touching and their sides are lined up exactly. They will only use whole patio stones and they want the entire patio covered in stones.

- (a) If they want to have their patio in the shape of a square, what are the dimensions of the largest possible patio they can build?
- (b) If they want to build a patio in the shape of a rectangle where the length of one side is twice as long as the length of the other side, what are the dimensions of the largest patio they can build?

#### Solution

(a) The following table shows the possible sizes of the patio if the patio is in the shape of a square.

Length	Width	Number of Stones Used
1	1	$1 \times 1 = 1$
2	2	$2 \times 2 = 4$
3	3	$3 \times 3 = 9$
4	4	$4 \times 4 = 16$
5	5	$5 \times 5 = 25$
6	6	$6 \times 6 = 36$
7	7	$7 \times 7 = 49$
8	8	$8 \times 8 = 64$
9	9	$9 \times 9 = 81$

Since there are only 75 patio stones available, the biggest square patio that Jody and Jillian can build is 8 patio stones long by 8 patio stones wide. Since each square patio stone has sides with length 50 cm, then 8 stones in a line have a length of  $8 \times 50 = 400$  cm or 4 metres. So the largest square patio made out of 75 or fewer patio stones has dimensions 4 meters long by 4 metres wide.

(b) To investigate a patio that has one side that is twice as long as the other side, we can name the shorter side Length and the longer side Width. The following table shows the possible sizes of the patio.

Length	Width	Number of Stones Used
1	2	$1 \times 2 = 2$
2	4	$2 \times 4 = 8$
3	6	$3 \times 6 = 18$
4	8	$4 \times 8 = 32$
5	10	$5 \times 10 = 50$
6	12	$6 \times 12 = 72$
7	14	$7 \times 14 = 98$

Since we only have 75 patio stones, the biggest patio Jody and Jillian can build that has one side twice as long as the other side is a rectangle that is 6 patio stones long by 12 patio stones wide. Since each square patio stone has sides with length 50 cm, then 6 stones in a line have a length of  $6 \times 50 = 300$  cm or 3 metres, and 12 stones in a line have a length of  $12 \times 50 = 600$  cm or 6 metres. So the largest patio with one side twice as long as the other side and made out of 75 or fewer patio stones, has dimensions 3 metres by 6 metres.

# Problem of the Week Problem A Perfect Punch

Su is going to make punch for her friends. She wants to mix 3 L of orange juice, 1 L of pop,  $\frac{1}{2}$  L of grape juice, and 300 mL of cranberry juice in a punch bowl.

- (a) To avoid spilling, Su plans to use a punch bowl with a capacity of at least 200 mL more than the liquid it holds. What is the smallest capacity that her punch bowl should have?
- (b) Su has cups that can each hold 300 mL of punch. How many of these cups can she fill with the punch she makes?







# Problem of the Week Problem A and Solution Perfect Punch

#### Problem

Su is going to make punch for her friends. She wants to mix 3 L of orange juice, 1 L of pop,  $\frac{1}{2}$  L of grape juice, and 300 mL of cranberry juice in a punch bowl.

- (a) To avoid spilling, Su plans to use a punch bowl with a capacity of at least 200 mL more than the liquid it holds. What is the smallest capacity that her punch bowl should have?
- (b) Su has cups that can each hold 300 mL of punch. How many of these cups can she fill with the punch she makes?

#### Solution

(a) The smallest capacity of the punch bowl is the sum of the volumes of each liquid, plus the 200 mL of extra space to avoid spilling.

One way to calculate this would be to convert all the volumes to millilitres.

- 3 L = 3000 mL
- 1 L = 1000 mL
- $\frac{1}{2}$  L = 500 mL

So the minimum capacity is 3000 + 1000 + 500 + 300 + 200 = 5000 mL.

Alternatively, we might notice that the sum of the volume of cranberry juice and the extra room for spillage is: 300 + 200 = 500 mL or  $\frac{1}{2}$  L.

So the minimum capacity is  $3 + 1 + \frac{1}{2} + \frac{1}{2} = 5$  L.

(b) The volume of punch is 3000 + 1000 + 500 + 300 = 4800 mL.

We can use skip counting to figure out how many cups of punch Su can fill: 300, 600, 900, 1200, 1500, 1800, 2100, 2400, 2700, 3000, 3300, 3600, 3900, 4200, 4500, 4800

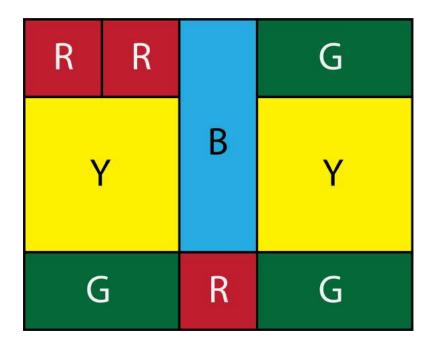
We can see from this that Su can fill 16 cups with punch.

Alternatively, we can calculate the number of cups of punch by dividing  $4800 \div 300 = 16$  cups.



# Problem of the Week Problem A Tricky Tiles

Arvin has red, yellow, green, and blue tiles. The red and yellow tiles are squares, and the green and blue tiles are rectangles. All tiles of the same colour have the same dimensions. Arvin places his tiles next to each other in order to create the following rectangle.



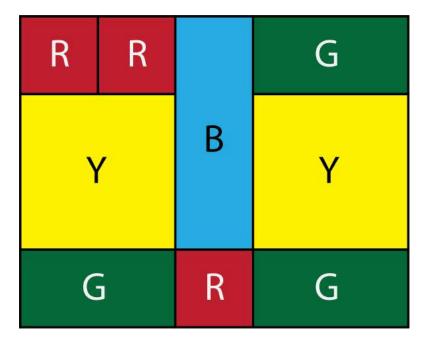
If Arvin wanted to use only red squares to make a rectangle with the same dimensions, how many red squares would be need?



### Problem of the Week Problem A and Solution Tricky Tiles

#### **Problem**

Arvin has red, yellow, green, and blue tiles. The red and yellow tiles are squares, and the green and blue tiles are rectangles. All tiles of the same colour have the same dimensions. Arvin places his tiles next to each other in order to create the following rectangle.



If Arvin wanted to use only red squares to make a rectangle with the same dimensions, how many red squares would be need?

#### Solution

We can see from the diagram that the side length of a yellow square is twice the side length of a red square. This means that if we arrange four red squares, two across and two down, we will form a square with the same dimensions as a yellow square.

We also see from the diagram that the shorter side of a green rectangle is the same length as the side length a red square, and the longer side of a green rectangle is the same length as the side length of a yellow square. Since the side length of a yellow square is twice the side length of a red square, if we arrange two red squares beside each other, we will form a rectangle with the same dimensions as a green rectangle.

From the diagram we see that the shorter side of a blue rectangle is the same length as the side length a red square, and the longer side of a blue rectangle is the same length as the sum of the side length of a yellow square plus the side length of a red square. Since the side length of a yellow square is twice the side length of a red square, if we arrange three red squares in a line, we will form a rectangle with the same dimensions as a blue rectangle.

#### In summary:

- 1 yellow square = 4 red squares
- 1 green rectangle = 2 red squares
- 1 blue rectangle = 3 red squares

We can create a rectangle with the same dimensions as shown in the diagram by replacing each yellow, green, and blue tile with the equivalent number of red squares.

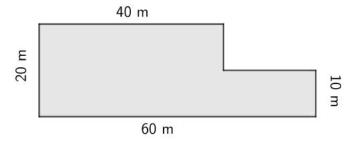
Tile Colour	Number in Diagram	Equivalent Number of Red Squares
yellow	2	$2 \times 4 = 8$
green	3	$3 \times 2 = 6$
blue	1	$1 \times 3 = 3$

Including the three red squares in the original diagram, we see that the rectangle can be formed using 8+6+3+3=20 red squares.

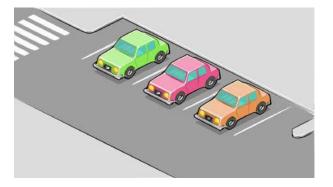


### Problem of the Week Problem A Perimeter and Area

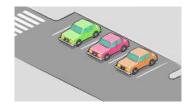
The diagram below shows the shape, and some of the dimensions, of an L-shaped parking lot. All angles in the diagram are right angles.



- (a) What is the perimeter of the parking lot?
- (b) What is the area of the parking lot?



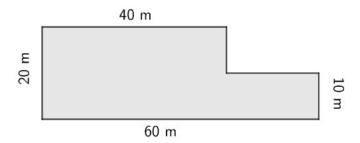




# Problem of the Week Problem A and Solution Perimeter and Area

#### Problem

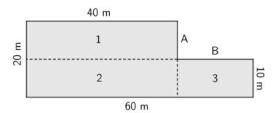
The diagram below shows the shape, and some of the dimensions, of an L-shaped parking lot. All angles in the diagram are right angles.



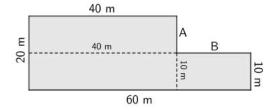
- (a) What is the perimeter of the parking lot?
- (b) What is the area of the parking lot?

#### Solution

To determine the perimeter and area of the parking lot, we add two dashed lines that extend the sides labelled A and B below. This divides the L-shaped lot into three rectangles, labelled 1, 2, and 3, as shown below.

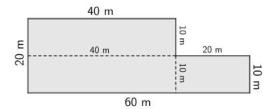


Since all angles are right angles, we know that the dashed line that is an extension of side B is 40 m in length, and the dashed line that is an extension of side A is 10 m in length.





We can now see that the length of side A is 20 - 10 = 10 m, and the length of side B is 60 - 40 = 20 m. A diagram with all of these dimensions included is shown below.



We now have enough information to calculate the perimeter and area of the parking lot.

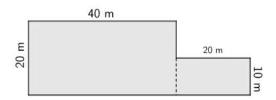
- (a) The perimeter of the parking lot is 20 + 40 + 10 + 20 + 10 + 60 = 160 m.
- (b) The area of the parking lot is equal to the sum of the areas of the three rectangles, rectangles 1, 2, and 3.

Rectangles 1 and 2 are each 40 m by 10 m. Therefore, rectangles 1 and 2 each have an area of  $40 \times 10 = 400$  m<sup>2</sup>.

Rectangle 3 is 20 m by 10 m. Therefore, rectangle 3 has an area of  $20 \times 10 = 200 \text{ m}^2$ .

Therefore, the total area of the parking lot is  $400 + 400 + 200 = 1000 \text{ m}^2$ .

Alternatively, we could divide the diagram into two parts, a 40 m by 20 m rectangle and a 20 m by 10 m rectangle.



The area of the 40 m by 20 m rectangle is  $40 \times 20 = 800 \text{ m}^2$ .

The area of the 20 m by 10 m rectangle is  $20 \times 10 = 200 \text{ m}^2$ .

Therefore, the total area of the parking lot is  $800 + 200 = 1000 \text{ m}^2$ .

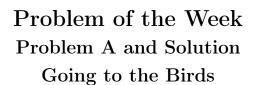


### Problem of the Week Problem A Going to the Birds

The following table shows some information about some of the world's tallest birds. However, the table is incomplete.

Bird	Average Height (in cm)
Mute Swan	
Ostrich	
King Penguin	92
Andean Condor	
Emu	
Southern Cassowary	155
Wandering Albatross	

- (a) Use the following information about the average heights of these birds to fill in the rest of the table:
  - The Andean Condor is 18 cm taller than the King Penguin.
  - The Mute Swan is 55 cm shorter than the Southern Cassowary.
  - The Wandering Albatross is 7 cm taller than the Mute Swan.
  - The Ostrich is 1 m taller than the Andean Condor.
  - The Southern Cassowary is 20 mm taller than the Emu.
- (b) List the birds in order from shortest to tallest.



#### Problem

The following table shows some information about some of the world's tallest birds. However, the table is incomplete.

Bird	Average Height (in cm)
Mute Swan	
Ostrich	
King Penguin	92
Andean Condor	
Emu	
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Wandering Albatross	

- (a) Use the following information about the average heights of these birds to fill in the rest of the table:
  - The Andean Condor is 18 cm taller than the King Penguin.
  - The Mute Swan is 55 cm shorter than the Southern Cassowary.
  - The Wandering Albatross is 7 cm taller than the Mute Swan.
  - The Ostrich is 1 m taller than the Andean Condor.
  - The Southern Cassowary is 20 mm taller than the Emu.
- (b) List the birds in order from shortest to tallest.

#### Solution

(a) Since the Andean Condor is 18 cm taller than the King Penguin, the Andean Condor is 92 + 18 = 110 cm tall.

Since the Mute Swan is 55 cm shorter than the Southern Cassowary, the Mute Swan is 155 - 55 = 100 cm tall.

Since the Wandering Albatross is 7 cm taller than the Mute Swan, the Wandering Albatross is 100 + 7 = 107 cm tall.

Since the Ostrich is 1 m taller than the Andean Condor, and 1 m = 100 cm, the Ostrich is 110 + 100 = 210 cm tall.

Since the Southern Cassowary is 20 mm taller than the Emu, and 20 mm = 2 cm, the Emu is 155 - 2 = 153 cm tall.

The completed version of the table is:

Bird	Average Height (in cm)
Mute Swan	100
Ostrich	210
King Penguin	92
Andean Condor	110
Emu	153
Southern Cassowary	155
Wandering Albatross	107

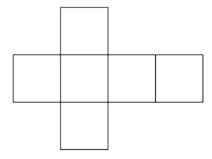
(b) We can rank the birds from shortest to tallest using the completed table from part (a). From shortest to tallest, the birds are:

King Penguin, Mute Swan, Wandering Albatross, Andean Condor, Emu, Southern Cassowary, Ostrich

# Problem of the Week Problem A Navid's Nets

When Navid recycles boxes, he flattens them. Since he is learning about nets in school, he notices there are many different ways to flatten a cube-shaped box into a net.

A net is a pattern that can be cut out and then folded together to create a 3D shape. For example, consider the net shown.



When the net is folded together, it makes a cube.



Navid draws all the possible nets for a cube and finds that there are eleven different nets. Draw as many of the nets as you can. Can you draw them all?



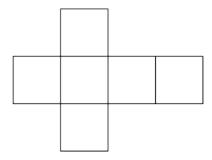


# Problem of the Week Problem A and Solution Navid's Nets

#### Problem

When Navid recycles boxes, he flattens them. Since he is learning about nets in school, he notices there are many different ways to flatten a cube-shaped box into a net.

A net is a pattern that can be cut out and then folded together to create a 3D shape. For example, consider the net shown.



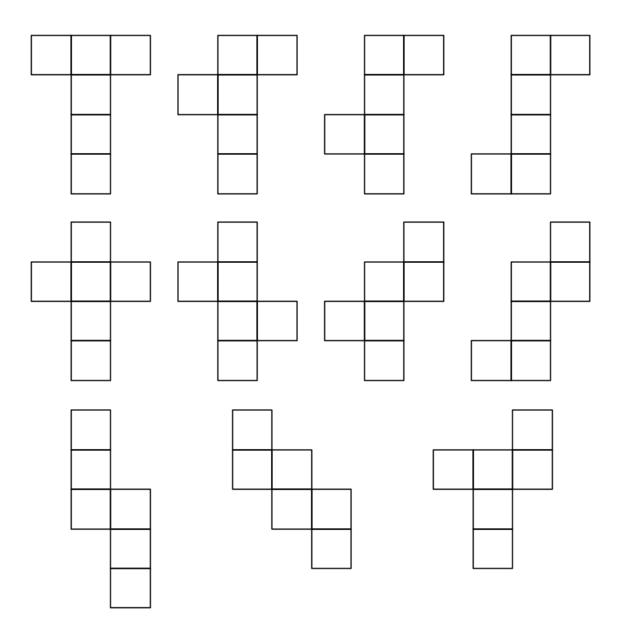
When the net is folded together, it makes a cube.

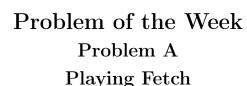


Navid draws all the possible nets for a cube and finds that there are eleven different nets. Draw as many of the nets as you can. Can you draw them all?

### Solution

The eleven possible nets for a cube are shown. Note that the nets are all considered different because for each net, flipping or rotating the net does not produce any of the other nets.





Leslie loves playing fetch with her dog Spencer. Spencer always starts by sitting beside Leslie before Leslie throws the ball. When Leslie throws the ball, Spencer runs to it and brings the ball back to the same spot. Leslie throws the ball three times.

- The first time she throws it 8 metres.
- The second time she throws it twice as far as the first time.
- The third time she throws it 5 metres less than the second time.

How far does Spencer run in total?







# Problem of the Week Problem A and Solution Playing Fetch

#### Problem

Leslie loves playing fetch with her dog Spencer. Spencer always starts by sitting beside Leslie before Leslie throws the ball. When Leslie throws the ball, Spencer runs to it and brings the ball back to the same spot. Leslie throws the ball three times.

- The first time she throws it 8 metres.
- The second time she throws it twice as far as the first time.
- The third time she throws it 5 metres less than the second time.

How far does Spencer run in total?

#### Solution

On the first throw, Spencer runs  $2 \times 8 = 16$  metres.

The distance of the second throw is  $8 \times 2 = 16$  metres.

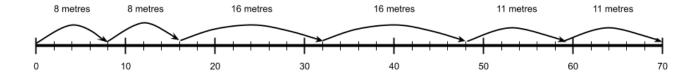
On the second throw, Spencer runs  $2 \times 16 = 32$  metres.

The distance of the third throw is 16 - 5 = 11 metres.

On the third throw, Spencer runs  $2 \times 11 = 22$  metres.

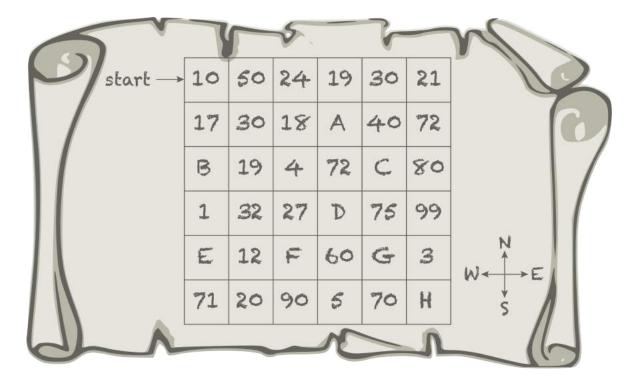
The total distance Spencer runs is: 16 + 32 + 22 = 70 metres.

We can also show the distance Spencer runs on a number line.



# Problem of the Week Problem A Treasure Map

Robbie found a treasure map that contains a grid filled with numbers and letters.



The treasure hunt starts in the northwest corner of the grid, on the square that contains the number 10. The squares with numbers are used to determine which way to go, using the following rules.

- If the number is even and a multiple of 5, then move one square east.
- If the number is even, but not a multiple of 5, then move one square south.
- If the number is a multiple of 5, but not even, then move one square north.
- If the number is neither even nor a multiple of 5, then move one square west.

The squares with letters represent possible locations of the treasure. If Robbie follows the rules, he will follow a path that reaches a square with a letter, which is where the treasure is hidden. Where is the treasure hidden?

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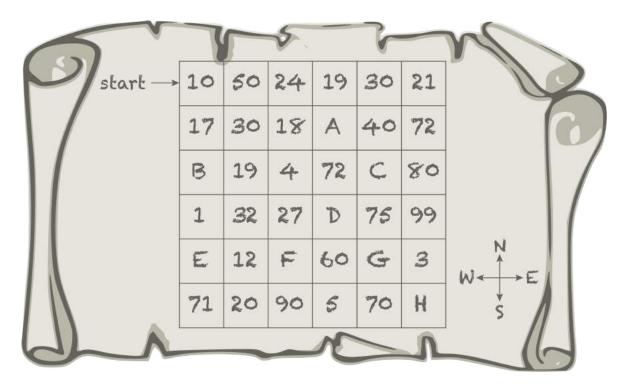
THEMES COMPUTATIONAL THINKING, GEOMETRY



#### Problem of the Week Problem A and Solution Treasure Map

#### Problem

Robbie found a treasure map that contains a grid filled with numbers and letters.



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The squares with letters represent possible locations of the treasure. If Robbie follows the rules, he will follow a path that reaches a square with a letter, which is where the treasure is hidden. Where is the treasure hidden?

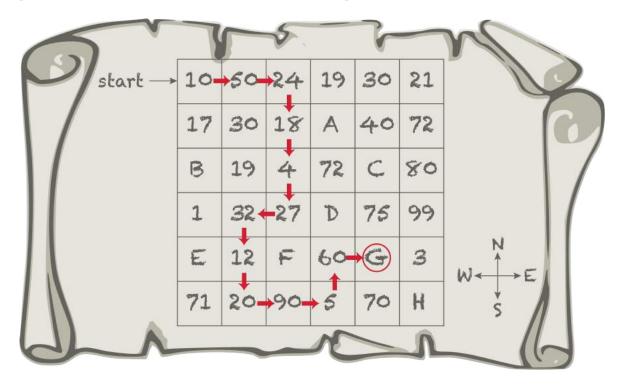
Not printing this page? You can use our interactive worksheet.

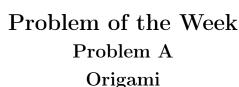
#### Solution

Starting in the northwest corner of the map, here are the steps Robbie would follow to find the hidden treasure.

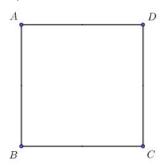
- 10 is an even number and a multiple of 5, so move one square east.
- 50 is an even number and a multiple of 5, so move one square east again.
- 24 is an even number but not a multiple of 5, so move one square south.
- 18 is an even number but not a multiple of 5, so move one square south again.
- 4 is an even number but not a multiple of 5, so move one square south again.
- 27 is neither even nor a multiple of 5, so move one square west.
- 32 is an even number but not a multiple of 5, so move one square south.
- 12 is an even number but not a multiple of 5, so move one square south again.
- 20 is an even number and a multiple of 5, so move one square east.
- 90 is an even number and a multiple of 5, so move one square east again.
- 5 is a multiple of 5 but not an even number, so move one square north.
- 60 is an even number and a multiple of 5, so move one square east.
- Robbie has reached a square with a letter, so the treasure is hidden in location G.

The path to the hidden treasure is shown on the map below.

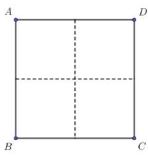




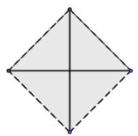
Laila starts with a square piece of paper. Starting at one corner and moving around the square, she labels the corners A, B, C, and D.



Laila folds the paper in half, by folding side AB onto side DC, to form a rectangle. She opens up the paper and folds it again to form another rectangle by folding side AD onto side BC. When she opens up the paper this time, she sees two creases in the paper as shown below.

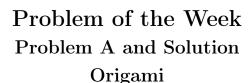


The centre of the square is the point where the two creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another smaller square made up of four triangular regions as shown below.



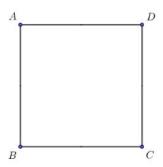
What fraction of the area of the original square is the area of this smaller square? Justify your answer.

THEMES GEOMETRY, NUMBER SENSE

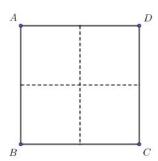


#### Problem

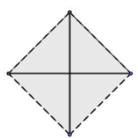
Laila starts with a square piece of paper. Starting at one corner and moving around the square, she labels the corners A, B, C, and D.



Laila folds the paper in half, by folding side AB onto side DC, to form a rectangle. She opens up the paper and folds it again to form another rectangle by folding side AD onto side BC. When she opens up the paper this time, she sees two creases in the paper as shown below.



The centre of the square is the point where the two creases intersect. Now, she takes each corner of the square and folds the paper so that each corner touches the centre of the square. Folding all four corners in this way forms another smaller square made up of four triangular regions as shown below.

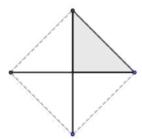


What fraction of the area of the original square is the area of this smaller square? Justify your answer.

#### Solution

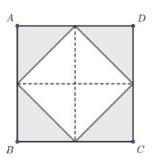
The smaller square has an area that is  $\frac{1}{2}$  the area of the original square. A justification of this is given below.

Consider the following image of the smaller square with only one of the four triangular regions shaded.



Underneath the shaded triangle is a region of the original square that is exactly the same size. That is true for all four of the triangles that were formed by having the corners meet at the centre of the original square.

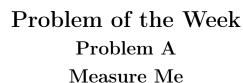
Let's now shade all four of the triangular regions in the smaller square, and then open up the paper again.



We can make the following observations:

- The shaded triangles each have a matching unshaded triangle.
- The shaded triangles make up the area of the smaller square.
- The area of the original square is equal to the area of the shaded triangles plus the area of the smaller square. This means the area of the original square is 2 times the area of the smaller square.

It follows that the area of the smaller square is  $\frac{1}{2}$  the area of the original square.



Arezoo, Femke, Jackson, Nya, and Elisabeth made a table that recorded their heights at different stages of their lives.

Name	Height at Birth	Height at Age 6	Adult Height
Arezoo	44 cm	106 cm	1.51 m
Femke	47 cm	104 cm	153 cm
Jackson	49 cm	108 cm	2 m
Nya	48 cm	1.11 m	189 cm
Elisabeth	52 cm	1 m and 19 cm	1.84 m

- (a) Who grew the most between birth and age 6? Who grew the least between birth and age 6? Justify your answers.
- (b) Using estimation, arrange the list of friends from least growth to most growth from age 6 to adulthood.







# Problem of the Week Problem A and Solution Measure Me

#### Problem

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- (a) Who grew the most between birth and age 6? Who grew the least between birth and age 6? Justify your answers.
- (b) Using estimation, arrange the list of friends from least growth to most growth from age 6 to adulthood.

#### Solution

(a) One way to solve this is to convert all of the measurements into centimetres and then find the difference between each person's height at age 6 and at birth.

Name	Height at Birth	Height at Age 6	Height Difference
Arezoo	44 cm	106 cm	106 - 44 = 62  cm
Femke	47 cm	104 cm	104 - 47 = 57  cm
Jackson	49 cm	108 cm	108 - 49 = 59  cm
Nya	48 cm	111 cm	111 - 48 = 63  cm
Elisabeth	52 cm	119 cm	119 - 52 = 67  cm

Elisabeth grew the most between birth and age 6.

Femke grew the least between birth and age 6.

(b) We will estimate the growth of each person by rounding numbers to the nearest 10 cm.

Name	Approximate Height at Age 6	Approximate Adult Height	Approximate Difference
Arezoo	110 cm	150 cm	150 - 110 = 40  cm
Femke	100 cm	150 cm	150 - 100 = 50  cm
Jackson	110 cm	200 cm	200 - 110 = 90  cm
Nya	110 cm	190 cm	190 - 110 = 80  cm
Elisabeth	120 cm	180 cm	180 - 120 = 60  cm

When we list the friends in order by approximate growth between age 6 and adulthood, from least growth to most growth, we get:

Arezoo, Femke, Elisabeth, Nya, and Jackson

It turns out that if we determine the actual difference between each person's adult height and height at age 6, and then list the friends in order from least growth to most growth, we would get the same ordering. However, this will not always be the case. For example, if Arezoo's adult height had been 1.54 m and Femke's adult height had been 151 cm, then the order when approximating would have been the same as above, but the actual order would have been Femke, Arezoo, Elisabeth, Nya, Jackson.

# Algebra (A)





## Problem of the Week Problem A

### **International Jumping Relay**

The International Jumping Relay Race will take place this year. In the race, each team has four members. The first member of each team starts the race and jumps until they tag the next member of their team. When a team member is tagged, they start jumping towards the next team member to tag them, except for the last team member who starts jumping towards the finish line.

BeHappy Bunny and Kicky Kangaroo are on different teams and will each be jumping in the last leg of the race. They are waiting 24 m away from the finish line for their teammates to tag them.

BeHappy Bunny is tagged first, and jumps three times before Kicky Kangaroo is tagged. Bunny's fourth jump and Kangaroo's first jump happen at the same time. BeHappy Bunny leaps 2 m forward with every jump, Kicky Kangaroo leaps 3 m forward with every jump, and the two animals take off and land at the same time on each jump.

- (a) Which animal will get to the finish line first? Justify your answer.
- (b) Will the two animals ever land at the same spot (not necessarily at the same time)? If so, how far away from the finish line are these spots?







# Problem of the Week Problem A and Solution International Jumping Relay

#### Problem

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- (a) Which animal will get to the finish line first? Justify your answer.
- (b) Will the two animals ever land at the same spot (not necessarily at the same time)? If so, how far away from the finish line are these spots?

#### Solution

One way to solve this problem is to make a table that keeps track of how far each animal has travelled after each jump. Recall that BeHappy Bunny jumps three times before Kicky Kangaroo jumps for the first time.

Bunny's Jump	Distance Jumped	Distance Jumped
Number	by Bunny (m)	by Kangaroo (m)
1	2	0
2	4	0
3	6	0
4	8	3
5	10	6
6	12	9
7	14	12
8	16	15
9	18	18
10	20	21
11	22	24
12	24	27



(a) Based on the table, we see that Kicky Kangaroo will get to the finish line, which is 24 m away, on Bunny's 11<sup>th</sup> jump. Bunny will get to the finish line on their 12<sup>th</sup> jump. So Kicky Kangaroo will get to the finish line first.

Another way to determine the winner is to see how many jumps it takes for each to travel 24 m. We can do this with division:

BeHappy Bunny will take  $24 \div 2 = 12$  jumps to get to the finish line. Kicky Kangaroo will take  $24 \div 3 = 8$  jumps to get to the finish line.

Since BeHappy Bunny jumps three times before Kicky Kangaroo starts, we can say that Bunny takes 12-3=9 jumps to get to the finish line from the time that Kangaroo starts jumping. However, this is one more jump than it takes Kangaroo to get to the finish line. So Kicky Kangaroo will get to the finish line first.

(b) There are four spots where both animals will land. These are exactly 6 m, 12 m, 18 m, and 24 m from where the animals were waiting to be tagged. (Notice that these numbers are all common multiples of 2 and 3, which represent each animal's jumping distance.)

Therefore, the spots where they both land are 24 - 6 = 18 m, 24 - 12 = 12 m, 24 - 18 = 6 m, and 24 - 24 = 0 m from the finish line.



### Problem of the Week Problem A **Puppy Plans**

Robbie wants to get a puppy, so he decides to help pay the costs. He wants to donate \$60 to the animal shelter he gets the puppy from, and he needs to contribute \$10 every month to help buy the puppy's food.

- (a) How much money does Robbie need to help pay the cost of getting the puppy and feeding the puppy in the first year?
- (b) Robbie needs to save the money calculated in part (a) before getting the puppy. He has \$87 in his piggy bank. He earns a \$7 allowance every week for doing his chores. How long will it take Robbie to earn the rest of the money he needs?







#### Problem of the Week Problem A and Solution Puppy Plans

#### Problem

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#### Solution

- (a) Since there are 12 months in a year and Robbie wants to contribute \$10 each month, he needs  $12 \times 10 = $120$  for puppy food in the first year. Counting the shelter donation, Robbie needs \$60 + \$120 = \$180 in total.
- (b) We can make a table to calculate how long it will take for Robbie to save enough money to have at least \$180. He starts with \$87 and will increase his total by \$7 each week.

Week	Total Saved (in \$)
0	87
1	94
2	101
3	108
4	115
5	122
6	129
7	136
8	143
9	150
10	157
11	164
12	171
13	178
14	185

So it will take Robbie 14 weeks to earn enough money to help cover the shelter donation and the costs of feeding the puppy in the first year.

Alternatively, since Robbie has already saved \$87, he needs to earn \$180 - \$87 = \$93 more. We calculate how long it will take to earn the remaining amount using division. Since  $93 \div 7 = 13$  with a remainder of 2, we round up to determine that it will take 14 weeks to earn the additional \$93 Robbie needs.

# Problem of the Week Problem A Black Box Calculations

The CEMC has created black box machines that process numbers. The machines accept an input number, then do calculations based on that number, then produce an output.

Ronit set up his black box machine to first multiply the input number by 2, and then add 8 to the result.

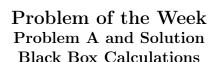


For example, if Ronit inputs the number 10, his black box machine will first multiply it by 2 to produce the number 20. Then it will take the number 20 and add 8 to produce the final output of 28.

Amrita set up her black box machine to first multiply the input number by 4, and then subtract 6 from the result.



- (a) Amrita inputs the number 10 into her black box machine. What number will the machine output?
- (b) Ronit and Amrita are the same age and they are older than 3. They each put their age number into their black box machine and were surprised to get the same output as each other. How old are Ronit and Amrita?
- (c) Write two black box machine setups that have an input of 10 and and output of 16. You can use any two of the following operations: addition, subtraction, multiplication, or division.



#### Problem

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- (a) Amrita inputs the number 10 into her black box machine. What number will the machine output?
- (b) Ronit and Amrita are the same age and they are older than 3. They each put their age number into their black box machine and were surprised to get the same output as each other. How old are Ronit and Amrita?
- (c) Write two black box machine setups that have an input of 10 and and output of 16. You can use any two of the following operations: addition, subtraction, multiplication, or division.

#### Solution

- (a) If Amrita inputs the number 10, the black box machine will first multiply it by 4 to produce the number 40. Then it will take the number 40 and subtract 6 to produce the final output of 34. So her black box machine will output 34.
- (b) One way to solve this problem is to create a table for each machine and keep track of the output produced for different inputs, starting at 4.

Ronit's	Machine

Input	Result after multiplying by 2	Result after adding 8	Output
4	$4 \times 2 = 8$	8 + 8 = 16	16
5	$5 \times 2 = 10$	10 + 8 = 18	18
6	$6 \times 2 = 12$	12 + 8 = 20	20
7	$7 \times 2 = 14$	14 + 8 = 22	22

#### Amrita's Machine

Input	Result after multiplying by 4	Result after subtracting 6	Output
4	$4 \times 4 = 16$	16 - 6 = 10	10
5	$5 \times 4 = 20$	20 - 6 = 14	14
6	$6 \times 4 = 24$	14 - 6 = 18	18
7	$7 \times 4 = 28$	28 - 6 = 22	22

From these tables we see that when the input is 7, the output is 22 for both machines. So one possibility is that Ronit and Amrita are 7 years old.

Is it possible that Ronit and Amrita are a different age? Looking at the Output columns in the tables, we see that the output from Ronit's black box machine increases by 2 as the input increases by 1. However, the output from Amrita's black box machine increases by 4 as the input increases by 1. Since the output from Amrita's black box machine is increasing faster than the output from Ronit's black box machine, this tells us that their outputs will not be the same for any other possible ages.

So Ronit and Amrita must both be 7 years old.

- (c) There are many possible setups that would produce an output of 16 given an input of 10. Some setups are shown below.
  - First divide by 5, then add 14.
  - First multiply by 2, then subtract 4.
  - First add 22, then divide by 2.
  - First subtract 2, then multiply by 2.

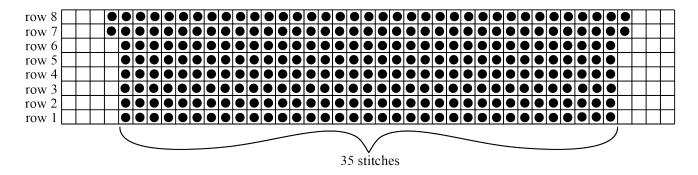
**EXTENSION:** We could describe the setup of each machine by writing a mathematical formula that describes the operations for the black box. The formula would use a letter such as n to represent the input number. Can you write a mathematical formula for each of the two machine setups from part (a)?



### Problem of the Week Problem A Knitting Patterns

Knitting is a process of making rows of stitches out of yarn. To knit a rectangular shape, the number of stitches in each row is always the same. However, to knit different shapes, the number of stitches in each row can be increased or decreased as you work through the pattern.

A particular pattern starts with 35 stitches in the first row. After every 6 rows, 2 more stitches are added to the row; one at the beginning and one at the end. So, the 7<sup>th</sup> row would have 37 stitches, and the 13<sup>th</sup> row would have 39 stitches. The first 8 rows of the knitting pattern are shown below, with dots representing the stitches. Note that knitting patterns are read from the bottom to the top.



If we follow this pattern, we will create an item that resembles a trapezoid.

- (a) How many stitches will be in the 20<sup>th</sup> row of the pattern?
- (b) The length of our finished item is the distance from the first row to the last row. Our finished item will have a length of 30 cm. If every 6 rows of knitting add 3 cm of length, how many rows will there be in total?
- (c) How many stitches will be in the last row found in part (b)?



ALGEBRA, NUMBER SENSE THEMES

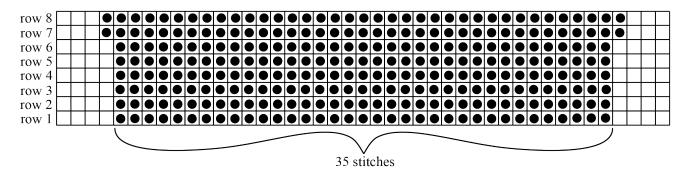


## Problem of the Week Problem A and Solution **Knitting Patterns**

#### Problem

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- (c) How many stitches will be in the last row found in part (b)?





#### Solution

(a) We can make a table showing the number of stitches in each row as we follow the pattern.

Rows	Number of Stitches
1 - 6	35
7 - 12	37
13 - 18	39
19 - 24	41

Since 20 is between 19 and 24, there will be 41 stitches in the 20<sup>th</sup> row.

(b) We can make a table showing the row number and the length of the item after that row has been knitted. Here we will use skip counting and look at the length after every  $6^{\rm th}$  row.

Row	Length (in cm)
6	3
12	6
18	9
24	12
30	15
36	18
42	21
48	24
54	27
60	30

After 60 rows, our finished item will have a length of 30 cm.

(c) We can make another table showing the number of stitches in each row, again using skip counting and looking at every 6<sup>th</sup> row up to row 60.

Row	Number of Stitches
6	35
12	37
18	39
24	41
30	43
36	45
42	47
48	49
54	51
60	53

The 60<sup>th</sup> row will have 53 stitches.

# Problem of the Week Problem A Deciding About Data

The Webb family is trying to decide on a new monthly internet plan. There are three choices:

Plan A: \$10 for the first 10 GB of data, and each additional 2 GB costs \$5.

Plan B: \$40 for the first 20 GB of data, and each additional 10 GB costs \$10.

Plan C: \$80 for unlimited GB of data.

Note that for Plan A, additional data has to be purchased in 2 GB increments. Similarly, for Plan B, additional data has to be purchased in 10 GB increments.

After keeping track of data used, the family decides they will use between 25 GB and 40 GB of data each month. Which plan should the Webb family choose? Justify your answer.





#### Problem of the Week Problem A and Solution Deciding About Data

#### Problem

The Webb family is trying to decide on a new monthly internet plan. There are three choices:

Plan A: \$10 for the first 10 GB of data, and each additional 2 GB costs \$5.

Plan B: \$40 for the first 20 GB of data, and each additional 10 GB costs \$10.

Plan C: \$80 for unlimited GB of data.

Note that for Plan A, additional data has to be purchased in 2 GB increments. Similarly, for Plan B, additional data has to be purchased in 10 GB increments.

After keeping track of data used, the family decides they will use between 25 GB and 40 GB of data each month. Which plan should the Webb family choose? Justify your answer.

#### Solution

One way to make a decision is to make a table showing how much each plan would cost for various amounts of data use.

Data Used	Total Cost (in \$)	Total Cost (in \$)	Total Cost (in \$)
(in GB)	for Plan A	for Plan B	for Plan C
10	10	40	80
12	15	40	80
14	20	40	80
16	25	40	80
18	30	40	80
20	35	40	80
22	40	50	80
24	45	50	80
26	50	50	80
28	55	50	80
30	60	50	80
32	65	60	80
34	70	60	80
36	75	60	80
38	80	60	80
40	85	60	80

Notice that at 25 GB, with Plan A they would need to get 26 GB of data. This would cost \$50. So, comparing the cost of the plans, we notice that Plan B is the lowest price from 25 GB to 40 GB. Therefore, the family should choose Plan B.

(Note that if the family uses less than 25 GB per month, then Plan A is better. Also, if they use more than 40 GB, then it looks like Plan B is better at first, but Plan C will eventually be better.)

# Problem of the Week Problem A Aquarium Issues

Jacques got an aquarium for his birthday and wants to fill it with some guppies and tetras. Guppies are sold in groups of four and tetras are sold in groups of three. He can buy four guppies for \$12, and three tetras for \$7.

If Jacques spent exactly \$50 on fish, how many of each fish did he buy?







# Problem of the Week Problem A and Solution Aquarium Issues

#### Problem

Jacques got an aquarium for his birthday and wants to fill it with some guppies and tetras. Guppies are sold in groups of four and tetras are sold in groups of three. He can buy four guppies for \$12, and three tetras for \$7.

If Jacques spent exactly \$50 on fish, how many of each fish did he buy?

#### Solution

Since 50 is not a multiple of 12 or a multiple of 7, then Jacques must have bought some of each fish in order to spend exactly \$50. One way to solve this problem is to make a table to keep track of how much money it costs for multiples of four guppies, and how much of the \$50 would be left to buy tetras.

Number of	Cost for Guppies	Money Leftover
Guppies	(in \$)	for Tetras (in \$)
4	$1 \times 12 = 12$	50 - 12 = 48
8	$2 \times 12 = 24$	50 - 24 = 26
12	$3 \times 12 = 36$	50 - 36 = 14
16	$4 \times 12 = 48$	50 - 48 = 2

Now we look for a multiple of seven in the leftover money, because each group of tetras costs \$7. The only multiple of seven in the leftover money is 14. Since  $14 = 2 \times 7$ , Jacques must have bought two groups of tetras.

This means Jacques bought 12 guppies and  $2 \times 3 = 6$  tetras.



### Problem of the Week Problem A Crafty Construction

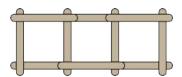
Sam is making square picture frames using popsicle sticks. He has one box of 50 popsicle sticks and wants to make 13 frames.

(a) Sam plans to make individual frames using four popsicle sticks as shown.



Will he have enough popsicle sticks to make 13 frames? Justify your answer.

(b) Sam changes his mind and instead of individual frames, he decides to connect the frames in a row. He starts by making a frame using four popsicle sticks, and then uses three popsicle sticks to create another frame attached to this frame. He then uses three more popsicle sticks to create another frame attached to the first two frames, so he has three connected frames, as shown.



Sam plans to continue this process, using three more populate sticks for each frame, until he has 13 frames connected in a row. Will he have enough popsicle sticks to make 13 frames? Justify your answer.

- (c) Can you draw another layout of the 13 frames that Sam could have built using at most 50 popsicle sticks? How many popsicle sticks does your layout use?
- (d) Can you draw a layout of the 13 frames that uses fewer than 35 popsicle sticks?



## Problem of the Week Problem A and Solution Crafty Construction

#### Problem

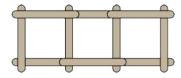
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Sam plans to continue this process, using three more popsicle sticks for each frame, until he has 13 frames connected in a row. Will he have enough popsicle sticks to make 13 frames? Justify your answer.

- (c) Can you draw another layout of the 13 frames that Sam could have built using at most 50 popsicle sticks? How many popsicle sticks does your layout use?
- (d) Can you draw a layout of the 13 frames that uses fewer than 35 popsicle sticks?

#### Solution

(a) Since each frame requires 4 popsicle sticks, the total number of popsicle sticks required for 13 frames is equal to  $13 \times 4 = 52$ .

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

Number of Frames	Number of Popsicle Sticks
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36
10	40
11	44
12	48
13	52

Either way, since it will take 52 popsicle sticks to make 13 individual frames, Sam will not have enough popsicle sticks to make all the frames.

(b) The first frame uses 4 popsicle sticks but the other 12 frames use only 3 popsicle sticks each. So the total number of popsicle sticks required can be found by adding 4 to  $12 \times 3$ . Since  $12 \times 3 = 36$ , this gives a total of 4 + 36 = 40 popsicle sticks.

Alternatively, we could make a table showing the number of frames and the number of popsicle sticks required.

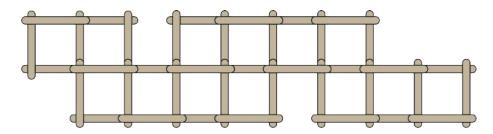
Number of Frames	Number of Popsicle Sticks
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28
10	31
11	34
12	37
13	40

Either way, since it will take 40 popsicle sticks to build a row of 13 connected frames, Sam will have enough popsicle sticks to make all the frames.

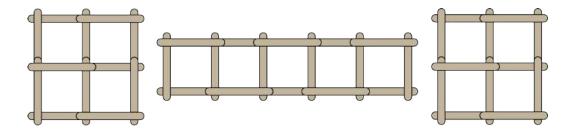


(c) There are many layouts of 13 frames you could make using at most 50 popsicle sticks. Two examples are shown.

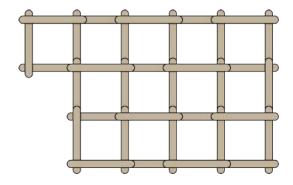
The following layout uses 39 popsicle sticks.



The following layout uses 40 popsicle sticks.



(d) The most efficient use of popsicle sticks is to share as many sides as possible with other frames. If you connect the frames to form a shape as close as possible to a square, you can make 13 frames using only 34 popsicle sticks, as shown.



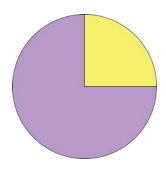
## Data Management (D)

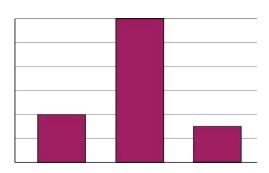




### Problem of the Week Problem A Missing Data

Jasmine created a circle chart and a bar chart based on data values, but forgot what numbers went with which chart. The charts are shown below. There are five values in total, two belong to the circle chart and three belong to the bar chart.





Here are the values:

V: 30, W: 40, X: 60, Y: 120, Z: 180

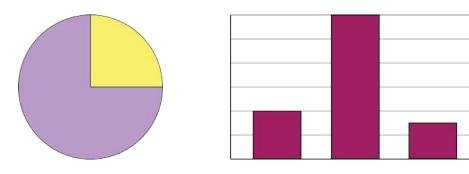
Label each chart with the letters matching the appropriate values. Then add values to the scale on the vertical axis of the bar chart.



### Problem of the Week Problem A and Solution Missing Data

#### Problem

Jasmine created a circle chart and a bar chart based on data values, but forgot what numbers went with which chart. The charts are shown below. There are five values in total, two belong to the circle chart and three belong to the bar chart.



Here are the values:

V: 30, W: 40, X: 60, Y: 120, Z: 180

Label each chart with the letters matching the appropriate values. Then add values to the scale on the vertical axis of the bar chart.

#### Solution

Comparing the size of the sections in the circle chart, we see that the smaller section appears to be one quarter of the whole circle and the larger section appears to be three quarters of the whole circle. This means the larger section is three times the size of the smaller section. Comparing the heights of the bars, we can see that the tallest bar is three times the height of the leftmost bar. We should look at the data to find pairs of numbers where one number is three times the other number.

From the values listed we see that  $3 \times 40 = 120$  and  $3 \times 60 = 180$ . So the circle chart values could be either 120 and 40 or 180 and 60.

• If the circle chart values are 120 and 40, then the remaining three values would be the bar chart values. This would mean that the tallest bar would be representing the value 180 and the leftmost bar would be representing the value 60. This means the rightmost bar would be representing the remaining value, which is 30. However, 30 is half as much as 60. This means the rightmost bar should be half the height of the leftmost bar. In the chart,

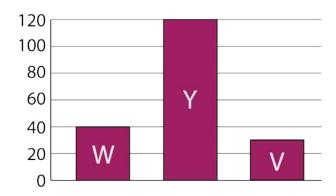


we can see that the rightmost bar is *more* than half the height of the leftmost bar. Therefore, these values cannot be correct.

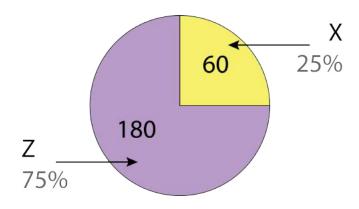
• If the circle chart values are 180 and 60, then the remaining three values would be the bar chart values. This would mean that the tallest bar would be representing the value 120 and the leftmost bar would be representing the value 40. This means the rightmost bar would be representing the remaining value, which is 30. Since the rightmost bar is more than half the height of the leftmost bar, the values of 40 and 30 would be reasonable.

Based on this logic, the leftmost bar represents the value 40. The top of the bar matches the second grid line. Since the grid lines are equally spaced, we can conclude that the distance between grid lines is  $40 \div 2 = 20$ .

Here is the labelled bar chart:



Here is the labelled circle chart:



# Problem of the Week Problem A Gym Schedules

At Spruce Glen Public School, each day is divided into nine blocks, which are each 30 minutes long. There are six classrooms that share the gym, according to the weekly schedule shown.

	Monday	Tuesday	Wednesday	Thursday	Friday
Block A	Room 1	Room 3	Room 2	Room 3	Room 6
Block B	Room 5	Room 5	Room 2	Room 3	Room 1
Block C	Room 3	Room 5	Room 2	Room 2	Room 3
Block D	Room 3	Room 5	Room 5	Room 2	Room 3
Block E	Room 6	Room 1	Room 5	Room 6	Room 3
Block F	Room 4	Room 2	Room 6	Room 1	Room 5
Block G	Room 4	Room 6	Room 1	Room 1	Room 2
Block H	Room 2	Room 4	Room 3	Room 4	Room 2
Block I	Room 2	Room 4	Room 4	Room 5	Room 4

- (a) Make a bar chart showing the total gym time per week for each room.
- (b) List the rooms in order from least to greatest total gym time per week.





# Problem of the Week Problem A and Solution Gym Schedules

#### Problem

At Spruce Glen Public School, each day is divided into nine blocks, which are each 30 minutes long. There are six classrooms that share the gym, according to the weekly schedule shown.

	Monday	Tuesday	Wednesday	Thursday	Friday
Block A	Room 1	Room 3	Room 2	Room 3	Room 6
Block B	Room 5	Room 5	Room 2	Room 3	Room 1
Block C	Room 3	Room 5	Room 2	Room 2	Room 3
Block D	Room 3	Room 5	Room 5	Room 2	Room 3
Block E	Room 6	Room 1	Room 5	Room 6	Room 3
Block F	Room 4	Room 2	Room 6	Room 1	Room 5
Block G	Room 4	Room 6	Room 1	Room 1	Room 2
Block H	Room 2	Room 4	Room 3	Room 4	Room 2
Block I	Room 2	Room 4	Room 4	Room 5	Room 4

(a) Make a bar chart showing the total gym time per week for each room.

(b) List the rooms in order from least to greatest total gym time per week.

#### Solution

(a) One way to draw the bar chart is to calculate the total gym time for each room. Then we can use the total times to determine the height of each bar. Remember that each block is 30 minutes long.

For example, if we add up the gym times for Room 1 in minutes we get:

• Monday time: 30 minutes

• Tuesday time: 30 minutes

• Wednesday time: 30 minutes

• Thursday time: 30 + 30 = 60 minutes

• Friday time: 30 minutes

Total time for Room 1 is 30 + 30 + 30 + 60 + 30 = 180 minutes



We could do these calculations for each room to get:

Total time for Room 2 is 60 + 30 + 90 + 60 + 60 = 300 minutes

Total time for Room 3 is 60 + 30 + 30 + 60 + 90 = 270 minutes

Total time for Room 4 is 60 + 60 + 30 + 30 + 30 = 210 minutes

Total time for Room 5 is 30 + 90 + 60 + 30 + 30 = 240 minutes

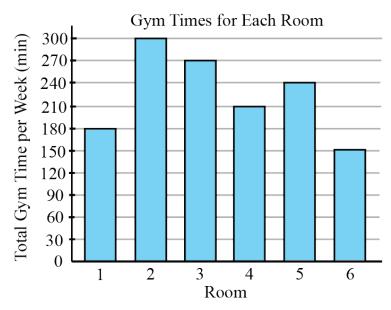
Total time for Room 6 is 30 + 30 + 30 + 30 + 30 = 150 minutes

Another way to build the bar chart is to count the total number of blocks that each class is in the gym. We can keep track of the number of blocks by using a tally chart.

Room	Number of Blocks
1	#
2	## ##
3	##
4	##
5	##
6	##

Then we can set our vertical axis to go up by 30 minute increments so that each tick mark in the bar chart represents one block in the gym.

The completed bar chart is shown.



(b) Looking at the size of the bars in the chart, the rooms in order from least to greatest total gym time per week is:

Room 6, Room 1, Room 4, Room 5, Room 3, Room 2

# Problem of the Week Problem A Candy Store Counting

Ricardo and Nadia went to the candy store. The clues below describe the types and amount of candies they bought.

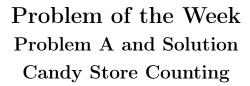
- They bought 8 packages of Sunbursts.
- They bought 4 more packages of Twinglers than Sunbursts.
- They bought 6 fewer packages of Nerts than Twinglers.
- They bought twice as many packages of W&Ws than Nerts.
- The number of packages of Sweet Patch Kids is equal to the total number of packages of Sunbursts and Nerts.
- The number of packages of Jolly Farmers is equal to the number of packages of Sweet Patch Kids minus the number of packages of Twinglers.

Complete the pictograph to show how many packages of each type of candy they purchased. Use one to represent two packages of candy. The first row has already been completed for you.

Type of Candy	Number of Packages
Sunbursts	
Twinglers	
Nerts	
W&Ws	
Sweet Patch Kids	
Jolly Farmers	

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THEMES DATA MANAGEMENT, NUMBER SENSE



#### Problem

Ricardo and Nadia went to the candy store. The clues below describe the types and amount of candies they bought.

- They bought 8 packages of Sunbursts.
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- The number of packages of Sweet Patch Kids is equal to the total number of packages of Sunbursts and Nerts.
- The number of packages of Jolly Farmers is equal to the number of packages of Sweet Patch Kids minus the number of packages of Twinglers.

Complete the pictograph to show how many packages of each type of candy they purchased. Use one to represent two packages of candy. The first row has already been completed for you.

Type of Candy	Number of Packages
Sunbursts	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
Twinglers	
Nerts	
W&Ws	
Sweet Patch Kids	
Jolly Farmers	

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### Solution

First we need to calculate how many packages of each kind of candy they bought. We know they bought 8 packages of Sunbursts.

- Buying 4 more packages of Twinglers than Sunbursts means they bought 8+4=12 packages of Twinglers.
- Buying 6 fewer packages of Nerts than Twinglers means they bought 12-6=6 packages of Nerts.
- Buying twice as many packages of W&Ws than Nerts means they bought  $6 \times 2 = 12$  packages of W&Ws.
- The number of packages of Sweet Patch Kids they bought is 8 + 6 = 14.
- The number of packages of Jolly Farmers they bought is 14 12 = 2.

To complete the pictograph, we use to represent two packages of candy. So we divide the number of packages for each type of candy by 2, in order to calculate the number of needed for that candy in the pictograph.

The completed pictograph is shown below.

Type of Candy	Nun	Number of Packages					
Sunbursts	65	65	65	65			
Twinglers	65	65	65	65	60	65	
Nerts	65	66	66				
W&Ws	65	66	66	60	65	60	
Sweet Patch Kids	65	65	65	65	65	65	65
Jolly Farmers	65						

**Key:** one represents 2 packages of candy.

### Computational Thinking (C)





## Problem of the Week Problem A What Number Am I?

I am a five-digit number made of the digits 0, 3, 4, 6, and 9. In my number, the following are true.

- The digit **4** is in a position whose place value is 10 times the place value of the position of the digit **9** and 100 times the place value of the position of the digit **0**.
- The digit **3** is in a position whose place value is 100 times the place value of the position of the digit **9**.
- The digit **0** is in a position whose place value is 10 times the place value of the position of the digit **6**.

What number am I?





### Problem of the Week Problem A and Solution What Number Am I?

#### Problem

I am a five-digit number made of the digits 0, 3, 4, 6, and 9. In my number, the following are true.

- The digit 4 is in a position whose place value is 10 times the place value of the position of the digit 9 and 100 times the place value of the position of the digit 0.
- The digit **3** is in a position whose place value is 100 times the place value of the position of the digit **9**.
- The digit **0** is in a position whose place value is 10 times the place value of the position of the digit **6**.

What number am I?

#### Solution

The positions in a five-digit number are: ones (units), tens, hundreds, thousands, and ten thousands.

Since the 4 is in a position whose place value is 10 and 100 times the place value of other positions, the 4 cannot be in the ones or tens position.

Since the 3 is in a position whose place value is 100 times the place value of another position, the 3 cannot be in the ones or tens position.

Since the 0 is in a position whose place value is 10 times the place value of another position, the 0 cannot be in the ones position.

So the ones position must contain either the 9 or the 6.

Let's assume that the 9 is in the ones position. Based on the first clue, the 4 must be in the tens position since  $10 \times 1 = 10$ . However, we have already stated that the 4 cannot be in the ones or tens position. So we cannot put the 9 in the ones position.

Therefore, we can conclude that the 6 is in the ones position. Based on the third clue, the 0 must be in the tens position since  $10 \times 1 = 10$ . Knowing that the 0 is in the tens position, we can use the first clue to conclude that the 4 must be in the thousands position, since  $100 \times 10 = 1000$ . Also, the 9 must be in the hundreds position, since  $10 \times 100 = 1000$ . Now the only position that is left, the ten thousands position, must contain the only digit left which is the 3. This is consistent with the second clue since  $100 \times 100 = 10000$ .

Therefore, the number must be 34 906.



### The Keys to Hidden Messages

There are many ways to encode messages. Here is an encoding using just zeros and ones.

0 0 0 1 1 0 0 1 0 1 1 0 1 1 0

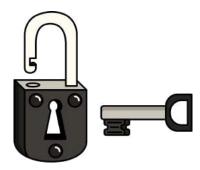
This single sequence could represent many different messages. To decode the information correctly, you need a key. The actual message that the encoding represents depends on the key you use to decode it.

Here are two keys which could be used to decode the message:

Alpha Key						
Letter	Code					
С	1	0	0			
D	1	0	1	0		
Е	0	1				
О	1	1	1			
R	1	0	1	1		
S	0	0				
Т	1	1	0			

${ m Beta} \; { m Key}$						
Letter	(	Co	$\mathrm{d}\epsilon$	9		
С	0	0	0	1		
D	0	1	1	0		
Е	1	1	1	0		
О	1	0	0	1		
R	1	0	1	0		
S	0	1	0	1		
Т	1	0	1	1		

- (a) What is the message if you decode it using the **Alpha Key**?
- (b) What is the message if you decode it using the **Beta Key**?





# Problem of the Week Problem A and Solution The Keys to Hidden Messages

#### Problem

There are many ways to encode messages. Here is an encoding using just zeros and ones.

#### 0 0 0 1 1 0 0 1 0 1 1 0 1 1 1 0

This single sequence could represent many different messages. To decode the information correctly, you need a key. The actual message that the encoding represents depends on the key you use to decode it.

Here are two keys which could be used to decode the message:

Alpha Key						
Letter	Code					
С	1 0 0					
D	1 0 1 0					
Е	0 1					
О	1 1 1					
R	1 0 1 1					
S	0 0					
Т	1 1 0					

${f Beta}  {f Key}$					
Letter	(	$\mathbb{C}\mathbf{c}$	de	9	
С	0	0	0	1	
D	0	1	1	0	
Е	1	1	1	0	
О	1	0	0	1	
R	1	0	1	0	
S	0	1	0	1	
Т	1	0	1	1	

- (a) What is the message if you decode it using the **Alpha Key**?
- (b) What is the message if you decode it using the **Beta Key**?

#### Solution

We can use pattern matching to decode the message with each key.

(a) The first row of the following table contains the message broken up into six pieces. Notice that each of these pieces is a code in the Alpha Key. The second row of the table contains the letter corresponding to each code.

0 0	0 1	1 0 0	1 0 1 1	0 1	1 1 0
S	E	С	R	E	T

Therefore, the message is "SECRET" when decoded using the Alpha Key.

(b) The table below was created using a similar process as in part (a), except instead for the Beta Key.

0 0 0 1	1 0 0 1	0 1 1 0	1 1 1 0
С	О	D	Ε

Therefore, the message is "CODE" when decoded using the Beta Key.

### Problem of the Week Problem A Black Box Calculations

The CEMC has created black box machines that process numbers. The machines accept an input number, then do calculations based on that number, then produce an output.

Ronit set up his black box machine to first multiply the input number by 2, and then add 8 to the result.

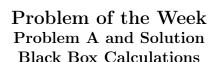


For example, if Ronit inputs the number 10, his black box machine will first multiply it by 2 to produce the number 20. Then it will take the number 20 and add 8 to produce the final output of 28.

Amrita set up her black box machine to first multiply the input number by 4, and then subtract 6 from the result.



- (a) Amrita inputs the number 10 into her black box machine. What number will the machine output?
- (b) Ronit and Amrita are the same age and they are older than 3. They each put their age number into their black box machine and were surprised to get the same output as each other. How old are Ronit and Amrita?
- (c) Write two black box machine setups that have an input of 10 and and output of 16. You can use any two of the following operations: addition, subtraction, multiplication, or division.



#### Problem

The CEMC has created black box machines that process numbers. The machines accept an input number, then do calculations based on that number, then produce an output.

Ronit set up his black box machine to first multiply the input number by 2, and then add 8 to the result.



For example, if Ronit inputs the number 10, his black box machine will first multiply it by 2 to produce the number 20. Then it will take the number 20 and add 8 to produce the final output of 28.

Amrita set up her black box machine to first multiply the input number by 4, and then subtract 6 from the result.



- (a) Amrita inputs the number 10 into her black box machine. What number will the machine output?
- (b) Ronit and Amrita are the same age and they are older than 3. They each put their age number into their black box machine and were surprised to get the same output as each other. How old are Ronit and Amrita?
- (c) Write two black box machine setups that have an input of 10 and and output of 16. You can use any two of the following operations: addition, subtraction, multiplication, or division.

#### Solution

- (a) If Amrita inputs the number 10, the black box machine will first multiply it by 4 to produce the number 40. Then it will take the number 40 and subtract 6 to produce the final output of 34. So her black box machine will output 34.
- (b) One way to solve this problem is to create a table for each machine and keep track of the output produced for different inputs, starting at 4.

Ronit's	Machine

Input	Result after multiplying by 2	Result after adding 8	Output
4	$4 \times 2 = 8$	8 + 8 = 16	16
5	$5 \times 2 = 10$	10 + 8 = 18	18
6	$6 \times 2 = 12$	12 + 8 = 20	20
7	$7 \times 2 = 14$	14 + 8 = 22	22

#### Amrita's Machine

Input	Result after multiplying by 4	Result after subtracting 6	Output
4	$4 \times 4 = 16$	16 - 6 = 10	10
5	$5 \times 4 = 20$	20 - 6 = 14	14
6	$6 \times 4 = 24$	14 - 6 = 18	18
7	$7 \times 4 = 28$	28 - 6 = 22	22

From these tables we see that when the input is 7, the output is 22 for both machines. So one possibility is that Ronit and Amrita are 7 years old.

Is it possible that Ronit and Amrita are a different age? Looking at the Output columns in the tables, we see that the output from Ronit's black box machine increases by 2 as the input increases by 1. However, the output from Amrita's black box machine increases by 4 as the input increases by 1. Since the output from Amrita's black box machine is increasing faster than the output from Ronit's black box machine, this tells us that their outputs will not be the same for any other possible ages.

So Ronit and Amrita must both be 7 years old.

- (c) There are many possible setups that would produce an output of 16 given an input of 10. Some setups are shown below.
  - First divide by 5, then add 14.
  - First multiply by 2, then subtract 4.
  - First add 22, then divide by 2.
  - First subtract 2, then multiply by 2.

**EXTENSION:** We could describe the setup of each machine by writing a mathematical formula that describes the operations for the black box. The formula would use a letter such as n to represent the input number. Can you write a mathematical formula for each of the two machine setups from part (a)?



### Problem of the Week Problem A Adding and Subtracting

Follow the steps below.

**Step 1:** Pick two different three-digit numbers. Label the larger number A and the smaller number B.

**Step 2:** Subtract B from 999 and label the difference C.

**Step 3:** Add A and C and label the sum D.

**Step 4:** Subtract 1000 from D and label the difference E.

**Step 5:** Add 1 to E and label the sum F.

- (a) What is the connection between the number F and the numbers A and B?
- (b) Try the same steps with different numbers. Do you think you will always get the same result? Why or why not?















# Problem of the Week Problem A and Solution Adding and Subtracting

#### Problem

Follow the steps below.

**Step 1:** Pick two different three-digit numbers. Label the larger number A and the smaller number B.

**Step 2:** Subtract B from 999 and label the difference C.

**Step 3:** Add A and C and label the sum D.

**Step 4:** Subtract 1000 from D and label the difference E.

**Step 5:** Add 1 to E and label the sum F.

- (a) What is the connection between the number F and the numbers A and B?
- (b) Try the same steps with different numbers. Do you think you will always get the same result? Why or why not?

#### Solution

- (a) We can work through this procedure with any two, random, three-digit numbers. Let's try starting with 814 and 275.
  - **Step 1:** The larger number 814 is labelled A, and the smaller number 275 is labelled B.
  - Step 2: The difference 999 B is 999 275 = 724, and is labelled C.
  - **Step 3:** The sum A + C is 814 + 724 = 1538, and is labelled D.
  - Step 4: The difference D 1000 is 1538 1000 = 538, and is labelled E.
  - **Step 5:** The sum E + 1 is 538 + 1 = 539, and is labelled F.

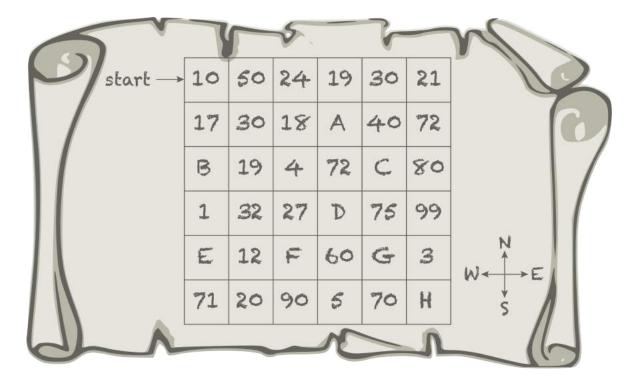
Notice that 814 - 275 = 539. So A - B = F.

- (b) If you try this procedure with any two three-digit numbers, it will always work out that A B = F. We will show this using algebra.
  - **Step 1:** We choose three-digit numbers A and B so that  $A \geq B$ .
  - **Step 2:** We calculate 999 B, and label this C. That is, C = 999 B.
  - **Step 3:** We add A + C. So, we calculate A + 999 B, and label this D. That is, D = A + 999 B.
  - **Step 4:** We subtract 1000 from D. So, we calculate A + 999 B 1000, and label this E. That is, E = A + 999 B 1000.
  - **Step 5:** We add 1 to *E*. So, we calculate A + 999 B 1000 + 1, and label this *F*. That is, F = A + 999 B 1000 + 1.

Now, we can simplify this expression to get A - B + 999 - 1000 + 1 = A - B, since 999 - 1000 + 1 = 0. So, F = A - B.

### Problem of the Week Problem A Treasure Map

Robbie found a treasure map that contains a grid filled with numbers and letters.



The treasure hunt starts in the northwest corner of the grid, on the square that contains the number 10. The squares with numbers are used to determine which way to go, using the following rules.

- If the number is even and a multiple of 5, then move one square east.
- If the number is even, but not a multiple of 5, then move one square south.
- If the number is a multiple of 5, but not even, then move one square north.
- If the number is neither even nor a multiple of 5, then move one square west.

The squares with letters represent possible locations of the treasure. If Robbie follows the rules, he will follow a path that reaches a square with a letter, which is where the treasure is hidden. Where is the treasure hidden?

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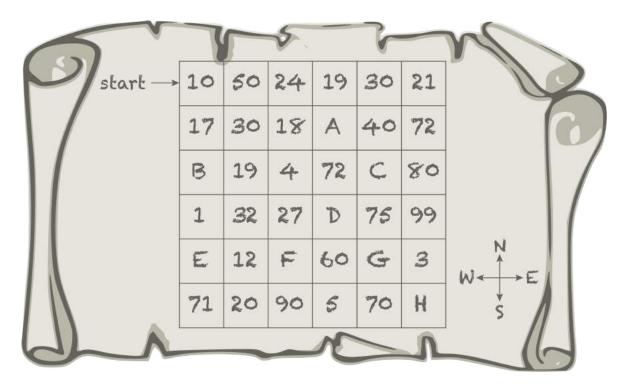
THEMES COMPUTATIONAL THINKING, GEOMETRY



### Problem of the Week Problem A and Solution Treasure Map

#### Problem

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- If the number is even and a multiple of 5, then move one square east.
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The squares with letters represent possible locations of the treasure. If Robbie follows the rules, he will follow a path that reaches a square with a letter, which is where the treasure is hidden. Where is the treasure hidden?

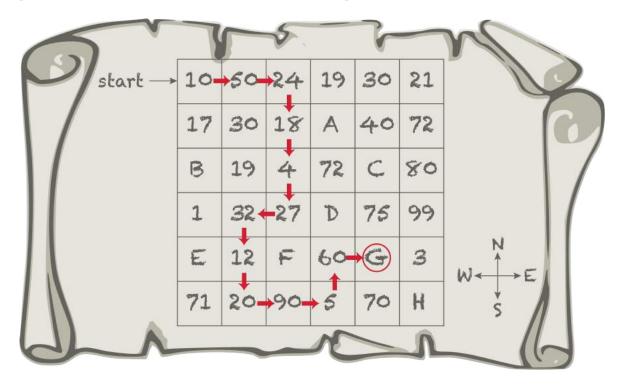
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#### Solution

Starting in the northwest corner of the map, here are the steps Robbie would follow to find the hidden treasure.

- 10 is an even number and a multiple of 5, so move one square east.
- 50 is an even number and a multiple of 5, so move one square east again.
- 24 is an even number but not a multiple of 5, so move one square south.
- 18 is an even number but not a multiple of 5, so move one square south again.
- 4 is an even number but not a multiple of 5, so move one square south again.
- 27 is neither even nor a multiple of 5, so move one square west.
- 32 is an even number but not a multiple of 5, so move one square south.
- 12 is an even number but not a multiple of 5, so move one square south again.
- 20 is an even number and a multiple of 5, so move one square east.
- 90 is an even number and a multiple of 5, so move one square east again.
- 5 is a multiple of 5 but not an even number, so move one square north.
- 60 is an even number and a multiple of 5, so move one square east.
- Robbie has reached a square with a letter, so the treasure is hidden in location G.

The path to the hidden treasure is shown on the map below.

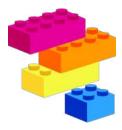


## Problem of the Week Problem A Indoor Recess

It has been a wet spring so far. It has rained so much that the students at Rising Up Academy were not able to go outside for recess for a whole week, from Monday to Friday. Instead the students chose from the following indoor activities: matching game, Pictionary, checkers, building blocks, and origami. Carlo did a different indoor activity each day.

Using the following clues, determine which indoor activity Carlo did each day.

- (1) On either Tuesday or Friday Carlo played with the building blocks.
- (2) Carlo did origami the day before he played the matching game.
- (3) On Thursday Carlo did not play Pictionary.
- (4) Carlo played the matching game two days before he played checkers.







## Problem of the Week Problem A and Solution Indoor Recess

#### Problem

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#### Solution

From clue (2), we know Carlo did origami the day before he played the matching game. From clue (4), we know Carlo played the matching game two days before he played checkers. So, putting these clues together tells us that we have two possibilities. One possibility is that Carlo did origami on Tuesday, played the matching game on Wednesday, and played checkers on Friday. Another possibility is that Carlo did origami on Monday, played the matching game on Tuesday, and played checkers on Thursday. Let's look at each of these possibilities.

- Suppose Carlo did origami on Tuesday, played the matching game on Wednesday, and played checkers on Friday. Then he must have played Pictionary and played with the building blocks on Monday and Thursday, in some order. However, from clue (3) we know that Carlo did not play Pictionary on Thursday and from clue (1) we know that Carlo did not play with the building blocks on Thursday. Therefore this possibility won't work.
- Suppose Carlo did origami on Monday, played the matching game on Tuesday, and played checkers on Thursday. Then he must have played Pictionary and played with the building blocks on Wednesday and Friday, in some order. We know from clue (1) that Carlo did not play with the building blocks on Wednesday. That means he must have played with the building blocks on Friday, and played Pictionary on Wednesday. We can verify that this order satisfies all four clues given in the problem.

Therefore, Carlo did origami on Monday, played the matching game on Tuesday, played Pictionary on Wednesday, played checkers on Thursday, and played with the building blocks on Friday.