



Problem of the Month

Problem 0: September 2023

Hint

- (a) $f(r)$ is undefined only when $r = -2$. For what value of r is $f(r) = -2$?
- (b) The sequence in part (i) is periodic. Can you show that the sequence is periodic for other values of r_1 ?
- (c) (ii) After substituting the expression for $f(r)$, multiply the numerator and denominator by $r + 1$. Try to find a common factor in the numerator and denominator.
- (iii) Use (ii) and the fact that when r is positive, $\left| \frac{1 - \sqrt{2}}{r + 1} \right| < \frac{1}{2}$. Try to establish the given inequality for a few small values of n and observe how knowing the inequality for n can help you to deduce it for $n + 1$.
- (d) Three of these sequences are periodic, one of them is constant (after the first term), and one of them always approaches the fixed value $\frac{3 - \sqrt{13}}{2}$ as long as there are no undefined values in the sequence.
-
-