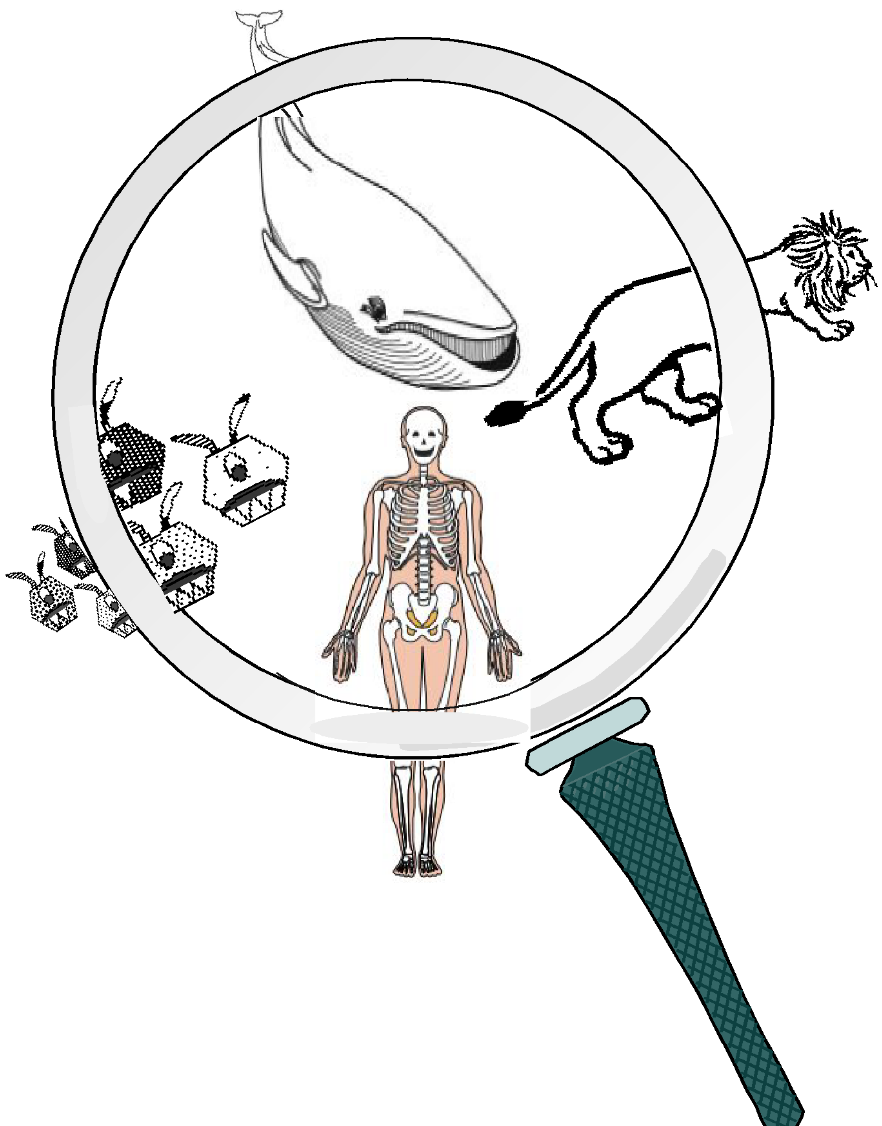
Invitations to Mathematics

Investigations in Number Sense and Estimation

"Numbers at Work"

Suggested for students at the Grade 5 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
Faculty of Mathematics, University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the “Extensions” and related activities included with individual activities/projects, provide ample scope for all students’ interests and ability levels. Related “Family Activities” can be used to involve the students’ parents/care givers.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

“Investigations in Number Sense and Estimation” is comprised of activities which explore the properties, estimation, and uses of whole numbers and fractions in mathematical and everyday settings. A reasonably level of numeracy is essential to navigating the complexities of the highly technical world in which we live. The activities in this unit develop many facets of number sense and apply them to a wide variety of practical situations.

Acknowledgements

Contributing Teachers

Nancy Dykstra (Waterloo County Board of Education)
Jo-Anne Judge (Waterloo County Board of Education)
Ron Sauer (Waterloo County Board of Education - retired)

Authors/Co-editors

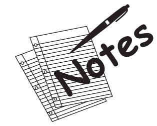
Bev Marshman (University of Waterloo)
Lorna Morrow (Mathematics Consultant)

We wish to acknowledge the support of the **Centre for Education in Mathematics and Computing,** and in particular, of Peter Crippin and Ian VanderBurgh. A special thank you goes to Linda Schmidt for prompt, accurate type-setting and creative diagrams, and for her patience in converting our efforts, both past and present, to new software.

Table of Contents

Preface i
Acknowledgements ii
Table of Contents iii
Overview 1
Common Beliefs 1
Essential Content 1
Curriculum expectations 2
Prerequisites 3
Logos 3
Marginal Problems Description 3
Materials 4
Letter to Parents 5
Activity 1: Using Numbers 7
Activity 2: Comparing & Ordering 10
Activity 3: Number Properties 16
Activity 4: Fractions 20
Activity 5: Estimation 24
BLM 1: Using Numbers 28
BLM 2: Mixed-up Numbers 29
BLM 3: A Number Barbecue 30
BLM 4: Decimal Tiles 31
BLM 5: Comparing Fractions 32
BLM 6: Fraction Lines 33
BLM 7: Out of Line 34
BLM 8: Putting Numbers in their Places 35
BLM 9: Dewey Decimals 36
BLM 10: Dewey Animals 37
BLM 11: Lining Up Letters 38
BLM 12: Devilish Differences – 1 39
BLM 13: Devilish Differences – 2 40
BLM 14: Flowing Division 41
BLM 15: About Halves 42
BLM 16: More Halves 43
BLM 17: Estimating Fractions 44
BLM 18: Sizing Fractions 45
BLM 19: About or Exact 46
BLM 20: Rough Estimates 47
BLM 21: Over and Under 48
BLM 22: A Range of Estimates 49
BLM 23: Product Bingo 50
Solutions & Notes 51
Suggested Assessment Strategies 63
Other Resources 70

Overview



Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and number sense and estimation specifically. Some of these beliefs are described below.

Numeracy involves an intuitive sense of the meanings of numbers and their various uses and interpretations. It is acquired slowly over a long period of time, and is fundamental, both to mathematics, and to the sciences which provide a quantitative understanding of the world around us.

While facility with number facts and algorithms is clearly important, the focus here is on developing students' thinking and reasoning abilities. This is achieved through investigation and sharing of ideas during group activities involving properties of numbers (whole numbers, decimals, and fractions), comparing and ordering, how numbers are used, and determining reasonable estimates. Problems using a variety of mechanisms (number lines, geometric quantities, mental manipulation, stories, games, etc) encourage flexibility in mthods os solution. Similarly, a variety of estimation situations increases students' awareness of the pervasive need for estimates in real life, and their ability to devise estimates competently. Students are encouraged not only to calculate in different ways but also to assess the reasonableness of their answers. In addition, by eliminating the need for boring computations, calculators can be used to permit students to focus on the process of obtaining solutions, and on their interpretation.

Throughout these activities, as they attempt to justify their conclusions using mathematical language, students deepen their insight into and understanding of how numbers relate to each other and to the world around them.

Essential Content

The activities herein explore numbers both in the abstract and in their connection to measures of real quantities, with the goal of developing students' ability to think and work flexibly with different kinds of numbers in a variety of contexts. In addition, there are Marginal Problems, Extensions in Mathematics, Cross-Curricular Activities, and Family Activities, which can be used prior to, during the activity, or following the activity. They are intended to suggest topics for extending the activity, assist integration with other subjects, and involve the family in the learning process.

During this unit, the student will:

- compare sizes of whole numbers (to millions), fractions, and decimals (to hundredths);
- explore number usage, place value, the nature of estimation, and estimation techniques;
- estimate products, sums, and differences (whole numbers and fractions), and averages;
- identify over and under estimates;
- practice skills in game situations;
- explore properties of number including equal distances on the number line;
- use mathematical language to express their results;
- work together to achieve success.

Notes

"Curriculum Expectations" are based on current Ontario curricula.

Overview

Curriculum Expectations

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Using Numbers**	• explore how numbers are used in real-world situations • determine appropriate values for a variety of quantities in specific contexts	• identify, investigate, and interpret the use of numbers
Activity 2 **Comparing and Ordering**	• compare and order decimal numbers between 0.01 and 100 000 • create fractions close to $\frac{1}{2}$, and close to 1, given either the denominator or the numerator • place fractions between 0 and 1 on a number line • solve puzzles requiring ordering of whole numbers and fractions • explore the Dewey Decimal system used by libraries	• compare and order decimals from 0.01 to 100 000 • compare and order fractions • solve problems involving decimals and fractions
Activity 3 **Number Properties**	• determine the end-points of intervals of equivalent length on the number line, using both fractions and decimals • use repeated differences to illustrate 'reduction to 0' • combine multiple divisions into a single equivalent division operation	• compare mixed numbers and proper and improper fractions • add and subtract decimal numbers to hundredths • use mental computation strategies to solve number problems
Activity 4 **Fractions**	• explore the use of fractions as a way of comparing real quantities • create suitable fractions to describe a specific situation • add, subtract, and multiply fractions with simple denominators (2, 4, 8). • place fractions with denominators 4, 5, 6, 8, 10, 20 on number lines with suitable units	• solve problems involving decimals and fractions • explain their thinking when solving problems involving fractions • order fractions on a number line
Activity 5 **Estimation**	• determine whether specific situations require an estimate or an exact number • use 'front-end-with-adjustment' estimation to approximate a grocery bill • determine whether given estimated products and sums of 2-digit and 3-digit numbers are over-estimates or under-estimates • estimate sums and products of 2-digit numbers	• explain their thinking when solving problems • use and explain estimation strategies to determine the reasonableness of solutions and problems • select and perform computation techniques appropriate to specific problems • recall multiplication and division facts to 144

Overview

Prerequisites

Although students should be able to deal with the activities in this book with an understanding of the previous grade's curriculum, it would help if they are familiar with the following:

- place value from tens to millions;
- rounding to the nearest ten or hundred;
- adding and subtracting fractions with the same denominator;
- comparing simple fractions with different denominators;
- the nature and use of estimates (i.e. what an estimate is and when an estimate is appropriate).

Logos

The following logos, which are located in the margins, identify segments related to, respectively:

Problem Solving Communication Assessment Use of Technology

Marginal Problems

Throughout the booklet you will see problems like this in the margin. These "Marginal Problems" may be used as warm-ups to a lesson, as quick 'tests' or reviews, as 'problems-of-the-day' or in any other way your experience tells you could be useful. Some 'Marginal Problems' deal with the same topic as the activity and some with other topics in Number Sense or Estimation.

If $4 \div \frac{1}{2} = 8$, is $4 \div \frac{1}{4}$ greater or less than 8? Why?

Materials

ACTIVITY	MATERIALS
Activity 1 **Using Numbers**	• Copies of BLM 1 • Copies of BLMs 2 and 3 (optional)
Activity 2 **Comparing and Ordering**	• Copies of BLMs 4, 5, and 7 • Copies of BLMs 6, 8, 9, and 10 (optional) • Scissors for each pair/group
Activity 3 **Number Properties**	• Copies of BLMs 11, 12, and 13 • Copies of BLM 14 (optional)
Activity 4 **Fractions**	• Copies of BLMs 15, 16, and 17 • Copies of BLM 18 (optional)
Activity 5 **Estimation**	• Copies of BLMs 19, 20, and 21 • A calculator for each pair/group • Newspapers • Copies of BLMs 22 and 23 (optional) • 10 - 12 game markers for each player (optional)

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week or so, students in our classroom will be participating in a unit titled “Numbers at Work”. The classroom activities will focus on expanding students’ understanding of numbers and estimation while exploring how numbers are related and how they are used. The emphasis will be on developing skill with mental manipulation, estimation, and computation.

You can assist your child in understanding the relevant concepts and acquiring useful skills by working together to perform number related tasks (e.g., comparing prices when shopping, estimating the total cost, calculating mileage for the family vechicle), and by helping to explore everyday ways numbers are used.

Various family activities have been included for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with measurement in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher’s Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Notes

Activity 1: Using Numbers

Focus of Activity:

- Identifying how and where numbers are used

‘BLM’ refers to the Black Line Masters which follow the Activity Notes

What to Assess:

- Reasonableness of answers
- Explanations of reasons for choices

Preparation:

- Make copies of BLM 1.
- Make copies of BLMs 2 and 3 (optional).

Activity:

Ask students how numbers might be used in everyday life. Ask how they might use numbers (e.g., buying something; deciding when to watch television; finding the right size to wear). Ask how numbers are used in school (e.g., as grades, recording attendance).

See page 3 for a description of “Marginal Problems”, an example of which is given here.

Write a sentence for each of the following numbers to show how it might be used in real life.

$2\frac{1}{4}$	20 000
0.10	4.75

Students should realize that they need not use these as ‘pure’ numbers, but may add units. For example, ‘0.10’ could be ‘\$0.10’ or $2\frac{1}{4}$ could be $2\frac{1}{4}$ cups of flour in a recipe.

To get the students started thinking about the many ways people use numbers and the different kinds of numbers they use, discuss with the class whether or not each of the following statements is true. Read each statement and allow students in pairs/ small groups to decide whether the statement is true or false, give an example of the usage described, and justify their choices.

1. Numbers over one million are used in sports. [TRUE; some professional athletes earn salaries in the millions.]

2. Carpenters use decimal numbers. [TRUE; especially when accurate small measures are needed.]

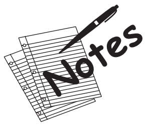
3. Weather forecasters use negative numbers. [TRUE; temperatures below zero are given as negative values.]

4. Musicians use fractions. [TRUE; this can lead to a discussion of quarter notes, half notes, $\frac{3}{4}$ time, etc.]

5. Trucking companies use numbers over 100 000. [TRUE; such a number might be used to record the number of small objects delivered, or the mass of heavy items or annual fuel usage.]

6. Servers in restaurants use decimal numbers. [TRUE; amounts of money use decimal numbers.]

Distribute copies of BLM 1 (Using Numbers). Allow students time to discuss the



Activity 1: Using Numbers

problems in their groups and come to some conclusions. When sharing answers with the class they should justify their responses.

Extensions in Mathematics:

Problem Solving

Jeremy has $12 and Jamie has $10. Each one gives the other person half of his own money. Who now has more? Explain.

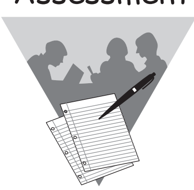
If Jeremy starts with $24 and Jamie with $20, would your answer be the same? Explain.

1. Distribute copies of BLM 2 (Mixed-up Numbers). Read through the first “newspaper article”, recording the numbers as they appear. Ask students what number should be in place of the ‘5’ (“...every year since 5.”) They should realize that 1947 is the only reasonable number to use here. Ask them which number is probably the attendance. Since ‘5312’ doesn’t logically fit anywhere else, this must be the attendance. Since the fair is held on a weekend, two numbers are needed for the dates that are close together. September 3 to 5 is a three-day term (Friday to Sunday). Students may suggest September 5 to 8, but this is a four day period and unlikely to be called a weekend. In addition, the ‘8’ is currently given as “September 8” and must therefore be wrong - i.e. the weekend cannot be from one date to September 8. This leaves the numbers ‘37’ and ‘8’. Ask which one is likely to be the number of layers in a cake. Students may like to imagine a 37-layer chocolate cake, but will realize that ‘37’ is the number of opponents and the cake has 8 layers.

   Have the students work in groups to complete the BLM, reminding them that all numbers are in the wrong places to start with. Tell them to look for hints that will help them place the numbers correctly.

   For example, in #2, the ‘Hexagonal Horrors’ will be 6-sided, and ‘7:15’ is in the form of a particular time. The numbers 36, 57, and 7 must be the percentages in some order, since their total is 100.

Assessment



   For example, in #3, the Canada Day that is yet to occur must be July 1, 2010. The last time the game was played must be ‘1956’ since it is the only other available number identifying a year.

   For example, in #4, the dog walkers must come from 3rd Avenue, since the abbreviation ‘rd’ is used only with the number ‘3’. Since only one raccoon is mentioned, the number ‘1’ must refer to this animal.

Communication

   See “Solutions and Notes” for further analysis and reasonable answers.

   If students can justify their answers and convince their classmates, their answers should be accepted.

Activity 1: Using Numbers

Cross-curricular Activities:

1. After using BLM 3 (A Number Barbecue), (See Family Activities #1, below), have students work in groups to make up a similar story about planning a birthday party, or going to an amusement park, or taking a school trip. Have students leave blanks in the story but provide reasonable values on a separate sheet.

   Have groups exchange problems with each other.

Family Activities:

1. Distribute copies of BLM 3 (A Number Barbecue). Have students work with their families to complete the blanks marked “(______)”. When they return to school with these numbers, discuss the reasonableness of their answers and choose one value for each blank. Have students use those values to solve for the numbers that belong in the square brackets.

   Note that this will mean only one right answer for each set of square brackets to make the checking easier for the teacher. Alternatively allow calculator usage and have groups exchange papers to mark each other’s.

2. Have students discuss with family members the way they use numbers in their jobs/hobbies/daily life. *Some parents/guardians may be willing to come to talk to the class about ways they use numbers.*

Other Resources:

For additional ideas, see annotated “Other Resources” list on page 70, numbered as below.

2. “Developing Sense About Numbers”

Notes

Communication

Six times as many pennies as dimes make 96¢. How many of each coin are there?

Activity 2: Comparing & Ordering

'BLM' refers to the Black Line Masters which follow the Activity Notes

Focus of Activity:

- Ordering decimals
- Comparing fractions to $\frac{1}{2}$ and 1
- Relating number value to place value

What to Assess:

- Accuracy of work
- Ability to generalize
- Articulation of reasons for choices made
- Use of mathematical language

Preparation:

- Make copies of BLMs 4, 5, and 7.
- Provide scissors for each pair/group.
- Make copies of BLMs 6, 8, 9, and 10 (optional).

Activity:

This activity contains a number of BLMs, each of which deals with a different aspect of comparing numbers by value/size. For example, BLM 4 compares decimal numbers, BLM 5 compares fractions to $\frac{1}{2}$ or to 1, and BLM 7 involves placing an errant digit in the correct place value column. BLM 8 challenges students in a game format to sort numbers according to given criteria.

BLM 4: (Decimal Tiles)

For each problem, students have to place each of the ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in a box to make true inequality statements. Start with #1 and ask students what digits they could put in the boxes for a):

$$\square.3 > \square.5$$

Possible responses are 2 and 1, 3 and 1, 3 and 2, 4 and 1, 4 and 2, 4 and 3, etc. Students should realize that the 'decimal 3' and 'decimal five' are irrelevant. It is sufficient to have the digit in the first box greater than the digit in the second.

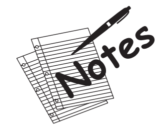
Since there are so many choices for (a), we leave it temporarily and investigate other parts of question 1.

For example, part (b):

$$\square.12 > 4.\square 2$$

Write a fraction you could add to $\frac{1}{10}$ to give a sum close to $\frac{1}{2}$.

Activity 2: Comparing & Ordering



Ask: What can go in the first box?
There are still several choices here, in that any digit from 5 to 9 in the first box will give a true statement. If '4' is used in the first box, the only possible choice for the second box will be '0'.

However, if we look at part (e),

$$1\square.4 < 11.\square$$

we see that the first box can contain only 0 or 1. Students should realize that they do not have to begin with the first part of the problem, but should look for a statement that gives a single choice or very few choices, allowing them to place digits with some confidence.

Problem Solving

The set of tiles along the bottom of the page should be cut out and used to solve each problem. The advantage of this is that students can change their choices without erasing. It has been found that children are far more willing to change their answers when they use the tiles. When they have placed all the digits, they can record their answers with pen or pencil.

Emphasize that, for each problem, all ten digits must be used. That is, no digit can be repeated in two boxes.

Students should be encouraged to use estimation for problem 3. For example, part (a),

$$2.9 \times \square.2 < 4.\square\,7$$

can be thought of as $3 \times \square < 4$. Then it can be seen that the first box could only be '0' or '1', since, if we write $2.9 \times \boxed{2}.2$ and estimate $3 \times 2 = 6$, we have a value which is greater than 4, and therefore cannot be less than $4.\square 7$, no matter what digit is put in the second box.

There are at least two different solutions for each problem. Students should be encouraged to find more than one solution for each. Calculators should be allowed for testing and checking.

Communication

A Challenge: Each group of students could be challenged to devise a similar problem to exchange with another group. Before the exchange, students should either show that there is only one solution, or should have two solutions available for checking. Each group solves another's problem, then gives it back for the composing group to check.

Notes

BLM 5 (Comparing Fractions)

Present a set of fractions with the same denominator to the class. For example,

$$\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}.$$

Ask students which fraction is closest to $\frac{1}{2}$. Students should realize that one of the fractions is actually equal to $\frac{1}{2}$. Ask which of the fractions is closest to 1. Then ask how far it is from 1. Students should realize that $\frac{5}{6}$ is just $\frac{1}{6}$ from 1, whereas each of the other fractions is further from 1 than this.

Repeat with $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$.

Discuss how, if the denominator is an even number, it is possible to write a fraction equal to $\frac{1}{2}$. Then list $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$. Students should realize that none of these is equal to $\frac{1}{2}$, but that $\frac{3}{7}$ and $\frac{4}{7}$ are both close to $\frac{1}{2}$. Students may suggest that the following fraction is equal to $\frac{1}{2}$:

$$\frac{3\frac{1}{2}}{7}$$

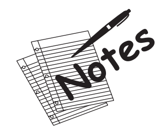
From this it can be seen that $\frac{3}{7}$ and $\frac{4}{7}$ are both close to $\frac{1}{2}$ and that each is one-half of a seventh away from $\frac{1}{2}$.

Although this way of writing fractions may be new to students, it is a valid form with which mathematicians are quite familiar. Allowing students to "play" with fractions in this way may help to remove some of the mystique which, unfortunately, fractions have for many students.

> What fraction could you subtract from 1 to get an answer less than $\frac{1}{2}$?
> Give three different answers.

Distribute copies of BLM 5. Read #1. Because of the previous discussions, students should recognize that the numerator for sevenths should be 3. Ask if it is possible to write a fraction equal to $\frac{1}{2}$ for any of the given denominators. They should remember from the introduction above that this is possible when the denominator is an even number. However, they should realize that the question asks for fractions less than $\frac{1}{2}$. Thus the correct response for eighths is $\frac{3}{8}$, not $\frac{4}{8}$.

Activity 2: Comparing & Ordering

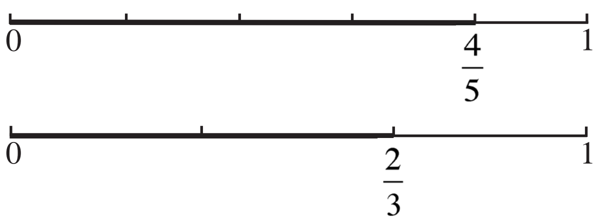


Caution them to read each problem carefully since some ask for fractions close to (but either lesss than or greater than) $\frac{1}{2}$ and some for fractions close to 1; some ask for numerators and some for denominators.

You may wish to provide copies of BLM 6 (Fraction Lines) to assist students with solutions for checking their answers. They should write the 'missing' fractions (e.g., $\frac{2}{3}$, $\frac{2}{4}$, $\frac{3}{4}$, etc.) in place.

Communication

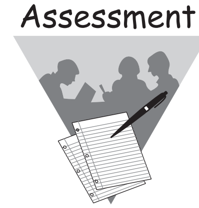
Allow students time to complete the problems. In discussing answers, help students to understand such generalizations as the following: the smaller the difference is between a fraction and $\frac{1}{2}$ (or 1), the closer that fraction is to $\frac{1}{2}$ (or 1). For example, in #3, the difference between $\frac{4}{5}$ and 1 is $\frac{1}{5}$, whereas the difference between $\frac{2}{3}$ and 1 is $\frac{1}{3}$. Since $\frac{1}{5}$ is less than $\frac{1}{3}$, then $\frac{4}{5}$ must be closer to 1 than $\frac{2}{3}$ is. This can be illustrated with fraction lines:



What fraction could you subtract from 1 to get an answer between $\frac{1}{2}$ and 1? Give three different answers.

The explanations students give for #6 can provide an opportunity for assessment. Are explanations rational? Do students use appropriate mathematical language? Is their arithmetic accurate?

Assessment



For example, $\frac{8}{9}$ is $\frac{1}{9}$ less than 1

$\frac{44}{45}$ is only $\frac{1}{45}$ less than 1

Thus, $\frac{44}{45}$ is closer to 1 than $\frac{8}{9}$.

Students who understand this principal will have no difficulty identifying $\frac{99}{100}$ as the given fraction closest to 1.

BLM 7: (Out of Line)

Students should use some knowledge of realistic numbers in various contexts, an understanding of place value, and some estimation skills.

Notes

Activity 2: Comparing & Ordering

In each case one digit is out of alignment. The other digits are in the correct order. For example, in #1, one could estimate 55¢ as $1 for every 2 items and therefore about $\$12\frac{1}{2}$ for 25 items. Therefore the correct answer is $13.75.

> Excerpt from BLM 7:
>
> 1. Twenty-five objects at 55¢ each cost $ $\overset{7}{1\,3\,5}$.

For #3, students should be able to decide which of 1925 or 1952 is reasonable, knowing the Queen has been on the throne for something over 50 years.

Problem Solving

> Excerpt from BLM 7:
>
> 3. Queen Elizabeth II was crowned in $\overset{5}{1\,9\,2}$.

For #6, the only possible answer is 200 since the leading digit should not be 0.

> Excerpt from BLM 7:
>
> 6. Your body has more than $\overset{2}{0\,0}$ bones.

Extensions in Mathematics:

1. Distribute copies of BLM 8 (Putting Numbers in Their Places). Read the instructions with the students. The example should help students place a ‘6’ in box *f*, and a ‘5’ in box *b*. Continuing with Game 1, it can be seen that a ‘4’ belongs in box *a*.

   Note that ‘between 3 and 6’ means 4 or 5. The ‘3’ and the ‘6’ are not included.

   Thus, clue *e* means only 3 or 4. Since the ‘4’ has already been placed in box *a*, the ‘3’ must belong in box *e*.

   The clue for box *c* (‘less than 2’) means that the ‘1’ must belong in box *c*.

   Therefore the ‘2’ must belong in box *d*.

   Students can record possible solutions in each box, and cross out various numbers as they are used elsewhere.

   Game 1 has just one solution. Games 2 and 3 have more than one solution. Game 4 has only one solution.

Activity 2: Comparing & Ordering

Cross-Curricular Activities:

1. BLMs 9 and 10 deal with the Dewey Decimal System, which has been used to classify books in libraries since 1876. BLM 9 provides some background to the system and BLM 10 is designed as an exercise for groups of students.

2. A further problem dealing with the Dewey Decimal System is given below.

   Use what you have learned about the Dewey Decimal System to match the following numbers with the titles of the books. Give reasons for your answers.

Number	Title
599.5248	The World of the Polar Bear
599.786	Endangered Pandas
599.756	Blue Whales
599.52	The Eastern Panther: mystery cat of the Appalachians
599.789168	Polar Bear versus Grizzly Bear
599.7524	Blue Whales and other Baleen Whales
599.789	Giant Pandas
599.786	The Way of the Tiger: natural history and conservation of the endangered big cat

   Two of these books have the same Dewey Decimal number.

Other Resources:

For additional ideas, see annotated “Other Resources” list on page 70, numbered as below.

4. “How Much is a Million?”

5. “How Big is Bill Gates’s Fortune?”

6. “A Game Involving Fraction Squares”

Activity 3: Number Properties

'BLM' refers to the Black Line Masters which follow the Activity Notes

Focus of Activity:

- Exploring properties of numbers (whole numbers and fractions), and operations (subtraction and division)

What to Assess:

- Accuracy in interpreting numbers and operations using the number line
- Ability to condense sequential divisions to one division
- Accuracy of computations

Preparation:

- Make copies of BLMs 11, 12, and 13.
- Make copies of BLM 14 (optional).

Activity:

If 3295 – 741 = 2554
what is
i) 3295 – 740?
ii) 3295 – 739?
iii) 3295 – 745?

Review with the class the idea of representing numbers using a number line. The important idea is that each number is a number of units from zero equal to the number itself. Using a 30-cm ruler to represent numbers from 0 to 30, it is easy to see that any number, say 21, is 21 cm from 0. Decimal numbers can also be illustrated on this ruler. For example, 10.5 is 10.5 cm from 0.

Actually, the unit used is arbitrary since the ruler could be used to represent numbers from 0 to 300 if millimetres are used as the units.

If you do not wish to name the actual units, you can simply speak of a number as being so many "units" from 0. For example, 5 is 5 "units" from 0 , or 3 "units" from 2, or 1 "unit" from 4.

Ask students such questions as:

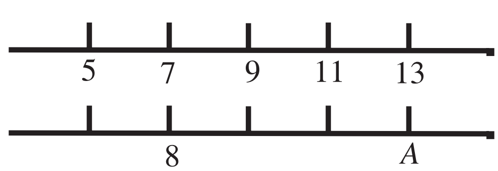
"How far apart are 2 and 4? 5 and 7? 7 and 5? $3\frac{1}{2}$ and $5\frac{1}{2}$?" [2 units]

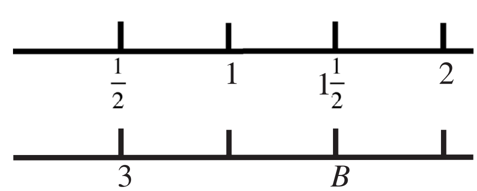
"What number is 2 units from 7? Is there more than one answer? [9 or 5]

"What number is the same distance from 11 as $7\frac{1}{4}$ is from $8\frac{1}{4}$? [12 or 10]

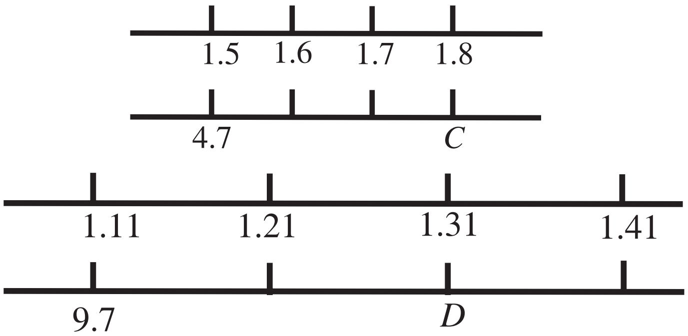
If 7132 – 894 = 6238
what is
i) 7133 – 894?
ii) 7130 – 894?
iii) 7132 – 894?

Display the following pieces of number lines on the blackboard or using the overhead projector, and ask students to give a value for each letter so that the intervals on the two number lines are equal. That is, $A = 14$, $B = 4$, $C = 5.0$, and $D = 10.0$.





Activity 3: Number Properties



Distribute copies of BLM 11 (Lining Up Letters). Have students complete question 1. With the whole class, list the "letter numbers" in order, least to the greatest. Have students complete question 2. The math term is "FRACTIONS". Assign question 3 which the students should complete on their own. The math term is "DECIMALS".

Optional: Explore with students how a calculator could be used to find the answers for question 3. For example, for a), enter $\boxed{27.3}\boxed{-}\boxed{10.1}\boxed{+}\boxed{81.5}\boxed{=}$ to get a display of 98.7. Some problems, such as d), differ slightly from this: enter $\boxed{32}\boxed{-}\boxed{16.9}\boxed{-}\boxed{50}\boxed{=}$ to get a display of -34.9, so the answer is 34.9. In this second calculation, students need to find the difference between L and 30. They should realize that the difference is the same magnitude no matter whether you subtract L from 30 or 30 from L. The sign (showing the direction of movement on the number line) will be '+' in the first case and ' – ' in the second, but the magnitude is 34.9 in each case.

Another way to approach d), for example, is to compare the '32' and the '50'. Moving from one number line to the other, the '32' given is increased by 18. Therefore the '16.9' must also be increased by 18, so that $L = 34.9$. Applying this method to a) shows that the '10.1' is increased by 71.4. Therefore the '27.3' must also be increased by 71.4 and

$$S = 27.3 + 71.4 = 98.7.$$

BLMs 12 and 13 (Devilish Differences - 1 and 2) explore differences between numbers in a different way. Beginning with numbers at the corners of the largest square, students find successive differences.

The difference between 1 and 2 is one. This is written at corner *A*. The difference between 2 and 5 is written at *B*, and so on. Continuing in this way, students eventually reach a square with '0' at each corner. Different numbers of squares will be needed for different problems.

When students reach question 4, you may want to suggest that they place the numbers consecutively around the largest square in the order written as in a) below, rather than randomly as in b).

a) b)

Notes

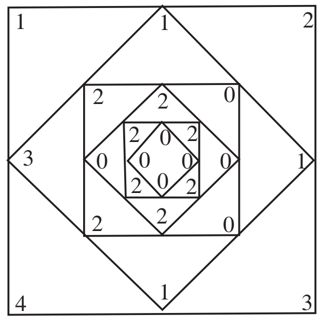
If 2954 – 617 = 2337
what is
i) 2950 – 617?
ii) 2950 – 610?
iii) 2950 – 613?

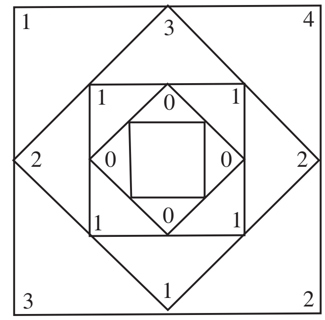
Write 3 other subtraction questions that will have the same answer as 3□52 – 879.

Students may find it easier to draw the squares on graph paper or geo paper.

Write 3 other addition questions with the same answer as $25\square9 + 876$.

Activity 3: Number Properties





However, allowing students to write the numbers in any order around the square shows that the order may affect the number of squares needed to reach zero, but it does not affect the end result as seen above.

For the challenge, encourage students to try various sets of numbers to see if they always reach zeroes. If each student begins with a different set of 4 numbers, the sample will be large enough to say with some validity that it is probably true that any set of 4 numbers will reach zeroes.

Extensions in Mathematics:

1. Distribute copies of BLM 14 (Flowing Division), which introduces students to the idea that two consecutive divisions can be condensed to one. If students are unfamiliar with flow charts, explain that the first flow chart means start with 24, divide by 2, and then divide the answer by 2.

Each flow chart in question 2 is equivalent to the corresponding flow chart in question 1. Thus, students can see that dividing by 2 twice consecutively is equivalent to dividing by 4 and that dividing by 3 and then by 2 is equivalent to dividing by 6. Students should realize that condensing two operations to 1 does not involve addition of the two but multiplication.

That is, $\boxed{\div 2}$ followed by $\boxed{\div 3}$ is NOT equivalent to $\boxed{\div 5}\ (2+3)$ but IS equivalent to $\boxed{\div 6}\ (2\times 3)$.

Understanding this principle enables students to do mental arithmetic as suggested in question 6 (A Challenge). For example, $240 \div 48$ can be thought of as $240 \div 6 \div 8$ which can be done mentally if students know their division facts.

In a similar way, multiplication can be broken down so that, for example, multiplication by 12 can be thought of as multiplication by 2 followed by multiplication by 6 or as $\times 3$ followed by $\times 4$ or as $\times 2$ followed by $\times 2$ again followed by $\times 3$. Facility with such mental arithmetic not only reinforces number facts, but also helps with estimation problems.

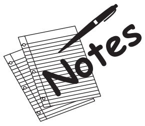
Activity 3: Number Properties

Other Resources:
For additional ideas, see annotated "Other Resources" list on page 70, numbered as below.

7. "Multiplication Games: How We Made and Used Them"

8. "The Influence of Ancient Egypt on Greek and Other Numeration Systems"

10. "Understanding Aztec and Mayan Numeration Systems"



'BLM' refers to the Black Line Masters which follow the Activity Notes

Separate a fraction into two parts. For example, $1\frac{1}{2}$ can be separated into 1 and $\frac{1}{2}$. Look for other ways of separating each of the following into 2 parts in two different ways:

$1\frac{1}{2}$ $\quad 2\frac{2}{5}$ $\quad \frac{7}{4}$

Assessment

Activity 4: Fractions

Focus of Activity:

- Ordering and comparing fractions

What to Assess:

- Appropriate use of fractions in real-life situations
- Identification of fractions between 0 and $\frac{1}{2}$, and between $\frac{1}{2}$ and 1
- Accuracy in ordering fractions with different denominators

Preparation:

- Make copies of BLMs 15, 16, and 17.
- Make copies of BLM 18 (optional).

Activity:

Distribute copies of BLM 15 (About Halves). Each pictured situation suggests a fraction. For example, (a) illustrates $\frac{5}{9}$. The important part of question 1 is the explanation a student gives for stating that this is more or less than one-half. For example, "Five is more than half of nine, so five ninths is more than $\frac{1}{2}$."

It should not take students long to complete this BLM, but it and BLM 16 (More Halves) are preparation for some of the ideas explored with BLM 17 (Estimating Sums of Fractions).

For questions 2 and 3 on BLM 15, students may describe fractions such as $\frac{1}{100}$ or $\frac{99}{100}$. If you wish more precision, ask them for fractions less than $\frac{1}{2}$ but close to it for question 2, and greater than $\frac{1}{2}$ but close to it for question 3.

BLM 16 is, in a sense, the reverse of BLM 15 since students are given a description of a fraction - - e.g., "almost $\frac{1}{2}$" - - and are asked to write a fraction fitting that description. Because the situations described are 'real life' examples, there is scope for considerable discussion. For example, in 2a), would $\frac{4}{10}$ be reasonable or should the fraction be $\frac{12}{30}$ to represent the 30 students in the class? What numbers would be reasonable? This could allow a review of the method of determining equivalent fractions.

Activity 4: Fractions

Students should realize that there is not one correct answer for each, but there are several.

You may wish to generalize the following: For any whole number, n:

1. *The closest fraction to* $\frac{1}{2}$ *that is still less than* $\frac{1}{2}$ *could be written as* $\frac{(n-1)}{2n}$.
2. *The closest fraction to 1 that is still less than one could be written as* $\frac{(n-1)}{n}$.

For example, the closest fraction to $\frac{1}{2}$ *but less than* $\frac{1}{2}$*, with a denominator of 12, is*

$\frac{n-1}{2\times n}=\frac{(6-1)}{2\times 6}$ *or* $\frac{5}{12}$.

The closest fraction to one but less than 1, with a denominator of 12, would be

$\frac{(12-1)}{12}$ *or* $\frac{11}{12}$.

Complete the following with proper fractions:

i) $\frac{1}{2}+\square<1$

ii) $\frac{1}{2}+\square>1$

BLM 17 is a task of estimating sums (except for the Challenge questions). Students should realize that if they add two fractions less than $\frac{1}{2}$, the sum must necessarily be less than 1 (part (b)). However, if the addends are greater than $\frac{1}{2}$, the sum must be greater than 1 (part (c)). Ask what relative size the sum would be if one addend is less than $\frac{1}{2}$ and one is greater. Students should realize that the answer may be less $\left(\text{e.g., } \frac{3}{4}+\frac{1}{8}\right)$ or greater $\left(\text{e.g., } \frac{7}{8}+\frac{1}{4}\right)$ than one, or even equal to 1 $\left(\text{e.g., } \frac{1}{4}+\frac{3}{4}\right)$.

Problem Solving

Question 2 can be extended to ask "How many different answers are there?" There are at least two correct answers for each part of #1, and exploring these will help students when they come to play the game.

Complete the following with proper fractions:

i) $\frac{3}{5}+\square<1$

ii) $\frac{3}{5}+\square>1$

The Game gives students more practice in estimating sums. Since the winner is the one whose total score is closest to 10, students should not be trying to get all their sums between 1 and $1\frac{1}{2}$ because these would give scores of '3'and after 5 turns the total would be more than 10. In introducing the game, ask students for examples that would give scores of 1 or 2. Other fractions such as thirds or fifths could be added to the set to give more possibilities.

Students may raise the question of what to do if the sum is exactly $\frac{1}{2}$ or exactly 1.

Activity 4: Fractions

Notes

Communication

The answer to an addition question is $3\frac{1}{2}$. Write four different questions that could have this answer.

Students should decide this as a class. Some choices are to consider a sum of $\frac{1}{2}$ as giving a score of 1 and a sum of 1 as giving a score of 2, or a sum of either $\frac{1}{2}$ or 1 giving a score of 0.

Students should be estimating the sums. Only if there is a major disagreement among those playing, should paper and pencil come into play. If anyone has a calculator that will add fractions, that could be used to check the answers.

The game can be extended further simply by changing “sum” in the rules to “difference”. Students may not at first realize that this makes it impossible to get a score of ‘3’ on one turn using the fractions given. You may wish to have students try to explain why this is so.

Extensions in Mathematics:

1. Distribute copies of BLM 18 (Sizing Fractions). Each number line given in problem 1 has the positions of “1” and “0” marked. In (b) and (c), students should add other marks to the number lines to indicate accurately where each fraction belongs. For example, to locate $\frac{3}{8}$ in part c) they will need to add a mark half-way between two of the given marks.

   Marking the answers on an acetate copy of the BLM makes it easy to check students’ work by simply matching the acetate copy to their copies.

   Question 2 on the BLM is self-checking. If students identify the places of the given fractions accurately, the letters will spell a math-related word, namely, (a) MATH, (b) EQUALS, (c) ONE HALF, (d) FRACTION, and (e) FOURTHS. Zero and 1 are marked on each line. Students should distinguish between the zero and the letter ‘O’.

   Students may find this easier to do if they first rewrite all of the given fractions with the same denominator, e.g., for e), $\frac{6}{8}, \frac{9}{8}, \frac{4}{8}, \frac{3}{8}, \frac{7}{8}, \frac{1}{8}, \frac{5}{8}$.

2. A similar exercise with decimals could be developed. Give a number line from, say 5.0 to 6.0 with tenths marked on it. Have students place listed decimals (e.g., 5.12, 5.09, 5.68) in appropriate places.

Activity 4: Fractions

Cross-curricular Activities:

1. Discuss whether the verb should be plural or singular when we are talking about only part of something. For example, 2h) on BLM 16:
   Nearly all of the students were at school last Friday.
   $\frac{27}{29}$ of the students (was/were) at school last Friday.

Family Activities:

1. Students could explore their homes to find situations in which knowing whether a fraction is less than or greater than $\frac{1}{2}$ could have relevance, (e.g., TV time, bed time).

Other Resources:

For additional ideas, see annotated "Other Resources" list on page 70, numbered as below.

6. "A Game Involving Fraction Squares"

9. "Building Percent Dolls: Connecting Linear Mesurement to Learning Ratio and Proportion"

Activity 5: Estimation

Focus of Activity:

- The nature of estimation (e.g., when an estimate is suitable, the difference between an exact value and an estimate, and ‘rough’ vs more accurate estimates)

What to Assess:

- Recognition of whether a given number is an estimate or an exact number or amount
- ‘Accuracy’ of estimates
- Recognition of situations for which an estimate is suitable

Preparation:

‘BLM’ refers to the Black Line Masters which follow the Activity Notes

- Make copies of BLMs 19, 20, and 21.
- Provide a calculator for each pair/group.
- Provide newspapers (or have students bring them in).
- Make copies of BLMs 22 and 23 (optional).
- Provide 10 - 12 game markers for each player (optional).

Activity:

For a similar activity, see the Grade 5 Measurement book in this series (“Inside and Out”), BLM 12 (“When to Estimate”)

This activity deals with the nature of estimation rather than specific techniques for estimating, but certain techniques can be introduced or reviewed with each BLM.

Distribute newspapers to each group. Have them circle numbers in headlines and in articles. Working together, read a few articles and decide whether the number used is an estimate or an exact value. Have students give reasons for their conclusions.

Alternatively, you may wish to give some headlines and ask students whether the numbers are exact or approximate. For example,

Population tops 230 million
Lottery winner receives over $100 000
The weight of Drumbo the elephant is 1205 kg.
Attendance at the game was 40 000
The temperature last night went down to –4° C.

Allow goups time to explore the newspapers further and report back to the class instances where they feel the numbers used are estimates, and their reasons for saying this.

Make a list of situations where exact values would be needed and situations where approximate values are suitable (e.g., daily school attendance)

Distribute copies of BLM 19 (About or Exact). Question 1 follows the discussion above. Question 2 can lead to a discussion of where we use estimates every day, and when such approximations are acceptable.

Activity 5: Estimation

BLM 20 (Rough Estimates) introduces the idea that estimates need not be as close as possible to the exact value. The idea of keeping a running total in one's head while shopping leads naturally to the technique of front-end estimating and then to front-end-with-adjustment.

For example, suppose the purchases have the following values: \$4.59, \$3.98, \$2.25 and \$1.59.

Front-end estimating will give \$4 + \$3 + \$2 + \$1 = \$10. This is an under-estimate since only the dollars are considered. We adjust this value through the following reasoning:

59¢ + 59¢ is about 1 more dollar (from \$4.59 and \$1.59)
98¢ is about another dollar (from \$3.98)

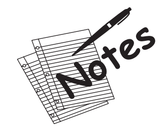
So we add \$2 to the first estimate and we now have a total of \$12.

Under some circumstances this may be an acceptable estimate, but if we are trying to determine whether or not we have enough money to buy these items, we should recognize that \$12 is still an under-estimate since we have so far ignored the 25¢ from \$2.25. An over-estimate of \$13 is more useful in this situation, particularly if we are going to be paying GST and PST as well. Question 2 on BLM 20 challenges the students to estimate and include the taxes as well. The technique given suggests approximating 14% by 15%, and then determining 15% by calculating 10% + $\frac{1}{2}$ of 10%, which is much easier.

Another example of front-end-with-adjustment is given on BLM 20. Students can refer to this while completing question 1.

Distribute copies of BLM 21: (Over and Under). This gives more practice in deciding if an estimate is over or under, although students will find that sometimes it is difficult to tell. Students should be encouraged to use estimation to answer these questions, and not work out the exact values. For example, a) can be estimated as $40 \times 25 = 1000$; thus the estimate is under the actual value, since one of the numbers is "rounded down", while the other remains the same. Similarly, b) can be estimated as $60 \times 80 = 4800$, obviously an over-estimate, since both numbers have been "rounded up". However, if c) is estimated as $70 \times 50 = 3500$, where one number is rounded down and one up, it is difficult to say whether this is an under or over estimate. You might want to work through 2 or 3 examples with the students and have them try to decide how the factors were rounded before multiplying to get the estimates given.

Addition should be somewhat easier. For example, a) could be estimated as $300 + 600 = 900$. Then, to determine whether this is an under or over estimate, note that in rounding 345 to 300, we have subtracted 45, and in rounding 562 to 600 we

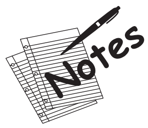


What two-digit number is equal to twice the sum of its digits?

Use of Technology

The use of calculators to check estimates is recommended.

Problem Solving



Activity 5: Estimation

have added 38, so we must have an under-estimate since we subtracted more than we added. Students should be encouraged to “take the numbers apart” in this way.

For further practice, have students work together to make up similar problems for their classmates. Before passing a problem on to another group, they should have analyzed it themselves and be prepared to justify their work.

Extensions in Mathematics:

1. Distribute copies of BLM 22 (A Range of Estimates) to pairs/groups. Even if students use calculators to check their estimates, they will be doing a good deal of estimating in trying to find numbers that meet the given conditions. There may be several answers for each problem (*see Solutions and Notes for possible answers*), so answers should be accepted if students can justify them.

   Ask students how they approached the problems. Did they start by finding two numbers whose sum met the condition given or did they look at the products first? How did they do the necessary estimating? For example, did they round to the nearest 10? Did they use front-end estimation?

   Students can check their answers using calculators.

Communication

2. Distribute copies of BLM 23 (Product Bingo). Aside from one Playing Board and one calculator per group, players need only a few markers, a different type or colour for each player. A simple way to provide these is to have students cut a sheet of paper into small squares and each write his/her own initials on a few.

   The game can be played with groups of 2-6 but is probably best with groups of no more than 4.

   On his/her turn, a student selects two numbers (factors) from the Factor Box, trying to select factors whose product is on the Board. If he/she selects two factors whose product is not on the board, the player loses his/her turn. As the students get better at the game, they start by identifying a product they want (in order to get 4 markers in a row) and examine the factors looking for a likely pair. To use less time for the game, have players try to get only 4 markers in a row.
   Students could take the game home to play with family members.

What two-digit number is equal to four times the sum of its digits?

Family Activities:

Note: For both games that follow, it is important that students estimate, and do not do the calculation. For this reason, it is sometimes necessary to set a time limit on each hand -- i.e., you must record your total (or product) within 10 seconds. As students improve their estimating abilities, this can be shortened.

Activity 5: Estimation

1. Estimating Sums: A game for 3 - 6 people. Remove the face cards from a deck of cards. Deal all cards out to players. (It will not matter if some players have more cards than others because the game is over when any one person runs out of cards). Each player keeps his/her cards in a face-down pile in front of him/her. At a word from the dealer, each player turns over one card. These cards should be placed face up in a central area where all players can see all cards easily. Each player then estimates the sum of the numbers showing, and writes it down. The sum is then calculated (with a calculator if desired). The player whose estimate is closest to the correct sum gets 1 point. Note that more than one player may earn a point on a hand. When any one player is out of cards the game ends. The winner is the one with the most points.

   The game can be made more challenging by making a set of cards with two-digit numbers on them, and including them in the deck.

   A variation on the game can be made by cutting prices out of sales flyers and pasting them onto a set of cards. The players will then be estimating "purchases" using the techniques they used for BLM 20.

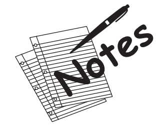
2. Estimating Averages: A game for 3 - 6 people. The rules for this game are similar to the rules for Estimating Sums, with the following exceptions.

   (i) A deck of 50 cards with the numbers from 1 to 50 is used. Students can make such decks out of cardboard, bristol board, or even heavy weight paper.

   (ii) When the cards are turned face up in the centre of the playing area, each player estimates the average of the numbers showing and writes it down. The correct average should be determined using a calculator. The player(s) closest to the actual average gain(s) 1 point.

Other Resources:
For additional ideas, see annotated "Other Resources" list on page 70, numbered as below.

13. "Mental Computation in the Middle Grades"

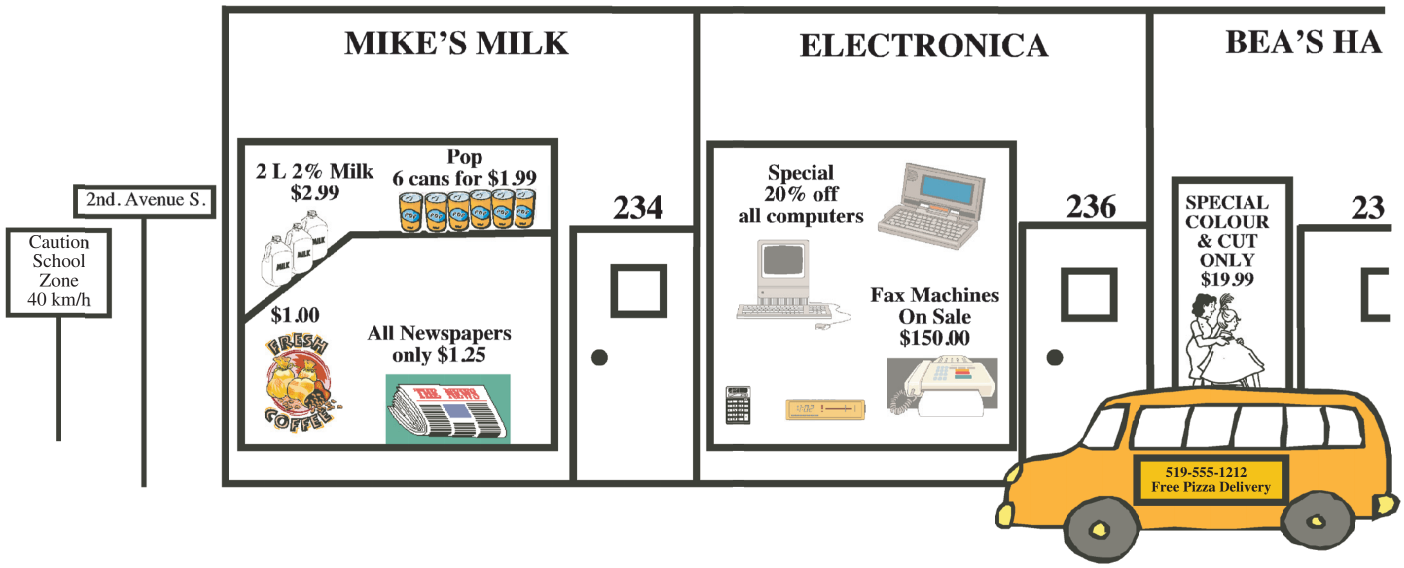


Technology

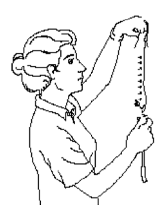
Twice as many dimes as nickels make 50¢. How many of each coin are there?

BLM 1: Using Numbers

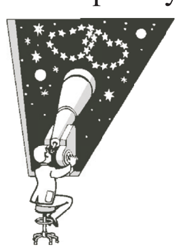
1. List as many ways as you can that numbers are used in this picture.

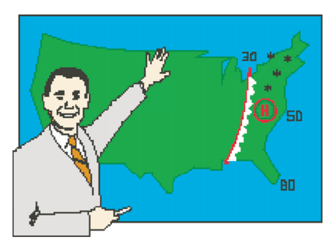


2. Give one way each of the following might use numbers in his/her job.
   a) nurse
   b) architect
   c) photographer
   d) artist
   e) furniture designer
   f) race car driver



3. Each of the following people might use numbers on the job. Select, from the box, a reasonable number that each person might use. Explain your choices.
   a) test pilot
   b) weather forecaster
   c) astronomer
   d) post office clerk
   e) sprinter
   f) grocery store cashier





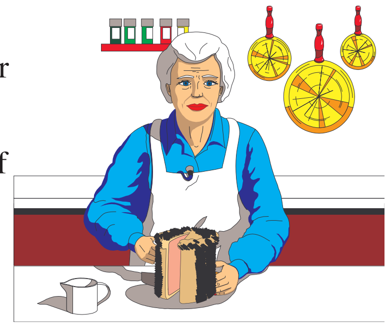
2 kg	50¢ each	1000 km/h
9.5 seconds	30^0 C	21 371 200 km

4. How would you use numbers when
   a) shopping?
   b) finding a book at the library?
   c) building a bird feeder?

LM 2: Mixed-up Numbers

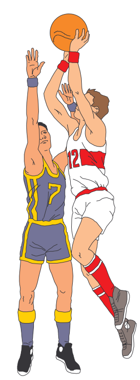
Somehow the numbers in each of the following newspaper articles got mixed up. Rewrite the articles putting the numbers in the correct places. No number is now in the correct place.

1. COTTAGE COUNTRY FAIR A SUCCESS
   The Cottage Country Fair, held on the weekend of September **37** to September **8**, recorded an attendance of **1947**. The Fair has been held every year since **5**. Once again Mrs. Cook won the baking contest, beating out a record number of **3** opponents with her **5312** layer chunky chocolate cake.



2. HORROR MOVIE NOT BAD
   The film *Hexagonal Horrors* will be shown at **3** p.m. on October **36**. The **7**-sided horrors are both furry and funny. At the preview last night, **7:15**% did not like the film, but **6**% gave it **57** stars. The other **31**% gave no opinion.

3. HIGH SCHOOL HOOPSTERS TRY AGAIN
   On Canada Day, July **5**, in the year **257**, the basketball team of **1** students from Hooper High will play basketball against **2** top professionals. The last time such a game was played, in **10**, the final score was **3** to **1956**. One high schooler scored **2010** free throws and Shorty Simpson got the only other Hooper High basket.



4. DOG-WALKING DONE DAILY
   Two **2**-year-olds from **1**rd Avenue Public School have started a dog-walking business. They charge $**6** per hour and will walk each dog once a day on weekdays and **12** times a day on weekends. Already, they have been hired by several families to walk **2.5** dogs and **3** raccoon.

BLM 3: A Number Barbecue

The problems below have many blanks. Your job is to find a reasonable number to put in the blanks marked with **round** brackets - e.g., (_____). You may have to go to the store to check prices or look in a cookbook to find what is needed to make cupcakes, or do some measuring to complete the blanks.

When all the blanks are filled, then calculate answers to put in the **square** brackets.

1. One day, Mr. Cook's class decided to have a barbecue. There were (_____) students in the class and each one wanted (_____) hot dogs. Wieners cost ($_____) for a package of (_____) wieners; hot dog rolls cost ($_____) for a package of (_____) rolls. The class decided that they needed to buy (_____) packages of wieners and (_____) packages of rolls. This would cost them [$______] for wieners and [$______] for rolls for a total of [$______]. Each student should pay [$______] for hot dogs.

2. Each student also wanted (_____) glasses of fruit juice. One litre of juice will give (_____) glasses, so the class had to buy [______] litres at ($_____) per litre. The orange juice would cost [$______], so each student should pay [$______] for juice.

3. Carlo's mother, who runs a bakery, agreed to help the class make cupcakes. It took (_____) egg(s) and (_____) cups of flour to make one dozen cupcakes. The class wanted (_____) cupcakes, so they decided to make (_____) dozen. They needed [______] eggs and [______] cups of flour. One kilogram of flour contains (______) cups, so they needed [______] kg. One dozen eggs cost ($_____) so the eggs they needed cost [______]. Altogether the eggs and flour cost [$______], so each student would have to pay [$______] for cupcakes.

4. The total each student paid was [$______]. They had a really good party.

BLM 4: Decimal Tiles

1. Use all ten tiles to complete the following to make true statements.

a) $\square.3 > \square.5$

b) $\square.12 > 4.\square 2$

c) $26.\square < 5\square.9$

d) $5.\square 7 < 5\square.6$

e) $1\square.4 < 11.\square$

2. Use all ten tiles to complete the following to make true statements.

a) $1\square.\square > \square 2.0$

b) $\square 3.0\square < \square 9.9$

c) $\square 7.93 < 99$

d) $\square.\square < \square.2$

3. Use all ten tiles to complete the following to make true statements.

a) $2.9 \times \square.2 < 4.\square 7$

b) $1.56 \times \square.\square < 7\square.0$

c) $3.08 \times \square.\square 5 < \square.41$

d) $9.36 \times \square.9 < \square 1.5$

Cut these number tiles apart and use them to help solve the problems.

0	1	2	3	4	5	6	7	8	9

BLM 5: Comparing Fractions

1. Write numerators for these fractions so that each fraction is close to, but less than, $\frac{1}{2}$.

$\frac{\_}{8}$ $\frac{\_}{9}$ $\frac{\_}{5}$ $\frac{\_}{7}$ $\frac{\_}{3}$ $\frac{\_}{10}$

Which of your fractions is closest to $\frac{1}{2}$? Which one is furthest from $\frac{1}{2}$? How do you know?

2. Write numerators for these fractions so that each fraction is close to, but greater than, $\frac{1}{2}$.

$\frac{\_}{8}$ $\frac{\_}{9}$ $\frac{\_}{5}$ $\frac{\_}{7}$ $\frac{\_}{3}$ $\frac{\_}{10}$

Which of your fractions is closest to $\frac{1}{2}$? How do you know?

3. Write numerators for these fractions so that each fraction is close to, but less than, 1.

$\frac{\_}{8}$ $\frac{\_}{9}$ $\frac{\_}{5}$ $\frac{\_}{7}$ $\frac{\_}{3}$ $\frac{\_}{10}$

Which of your fractions is closest to 1? Which is furthest from 1? How do you know?

4. Write denominators for these fractions so that each one is close to, but less than, $\frac{1}{2}$.

$\frac{9}{\_}$ $\frac{3}{\_}$ $\frac{6}{\_}$ $\frac{5}{\_}$ $\frac{4}{\_}$ $\frac{8}{\_}$

Which of your fractions is closest to $\frac{1}{2}$? Which one is furthest from $\frac{1}{2}$? How do you know?

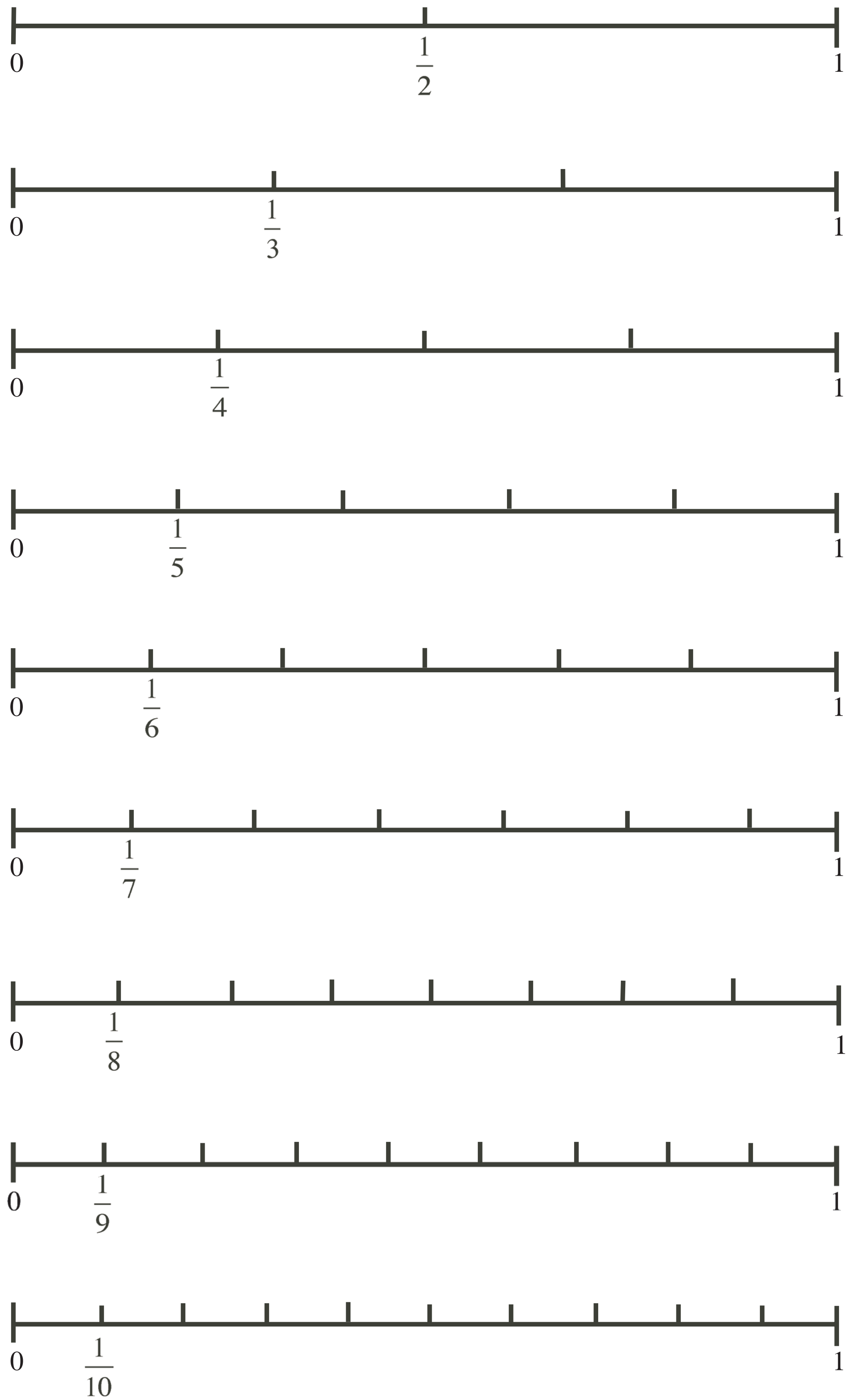
5. Write denominators for these fractions so that each one is close to, but greater than, 1.

$\frac{9}{\_}$ $\frac{3}{\_}$ $\frac{6}{\_}$ $\frac{5}{\_}$ $\frac{4}{\_}$ $\frac{8}{\_}$

6. Which of the following fractions is closest to 1? Explain how you know.

$\frac{8}{9}$ $\frac{44}{45}$ $\frac{27}{37}$ $\frac{99}{100}$ $\frac{11}{4}$

BLM 6: Fraction Lines



BLM 7: Out of Line

Jan's printer was acting funny. When he tried to write answers for his math problems, one of the digits in each number was printed on a different line. Help Jan by writing all the digits in the correct order.

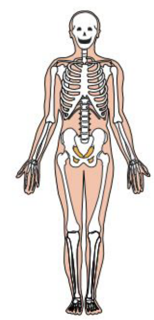
For example,

1
Thirteen bottles of cold drinks at 89¢ each cost 157¢ <u>$11.57</u>

7
1. Twenty-five objects at 55¢ each cost 135¢. ______________

1
2. The number of days in 23 weeks is 16. ______________

5
3. Queen Elizabeth II was crowned in 192. ______________



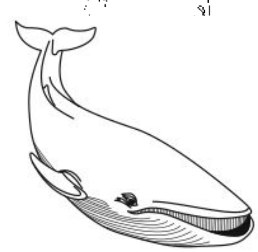
8
4. Each week has 16 hours. ______________

2
5. Your body has more than 00 bones. ______________

8
6. Mount Everest is 708 metres or more than 8 km high. ______________

2
7. A German Shepherd dog weighs about the same as an average Grade 5 student; that is, about 3 kg. ______________

5
8. The heart of a blue whale can weigh 70 kg or about 25 times the weight of an average 10-year old. ______________



BLM 8: Putting Numbers in their Places

In this game a different number belongs in each square of the game board. There are clues given to help you decide which number belongs in each square and the numbers you may use are listed.

For example, in Game 1 below, the second clue tells us that the number in square ***b*** must be more than 4. What choices do we have? The last clue says that the number in box ***f*** must be greater than 5. What number belongs in box ***f***? This means that box ***b*** must contain the number 5. Why? Complete the rest of Game 1.

Game 1: Clues. Use 1, 2, 3, 4, 5, 6

a. between 3 and 6
b. more than 4
c. less than 2
d. less than 4
e. between 2 and 5
f. more than 5

Game Board

a	b	c
d	e	f

Complete Games 2, 3 and 4 the same way. Games 3 and 4 use different sets of numbers.

Game 2: Clues. Use 1, 2, 3, 4, 5, 6

a. greater than 3
b. between 1 and 4
c. less than 3
d. the quotient of 8 and 2
e. less than 4
f. greater than 5

Game Board

a	b	c
d	e	f

Game 3: Clues. Use 6, 7, 8, 9, 10, 11

a. greater than 8
b. less than 7
c. between 8 and 11
d. a prime number
e. greater than 9
f. an even number

Game Board

a	b	c
d	e	f

Games 2 and 3 have more than 1 solution each. How many different solutions can you find?

Game 4: Clues. Use $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}, \frac{4}{3}, \frac{4}{5}, \frac{3}{2}$

a. less than $\frac{1}{2}$
b. between $\frac{1}{2}$ and 1
c. greater than 1
d. greatest of the given fractions
e. half of $\frac{1}{2}$
f. $\frac{1}{2}+\frac{1}{4}$
g. closest to 1
h. fraction not yet used

Game Board

a	b	c	d
e	f	g	h

BLM 9: Dewey Decimals

The Dewey Decimal System is used by libraries to classify and sort their books. The first edition was developed in 1876. The information here is from the 21st. edition of 1996.

There are 10 general classes, each with a number from 0 to 9 in the hundreds place:

000 - Generalities
100 - Philosophy, Psychology
200 - Religion
300 - Social Sciences
400 - Language
500 - Natural Sciences and Mathematics
600 - Technology (Applies Sciences)
700 - The Arts
800 - Literature
900 - Geography, History

The digit in the tens column indicates which of ten divisions the book belongs to. For example,

500 - general science
510 - mathematics
520 - astronomy
530 - physics

Each division is subdivided into ten sections. For example,

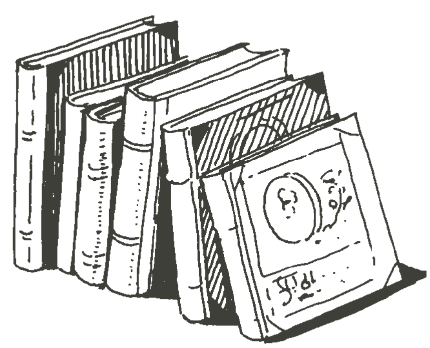
530 - general physics
531 - classical mechanics

Sections are further subdivided by the use of decimal fractions. There may be one or more digits to the right of the decimal. The following shows an example of the subdividing that may take place.

600 - Technology (Applied Sciences)
630 - Agriculture and related topics
636 - Animal husbandry
636.7 - Dogs or 636.8 - Cats

A more detailed example from Geography is given below:

------4 Europe
-----41 British Isles
----411 Scotland
----415 Ireland
-----42 England and Wales
----421 Greater London
----422 South Eastern England
-----43 Central Europe
----431 North Eastern Germany
----438 Poland
----439 Hungary



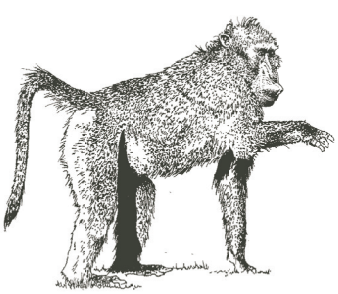
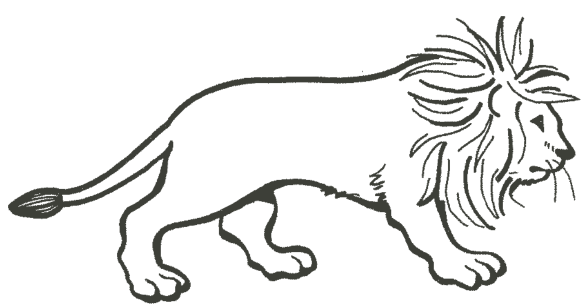
BLM 10: Dewey Animals

The following tables give Dewey Decimal Numbers for various animals. In the first chart, the numbers are in order of size. In the second, the animals are listed alphabetically.

Dewey Decimal Number	Animal
594.756	Tiger
595.386	Crabs
595.78139	Caterpillars
597	Bony Fishes
597.633	Cod
597.69	Flatfishes
597.783	Tuna
598.9	Birds of Prey
598.93	Ostrich
599.5246	Finback Whale
599.542	White Whale
599.752	Ocelot
599.757	Lion
599.759	Cheetah
599.7725	Coyote
599.78	Bears
599.789	Panda
599.865	Baboons
599.88	Great Ape
599.883	Orangutan
599.885	Chimpanzee

Dewey Decimal Number	Animal
599.865	Baboons
599.78	Bears
598.9	Birds of Prey
597	Bony Fishes
595.78139	Caterpillars
599.759	Cheetah
599.885	Chimpanzee
597.633	Cod
599.7725	Coyote
595.386	Crabs
599.5246	Finback Whales
597.69	Flatfishes
599.88	Great Ape
599.757	Lion
599.752	Ocelot
599.883	Orangutan
598.93	Ostrich
599.789	Panda
594.756	Tiger
597.783	Tuna
599.542	White Whale

1. What animals' numbers start with '599'? What animals numbers do not? What do the '599' animals have in common that the other animals do not?
2. What animals' numbers begin with '599.75'? What do these animals have in common?
3. Bony Fishes are numbered 597. Why are whales numbered 599 instead of 597?
4. What are the first three digits for birds?
5. What types of animals are numbered 599.8?


BLM 11: Lining Up Letters

1. Give a number for each letter so the two number lines show the same interval.

a) $2\frac{1}{4}$ — $5\frac{3}{4}$ 1 — ***F***	b) $3\frac{1}{3}$ — $7\frac{2}{3}$ ***I*** — 50	c) $12\frac{3}{7}$ — $19\frac{5}{7}$ 25 — ***C***	d) $17\frac{1}{3}$ — 20 ***O*** — 50
e) 11 — $15\frac{3}{8}$ $31\frac{1}{8}$ — ***T***	f) $3\frac{7}{9}$ — 14 ***N*** — 67	g) $4\frac{2}{5}$ — 9 $16\frac{1}{10}$ — ***A***	h) $23\frac{1}{2}$ — $40\frac{1}{4}$ $2\frac{1}{4}$ — ***R***

2. Write the letter numbers from #1 in order of size, least to greatest. Write the letters below the numbers as shown. If you are correct, the letters will spell a mathematical term.

Number: $4\frac{1}{2}$ _______ _______ _______ _______ _______ _______ _______

Letter: F _______ _______ _______ _______ _______ _______ _______

3. Repeat questions 1 and 2 using these number lines with decimal numbers.

a) 10.1 — 27.3 81.5 — ***S***	b) 1.2 — 6.7 10.3 — ***I***	c) 20.6 — 39.1 4 — ***M***	d) 16.9 — 32 ***L*** — 50
e) 17.6 — 36.25 ***C*** — 23.3	f) 15.5 — 36.05 3.67 — ***A***	g) 3.1 — 7.3 ***D*** — 6.1	h) 12.09 — 13.01 3.01 — ***E***

Number: _______ 3.93 _______ _______ _______ _______ _______ _______

Letter: _______ E _______ _______ _______ _______ _______ _______

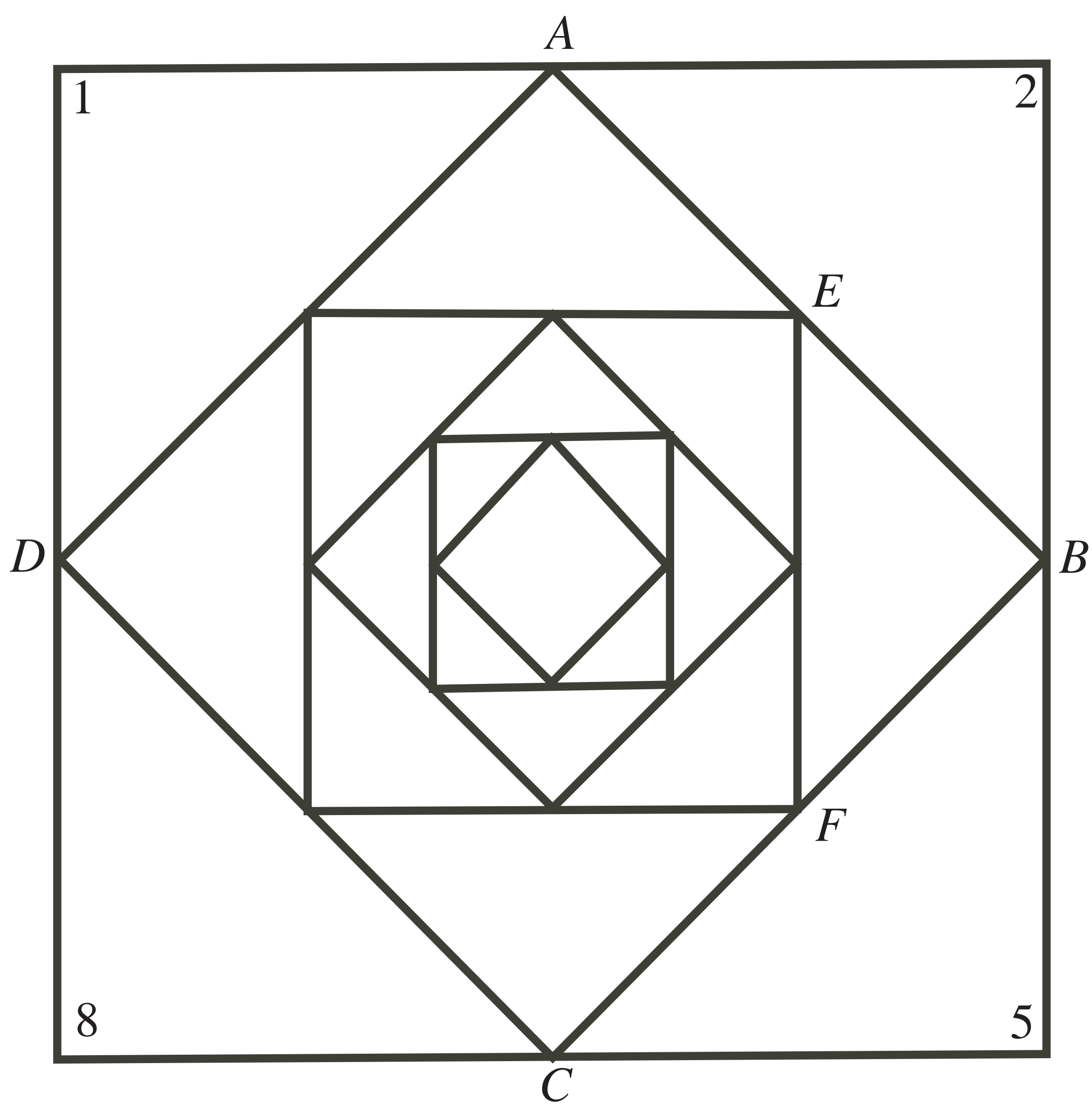
BLM 12: Devilish Differences - 1

1. The largest square below has the numbers 1, 2, 5, and 8 at its corners.
   Find the difference between 1 and 2 and write the answer at corner *A* of the next largest square.
   Write the difference between 2 and 5 at corner *B*, the difference between 5 and 8 at corner *C*, and the difference between 8 and 1 at corner *D*. You should now have a number at each corner of the second largest square.

   Now find the difference between the numbers at *A* and *B*, and write this difference at corner *E* of the third largest square. Write the difference between the numbers at *B* and *C* at corner *F*, and so on.

   Continue in this way until you have numbers at the corners of the smallest square.

   What do you notice about these numbers?



BLM 13: Devilish Differences - 2

2. Repeat, starting with 2, 5, 9, and 7 as shown on the right.

   Did you get the same result?

   Did you need all six squares?

3. Repeat, starting with 15, 23, 98, and 42 as shown below.

   Did you get the same result?

   Did you need all six squares?

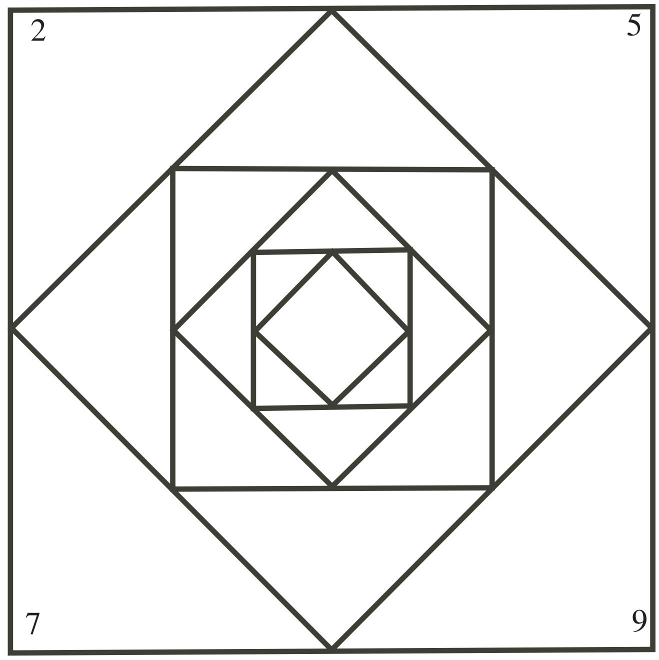
4. Draw your own squares and test the following sets of numbers. Describe your results.
   a) 1, 2, 3, 4
   b) 5, 13, 43, 29
   c) 99, 75, 33, 25

5. Will you get the same result with decimals? with fractions?
   Test using the following sets of numbers
   a) 3.5, 9.7, 8.1, 4.4
   b) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, 1$

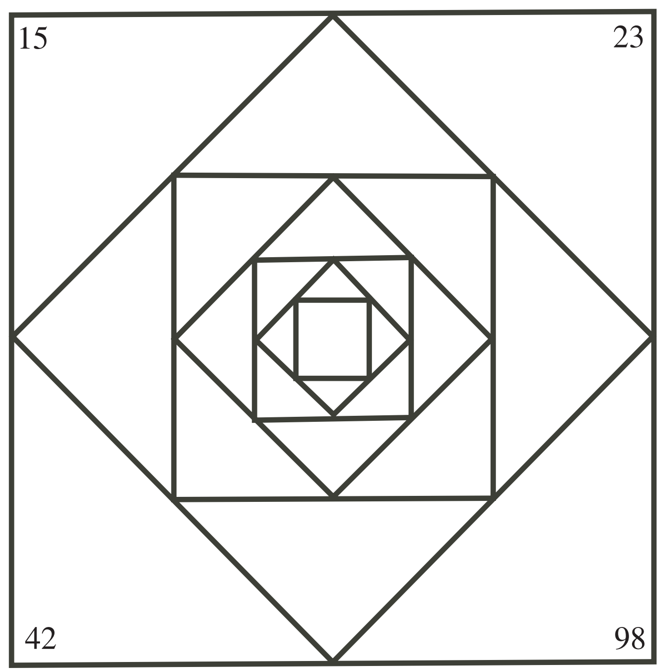
A Challenge:

Try your own set of any four numbers. Do you think the result will always be four zeroes?

2.

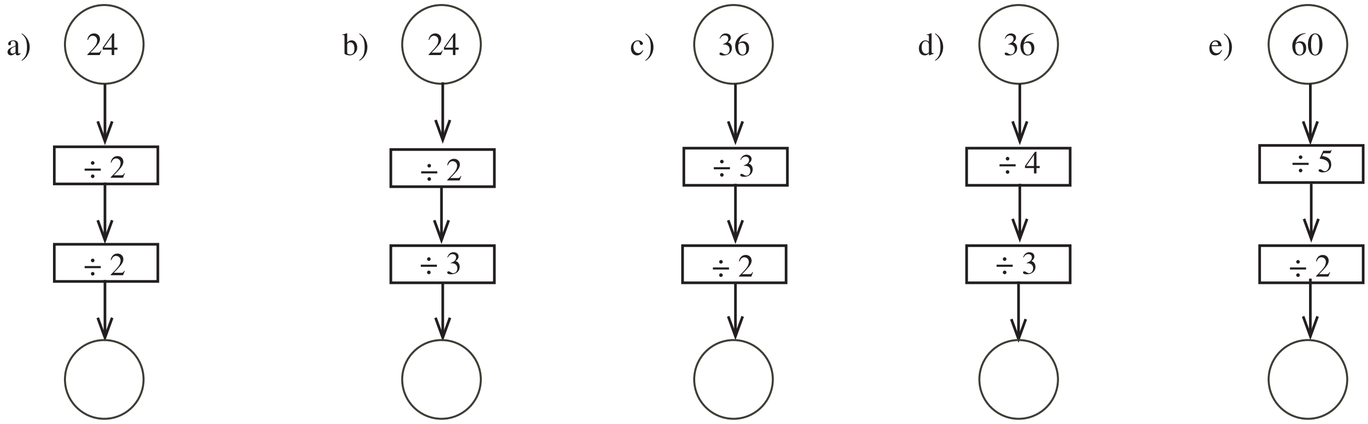


3.

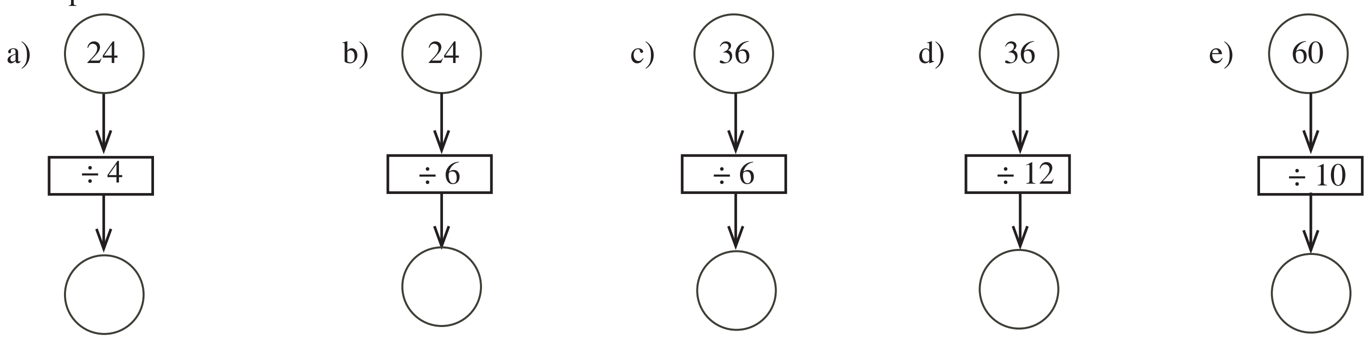


BLM 14: Flowing Division

1. Complete each flow chart.



2. Complete each flow chart.



3. Compare the results for questions 1 and 2. What do you notice? Why does this happen?

4. What single division would give the same result as each of the following combinations?
   a) divide by 2, then divide by 4
   b) divide by 4, then divide by 2
   c) divide by 3, then divide by 3 again
   d) divide by 5, then by 5 again
   e) divide by 3, then divide by 5
   f) divide by 2, then by 3, then by 3 again

5. What two or three divisions in a row would give the same result as each of the following single divisions?
   a) divide by 12
   b) divide by 40
   c) divide by 18
   d) divide by 25
   e) divide by 36
   f) divide by 48

A Challenge:

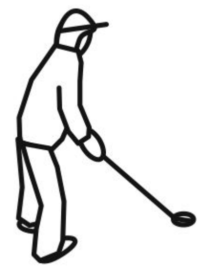
6. Use your answer to question 5 to help you complete the following mentally.
   a) $60 \div 12$
   b) $108 \div 12$
   c) $54 \div 18$
   d) $225 \div 25$
   e) $240 \div 48$
   f) $108 \div 36$

BLM 15: About Halves

1. For each of the following situations the cartoon suggests a fraction. Tell whether the fraction is more than $\frac{1}{2}$ or less than $\frac{1}{2}$. Explain why you think so.

(a) Cesar struck out 5 of 9 batters.

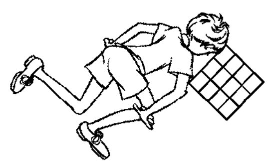
(b) Rosita scored par on 7 out of 18 holes.



(c) Jerry made 9 hits in 19 times at bat.

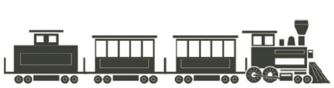
(d) Sally scored on 6 out of 10 free throws.

(e) Luis lost 12 of his 25 pennies down the storm drain.



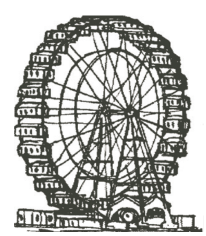
(f) Luigi’s mother put raisins in 22 of the 3 dozen cookies she baked for the shool sale.

(g) Logan saw that there were cattle in 15 of the 23 box cars on the train.



(h) Mei Lin’s cat had 7 kittens. Five of them were tabbies.

(i) Ari went on 11 of the 19 rides at the fair.



2. Describe 2 situations illustrating a fraction less than one-half.

3. Describe 2 situations illustrating a fraction greater than one-half.

BLM 16: More Halves

For each of the following sentences, write a fraction that will make sense. Two examples are done for you.

Example 1: Giulio missed (almost half) ____$\frac{4}{10}$____ of his soccer games when he broke his leg.

Example 2: Melinda took (more than half, but not quite all) ____$\frac{6}{7}$____ of her books back to the library.

1. Give another possible answer for each of the examples above.

2. Write a fraction for each of the following. Use a different fraction for each sentence.

a) (Almost half) _____ of Erica's classmates watched TV last night.

b) (Slightly more than half) _____ of Tuan's family moved here from Thailand.

c) Gorem gave (almost half) _____ of his allowance to "Feed the Children".

d) (More than half, but not quite all) _____ of the class passed the test.

e) (Almost none) _____of the class missed the field trip.

f) Julian found (almost half) _____ of the library books he wanted.

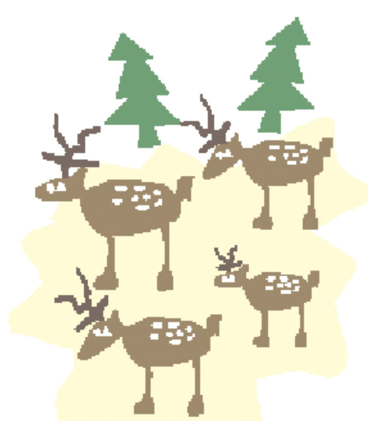
g) Mr. Teecher marked (more than half, but not all) _____of the homework in one hour.

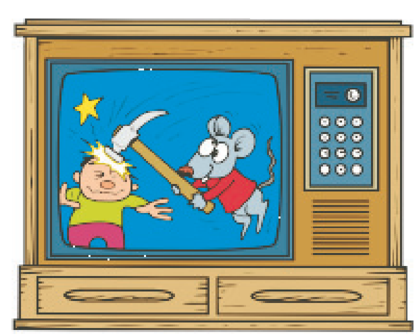
h) (Nearly all) _____ of the students were at school last Friday.

i) (Almost all) _____ of Santa's reindeer were sick in July.

j) Indra managed to stay on her horse over (almost all) _____ of the jumps.

k) (Nearly half) _____ of the items of e-mail that Jack received on Wednesday was "spam".





BLM 17: Estimating Fractions

1. Use fractions from the following set to complete the statements.

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$
$\frac{1}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{2}{4}$

a) Two fractions that are less than $\frac{1}{2}$ are _____ and _____.

b) Two fractions that are almost 1 if they are added together are _____ and _____.

c) Two fractions whose sum is greater than 1 are _____ and _____.

d) Two fractions whose difference is almost zero are _____ and _____.

Two Challenges:

e) Two fractions whose product is about $\frac{1}{2}$ are _____ and _____.

f) Three fractions whose sum is less than 1 are _____, _____, and _____.

2. Which parts of #1 have more than one correct answer? Give a second answer for each.

3. **Game: Fraction sum**

Choose two of the fractions in the box above. Add them. Determine your score by the following:

If the sum is between 0 and $\frac{1}{2}$, your score is 1.

If the sum is between $\frac{1}{2}$ and 1, your score is 2.

If the sum is between 1 and $1\frac{1}{2}$, your score is 3.

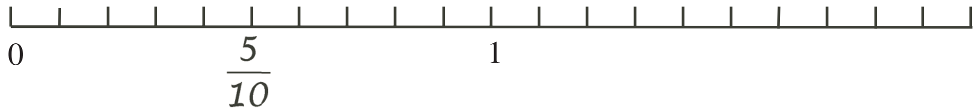
Play for 5 turns each. Try to get a total score close to 10. The winner is the one whose total score is closest to 10.

Names of Players	Scores					
	1	2	3	4	5	Total

BLM 18: Sizing Fractions

1. Write each fraction listed in the correct place on the number line. One is done for you.

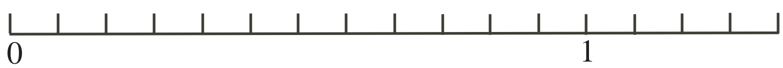
a) $\frac{5}{10}, \frac{7}{10}, \frac{1}{10}, \frac{14}{10}$



b) $\frac{2}{20}, \frac{6}{20}, \frac{17}{20}, \frac{10}{20}$

c) $\frac{3}{4}, \frac{3}{8}, \frac{1}{8}, \frac{7}{8}$

d) $\frac{2}{6}, \frac{5}{6}, \frac{1}{6}, \frac{7}{6}$



e) $\frac{3}{5}, \frac{2}{5}, \frac{6}{5}, \frac{1}{5}$

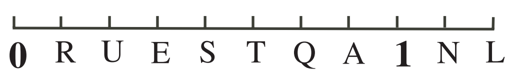
2. The place for each fraction is marked by a letter. Write the letter in the box under the correct fraction. If you are correct, the letters will spell out a word. Two letters are done for you. Not all letters will be used.

a)

$\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{8}$	$\frac{9}{8}$
	A	T	

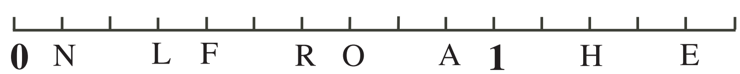
b)

$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{5}{4}$	$\frac{1}{2}$



c)

$\frac{7}{10}$	$\frac{1}{10}$	$\frac{7}{5}$	$\frac{12}{10}$	$\frac{9}{10}$	$\frac{3}{10}$	$\frac{2}{5}$



d)

$\frac{2}{5}$	$\frac{3}{5}$	$\frac{11}{10}$	$\frac{8}{10}$	$\frac{1}{5}$	$\frac{13}{10}$	$1\frac{1}{2}$	$\frac{3}{10}$

e)

$\frac{3}{4}$	$\frac{9}{8}$	$\frac{2}{4}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{8}$	$\frac{5}{8}$

BLM 19: About or Exact

1. Tell whether the number in each sentence below is likely an estimate or an exact number. Circle "about" if you think the number is an estimate and "exact" if you think the number is exact. Be prepared to give reasons for your choice.

a) Jerry drank 1 litre of milk every day.	exact	about
b) Mr. Jackson received 3 ties for Father's day.	exact	about
c) Miss McLachlan drives 45 km to work each day.	exact	about
d) The rock concert was attended by 4000 fans.	exact	about
e) When Angela was sick, her temperature was 39° C.	exact	about
f) The cake recipe called for 2 tablespoons of butter.	exact	about
g) A half-hour TV program has 8 minutes of commercials.	exact	about

2. There are several numbers in the following story. Decide if each number is probably an estimate or an exact number and write "about" or "exactly" in the blank next to that number. Compare your choices with your partner's, and discuss your reasons.

Elizabeth woke up at _________ 7:30 a.m. on a Friday. She got dressed and ate her breakfast in ________ 30 minutes. The school bus was due at ___________ 8:15 a.m. so she left her house at ___________ 8 o'clock for the 8 minute walk to the bus stop. She arrived at school at ___________ quarter to nine. She went right to her classroom because her class and the other two grade 5 classes were going to a concert that day. There were __________ 24 students in her class, __________ 26 in her friend Catherine's class, and ___________ 21 in the third Grade 5 class. Each bus had seats for __________ 34 people. Catherine and Elizabeth decided that _________ 3 buses would be needed, since _________ 3 parents and 1 teacher would be going with each class. The buses were finally loaded by _________ 9:30 a.m. and, after a trip of __________45 minutes, they arrived at the concert hall. Tickets for the concert cost __________ $5 for each student and __________ $9.50 for each adult. The concert was scheduled to take _________ $1\frac{1}{2}$ hours, after which they would return to the school. The teachers thought that they would probably be back at school by __________ 1:30 p.m.

BLM 20: Rough Estimates

Sometimes estimates should be close to the exact number or amount, but at other times a "rough" estimate can be used. For example, suppose you are shopping. You have $15.00 to spend, so you want a rough estimate to be sure that when you reach the cash register, you will have enough money to buy the items you have selected.

> One way to make a rough estimate is by using front-end-with-adjustment estimation.
>
> For example, suppose your purchases are $2.39 + $1.98 + $2.25 + $1.59.
> Front-end estimating will give $2 + $1 + $2 + $1 = $6.
> This is an under-estimate since only the full dollar amounts are used.
> Adjust the value in the following way: 39¢ + 59¢ is about $1.
> 98¢ is about $1.
> The new estimate is $6 + $1 + $1 = $8. Since you have $15.00 to spend, you will have enough for these purchases.

1. Make a rough estimate for each of the following, to see if your $15 is enough money or not. (Ignore the taxes).

a)	$3.50 + $2.95 + $1.99 + $4.49	yes	no
b)	$3.49 + $3.76 + $2.15 + $4.98	yes	no
c)	$3.98 + $3.44 + $2.19 + $6.50	yes	no
d)	$9.33 + $2.98 + $1.59	yes	no
e)	$2.44 + $4.32 + $4.39 + $ 0.98	yes	no

2. **A Challenge:**
In Ontario, the GST plus the PST totals 14%. That means for every dollar you spend, you will have to pay an extra 14¢. If you include the tax, for which of the purchases above will you have enough money?

> Using an estimated cost of $12, taxes can be estimated in the following way:
> The total tax is 14% which is roughly 15%, or 15¢ for every dollar.
> So the tax will be roughly 12 x 15¢ which equals $12\times10¢+\frac{1}{2}(12\times10¢)$
> $=120¢+60¢$
> $=180¢=\$1.80.$
> So the estimate of total cost is $13.80, which is still less than $15.00.

3. Use your calculator to check your estimates.

BLM 21: Over and Under

1. Sometimes when you make a rough estimate, it doesn't matter if the estimate or over (more than) or under (less than) the exact number or amount. For each of the following, circle the description that describes the estimate best.

Calculation	Estimate	Description		
a) 43 x 25	40 x 25 = 1000	OVER	UNDER	HARD TO TELL
b) 58 x 77	60 x 80 = 4800	OVER	UNDER	HARD TO TELL
c) 73 x 49	70 x 50 = 3500	OVER	UNDER	HARD TO TELL
d) 91 x 62	90 x 60 = 5400	OVER	UNDER	HARD TO TELL
e) 24 x 64	25 x 60 = 1500	OVER	UNDER	HARD TO TELL
f) 17 x 91	20 x 90 = 1800	OVER	UNDER	HARD TO TELL

2. If you circled "HARD TO TELL" for any of the above, explain why.

3. Now try some addition questions.

Calculation	Estimate	Description		
a) 345 + 562	300 + 600 = 900	OVER	UNDER	HARD TO TELL
b) 239 + 876	200 + 900 = 1100	OVER	UNDER	HARD TO TELL
c) 588 + 407	600 + 400 = 1000	OVER	UNDER	HARD TO TELL
d) 987 + 456	1000 + 500 = 1500	OVER	UNDER	HARD TO TELL
e) 888 + 755	900 + 800 = 1700	OVER	UNDER	HARD TO TELL

4. If you circled "HARD TO TELL" for any of the additions, explain why.

5. Did you find it easier to judge estimates with multiplication or with addition? Why?

BLM 22: A Range of Estimates

1. Choose two numbers from the box on the right that will fit the conditions described.

63		51
	87	
92		16
	28	
36		12
	77	
46		82
	56	
32		68
	71	

   a) Their sum is about 90 and their product is close to, but less than 2000. ___________

   b) Their sum is about 80 and their product is close to, but more than 1600. ___________

   c) Their sum is close to 120 and their product is close to 3800. ___________

   d) Their sum is close to, but less than 100, and their product is close to, but more than 900. ___________

   e) Their difference is about 70 and their sum is about 100. ___________

   f) Their difference is about 30 and their product is about 4000. ___________

2. For each question below, select two numbers from the box. Record these on a separate piece of paper. Estimate their sum and their product. Write these numbers in the blanks.

   a) Their sum is about ___________ and their product is close to, but less than ___________.

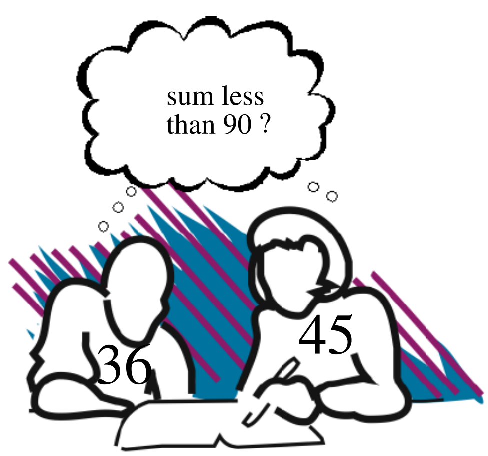
   b) Their sum is about ___________ and their product is close to, but more than ___________.

   c) Their difference is about ___________ and their sum is about ___________.

   d) Their difference is about ___________ and their product is about ___________.

   e) Their sum is close to, but less than ___________ and their product is close to, but more than ___________.

3. Trade your copy of #2 with the estimates included) with another group. Each group should try to decide which numbers from the box were used. Compare answers. Explain any differences.



BLM 23: Product Bingo

To play the game each player needs 10 - 12 markers to place on the board according to the rules below.

Playing Board

720	1080	432	900
144	648	1152	360
384	288	1440	540
2700	810	405	324

Players take turns.
On your turn, select two different factors from the box below that you think will have a product on the Playing Board. State the factors you chose and the number on the Board you think is their product.
Check with a calculator.
If you were correct, place one of your markers on the square containing the product. If you were not correct, you lose that turn.

The winner is the first to get four markers in a row, in any direction, on the Board.

Factors. Choose two of these factors on your turn.

6	15	30	54
8	16	36	60
9	18	45	72
12	24	48	90

Solutions & Notes

Marginal Problems

Page	Problem	Discussion
3	If $4 \div \frac{1}{2} = 8$, is $4 \div \frac{1}{4}$ greater or less than 8? Why?	Since the second divisor $\left(\frac{1}{4}\right)$ is less than the original divisor $\left(\frac{1}{2}\right)$, the quotient will be greater than 8. OR $4 \div \frac{1}{4}$ means "How many quarters in 4?" and there are more quarters than halves in 4.
7	Write a sentence for each of the following numbers to show how it might be used in real life. $2\frac{1}{4}$; 20 000; 0.10; 4.75.	Answers will vary. Possibilities are: I spent \$4.75 on candy; there were 20 000 people at the concert; I used $2\frac{1}{4}$ m of cloth to make my vampire costume; I bought a package of candy that weighed 0.10 kg.
8	Jeremy has \$12 and Jamie has \$10. Each one gives the other person half of his money. Who now has more? Explain.	Jeremy gives away \$6 and receives \$5, so he now has \$11. Jamie gives away \$5 and receives \$6, so he now also has \$11.
8	If Jeremy starts with \$24 and Jamie with \$20, would your answer be the same? Explain.	Although the amounts of money differ from the problem above, the ratios of the amounts remain the same. Once again, they will end with the same amount of money. To determine if students understand the problem ask what would happen if they started with \$36 and \$30. Ask for two other amounts of money they could start with and still end up with the same amount each.
9	Six times as many pennies as dimes makes 96¢. How many of each coin are there?	This problem lends itself to trial - and - adjustment and look - for - a - pattern strategies. The key idea is that 96¢ minus the number of pennies must be a multiple of 10¢. Six dimes and 36 pennies total 96¢.
11	Write a fraction you could add to $\frac{1}{10}$ to give a sum close to $\frac{1}{2}$.	Since $\frac{1}{2} = \frac{5}{10}$, you could add $\frac{3}{10}$ to $\frac{1}{10}$ to get a sum close to $\frac{1}{2}$. Other answers are possible $\left(\text{e.g., } \frac{6}{20}, \frac{1}{4}, \frac{1}{3}\right)$. Also, since the problem does not specify that the answer must be less than $\frac{1}{2}$, any fraction equal to $\frac{1}{2}$ is also suitable $\left(\text{e.g., } \frac{5}{10}, \frac{4}{8}\right)$ since that will give an answer slightly greater than $\frac{1}{2}$ but close to it.

Solutions & Notes

Page	Problem	Discussion
13	What fraction could you subtract from 1 to get an answer less than $\frac{1}{2}$? Give three different answers.	Any fraction greater than $\frac{1}{2}$ is suitable.
13	What fraction could you subtract from 1 to get an answer between $\frac{1}{2}$ and 1? Give three different answers.	Any fraction between 0 and $\frac{1}{2}$ is suitable.
17	If $3295 - 741 = 2554$, what is i) $3295 - 740$? ii) $3295 - 739$? iii) $3295 - 745$?	i) Subtract 1 less and the answer is 1 more (2555) ii) Subract 2 less and the answer is 2 more (2556) iii) Subtract 4 more and the answer is 4 less (2550)
17	If $7132 - 894 = 6238$, what is i) $7133 - 894$? ii) $7130 - 894$? iii) $7122 - 894$?	i) Increase the subtrahend by 1 and the answer is 1 more (6239) ii) Decrease the subtrahend by 2 and the answer is 2 less (6236) iii) Decrease the subtrahend by 10 and the answer is 10 less (6228)
18	If $2954 - 617 = 2337$, what is i) $2950 - 617$? ii) $2950 - 610$? iii) $2950 - 613$?	i) Decrease the subtrahend by 4 and the answer is 4 less (2333) ii) Decrease the minuend in i) by 7 and the answer is 7 more (2340) iii) Increase the minuend in ii) by 3 and the answer is 3 less (2337) Comparing iii) with the original we see that both subtrahend and minuend are decreased by 4 and the answer remains the same.
18	Write three other subtraction questions that will have the same answer as $3\square52 - 879$.	If both subtrahend and minuend are increased or decreased by the same amount, the answer will remain the same, e.g., $3\square\mathbf{50} - 87\mathbf{7}$ (decrease each by 2) $3\square\mathbf{42} - 8\mathbf{69}$ (decrease each by 10) $\mathbf{4}\square52 - \mathbf{1}869$ (increase each by 1000)

Solutions & Notes

Page	Problem	Discussion
19	Write 3 other addition questions that will have the same answer as $25\square 9+876$.	If one addend is increased and the other decreased by the same amount, the answer will remain the same, e.g., $25\square\mathbf{8}+87\mathbf{7}$ $2\mathbf{6}\square 9+\mathbf{7}76$ $\mathbf{1}5\square 9+\mathbf{1}876$
21	Separate a fraction into two parts. For example, $1\frac{1}{2}$ can be separated into 1 and $\frac{1}{2}$. Give two other ways of separating each of the following into two parts. $1\frac{1}{2}$ $2\frac{2}{5}$ $\frac{7}{4}$	Answers will vary. Possibilities are $1\frac{1}{2}=\frac{3}{4}+\frac{3}{4}$ or $1\frac{1}{4}+\frac{1}{4}$ $2\frac{2}{5}=1+1\frac{2}{5}$ or $1\frac{1}{5}+1\frac{1}{5}$ $\frac{7}{4}=1+\frac{3}{4}$ or $\frac{5}{4}+\frac{2}{4}$
22	Complete the following with proper fractions: i) $\frac{1}{2}+\square<1$ ii) $\frac{1}{2}+\square>1$	i) Any fraction between 0 and $\frac{1}{2}$ is suitable ii) Any fraction greater than $\frac{1}{2}$ is suitable
22	Complete the following with proper fractions: i) $\frac{3}{5}+\square<1$ ii) $\frac{3}{5}+\square>1$	i) Any fraction between 0 and $\frac{2}{5}$ is suitable $\left(\text{e.g., }\frac{1}{5},\frac{1}{6},\frac{1}{7}\right)$ ii) Any fraction greater than $\frac{2}{5}$ is suitable $\left(\text{e.g., }\frac{3}{5},\frac{3}{4},\frac{1}{2}\right)$
23	The answer to an addition question is $3\frac{1}{2}$. Write four different questions that could have this answer.	Possible answers: $3+\frac{1}{2}=?$ $3\frac{1}{4}+\frac{1}{4}=?$ $2\frac{1}{2}+1=?$ $1\frac{1}{4}+2\frac{1}{4}=?$ $3+\frac{4}{8}=?$ $1+1+1+\frac{1}{2}=?$

Solutions & Notes

Page	Problem	Discussion
26	What two - digit number is equal to twice the sum of its digits?	This problem lends itself to trial-and-adjustment and look-for-a-pattern strategies. $18 = 2\times(1+8) = 2\times 9$
27	What two - digit number is equal to four times the sum of its digits?	There are several answers: $12 = 4\times(1+2) = 4\times 3$ $24 = 4\times(2+4) = 4\times 6$ $36 = 4\times(3+6) = 4\times 9$ and $48 = 4\times(4+8) = 4\times 12$
28	Twice as many dimes as nickels make 50¢. How many of each coin are there?	Four dimes and two nickels make 50¢.

Activity 1: Using Numbers

BLM 1: Using Numbers

1. Answers will vary but may include speed limits, street signs, prices, quantities of goods, building numbers (addresses), telephone number.

2. Suggestions are given here. Students may identify other uses.
   a) A nurse uses numbers in taking temperature or blood pressure, and in checking dosages.
   b) An architect uses numbers in measuring sizes, pricing estimates, and bookkeeping.
   c) A photographer uses numbers when determining the size of pictures or measuring chemicals to develop pictures.
   d) An artist uses numbers to determine amount of paint needed, or amount of materials for sculpture or casting metals, and costs of such materials.
   e) A furniture designer uses numbers to show sizes of parts of the furniture so the manufacturer knows what size to make each part.
   f) A race car driver uses numbers to show the speed he/she is going, or what level of power is needed in an engine.

3. The numbers each person would use, and possible ways the number is used are given below:
   a) test pilot <u>1000 km/h</u>: speed of a plane
   b) weather forecaster <u>30</u>^{o} <u>C</u>: tomorrow's temperature
   c) astronomer <u>21 371 200 km</u>: distance between planets
   d) post office clerk <u>50¢ each</u>: cost of stamps
   e) sprinter <u>9.5 seconds</u>: time taken to run a race
   f) grocery store cashier <u>2 kg</u>: weight of a bag of sugar

4. Answers will vary.

Solutions & Notes

BLM 2: Mixed-up Numbers

The articles as they should be written, are given below.

1. The Cottage County Fair, held on the weekend of September **3** to September **5** recorded an attendance of **5312**. The fair has been held every year since **1947**. Once again Mrs. Cook won the baking contest, beating out a record number of **37** opponents with her **8** layer chunky chocolate cake.

2. The film Hexagonal Horrors will be shown at **7:15** p.m. on October **31**. The **6**-sided horrors are both furry and funny. At the preview last night **36**% did not like the film, but **57**% gave it **3** stars. The other **7**% gave no opinion.

   Students may interchange the percentages, depending on whether they think the audience liked the film or not. This leaves ‘3’ and ‘31’. Since October 31 is Halloween, this seems a reasonable date for the film preview, and 31 stars seems rather excessive.

3. On Canada Day, July **1**, in the year **2010**, the basketball team of **10** students from Hooper High will play basketball against **5** top professionals. The last time such a game was played, in **1956**, the final score was **257** to **3**. One high schooler scored **2** free throws and Shorty Simpson got the only other Hooper High basket.

   The student scoring free throws must have gotten at least 2, but it is unlikely that he got many more. Also, his/her 2 plus the one Shorty Simpson scored makes 3. No other numbers would fit in these blanks.

4. Two **12** year olds from **3**rd Avenue Public School have started a dog walking business. They charge **$2.50** per hour and will walk each dog once a day on weekdays and **2** times a day on weekends. Already, they have been hired by several families to walk **6** dogs and **1** raccoon.

   The ‘2.50’ makes no sense anywhere but as $2.50. Walking 6 dogs and 1 raccoon more than 2 times a day on the weekend would take most of the weekend, but students may suggest that they could walk them all at the same time and may suggest walking 2 dogs and 1 raccoon 6 times a day on weekends.

Activity 2: Comparing and Ordering

BLM 4: Decimal Tiles

Answers will vary. Here are two possibilities.

1. Solution 1:

   a) $\boxed{7}.3 > \boxed{2}.5$ b) $\boxed{9}.12 > 4.\boxed{8}2$ c) $26.\boxed{6} < 5\boxed{1}.9$ d) $5.\boxed{3}7 < 5\boxed{5}.6$

   e) $1\boxed{0}.4 < 11.\boxed{4}$

   Solution 2:

   a) $\boxed{2}.3 > \boxed{0}.5$ b) $\boxed{8}.12 > 4.\boxed{3}2$ c) $26.\boxed{9} < 5\boxed{4}.9$ d) $5.\boxed{6}7 < 5\boxed{7}.6$

   e) $1\boxed{1}.4 < 11.\boxed{5}$ Options for the first box in e) are limited: only a 0 or a 1 is suitable.

Solutions & Notes

2. Solution 1:

a) $1\boxed{7}.\boxed{0} > \boxed{1}2$ b) $\boxed{3}3.0\boxed{8} < \boxed{6}9.9$ c) $\boxed{2}7.93 < 99$ d) $\boxed{4}.\boxed{9} < \boxed{5}.2$

Solution 2:

a) $1\boxed{9}.\boxed{5} > \boxed{1}2.0$ b) $\boxed{6}3.0\boxed{4} < \boxed{8}9.9$ c) $\boxed{7}7.93 < 99$ d) $\boxed{2}.\boxed{0} < \boxed{3}.2$

The only suitable number for the third box in a) is 1.

3. Solution 1:
The first box in each of a) and c) has limited choices.

a) $2.9 \times \boxed{1}.2 < 4.\boxed{6}7$ b) $1.56 \times \boxed{2}.\boxed{4} < 7\boxed{8}.0$ c) $3.08 \times \boxed{0}.\boxed{5}5 < \boxed{3}.41$ d) $9.36 \times \boxed{7}.9 < \boxed{9}1.5$

Solution 2:

a) $2.9 \times \boxed{0}.2 < 4.\boxed{1}7$ b) $1.56 \times \boxed{3}.\boxed{9} < 7\boxed{7}.0$ c) $3.08 \times \boxed{2}.\boxed{4}5 < \boxed{8}.41$ d) $9.36 \times \boxed{5}.9 < \boxed{6}1.5$

BLM 5: Comparing Fractions

1. $\frac{3}{8}, \frac{4}{9}, \frac{2}{5}, \frac{3}{7}, \frac{1}{3}, \frac{4}{10}$. $\frac{4}{9}$ is closest to $\frac{1}{2}$, and $\frac{1}{3}$ is furthest from $\frac{1}{2}$.

2. $\frac{5}{8}, \frac{5}{9}, \frac{3}{5}, \frac{4}{7}, \frac{2}{3}, \frac{6}{10}$. $\frac{5}{9}$ is closest to $\frac{1}{2}$, and $\frac{2}{3}$ is furthest from $\frac{1}{2}$.

3. $\frac{7}{8}, \frac{8}{9}, \frac{4}{5}, \frac{6}{7}, \frac{2}{3}, \frac{9}{10}$. $\frac{9}{10}$ is closest to 1, and $\frac{2}{3}$ is furthest from 1.

4. $\frac{9}{19}, \frac{3}{7}, \frac{6}{13}, \frac{5}{11}, \frac{4}{9}, \frac{8}{17}$. $\frac{9}{19}$ is closest to $\frac{1}{2}$, and $\frac{3}{7}$ is furthest from $\frac{1}{2}$.

5. $\frac{9}{8}, \frac{3}{2}, \frac{6}{5}, \frac{5}{4}, \frac{4}{3}, \frac{8}{7}$. $\frac{9}{8}$ is closest to 1 because it is only $\frac{1}{8}$ distant from 1; $\frac{3}{2}$ is furthest from 1, at a distance of $\frac{1}{2}$.

6. $\frac{99}{100}$ is closest to 1 because it is only $\frac{1}{100}$ distant from 1. The other fractions are further from 1.

BLM 7: Out of Line

1. \$13.75; 2. 161 days; 3. 1952; 4. 168 h; 5. 200 bones; 6. 8708 m; 7. 32 kg 8. 750 kg

Solutions & Notes

BLM 8: Putting Numbers in Their Places

Game 1:

4 a	5 b	1 c
2 d	3 e	6 f

Game 2:

5 a	3, 2, or 3 b	1, 1, or 2 c
4 d	2, 3, or 1 e	6 f

Game 3;

9 or 10 a	6 b	10 or 9 c
7 d	11 e	8 f

Game 4:

$\frac{1}{3}$ a	$\frac{2}{3}$ b	$\frac{4}{3}$ c	$\frac{3}{2}$ d
$\frac{1}{4}$ e	$\frac{3}{4}$ f	$\frac{4}{5}$ g	$\frac{1}{2}$ h

BLM 10: Dewey Animals

1. Mammals' numbers start with '599'; however, students may not initially identify this, and will offer several characteristics these animals share.
2. Ocelot, lion, and cheetah start with 599.75; they are all members of the cat family (felines).
3. Whales are not fish.
4. '598' are the first three digits for birds.
5. Animals numbered 599.8 are all apes.

Cross-Curricular Activity

2.

Number	Correct Title
599.52	Blue Whales and other Baleen Whales
599.5248	Blue Whales
599.7524	The Eastern Panther: mystery cat of the Appalachians
599.756	The Way of the Tiger: natural history and conservation of the endangered big cat
599.786	The World of the Polar Bear
599.786	Polar Bear versus Grizzly Bear
599.789	Giant Pandas
599.789168	Endangered Pandas

Activity 3

BLM 11: Lining Up Letters

1, 2

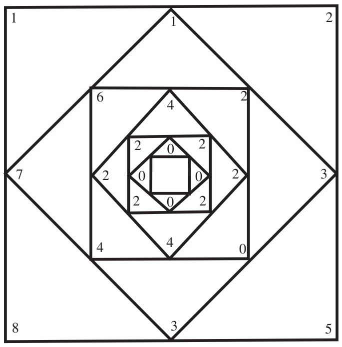
Number:	$4\frac{1}{2}$	19	$20\frac{7}{10}$	$32\frac{2}{7}$	$35\frac{1}{2}$	$45\frac{2}{3}$	$47\frac{1}{3}$	$56\frac{7}{9}$
Letter:	F	R	A	C	T	I	O	N

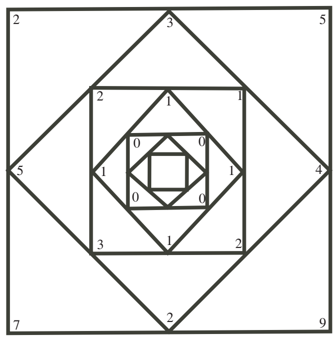
Solutions & Notes

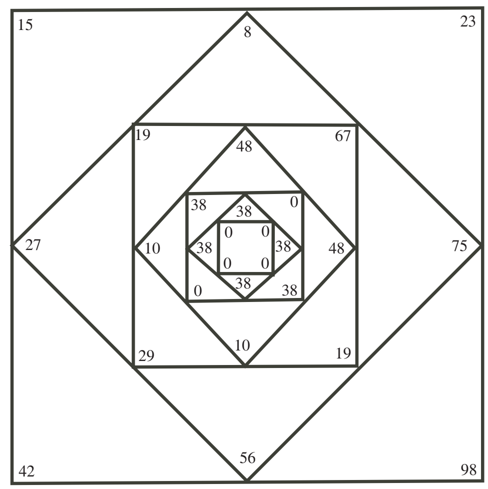
3.

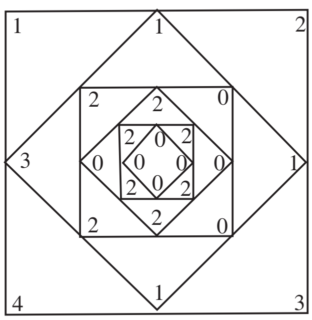
Number:	1.9	3.93	4.65	15.8	22.5	24.22	34.9	98.7
Letter:	D	E	C	I	M	A	L	S

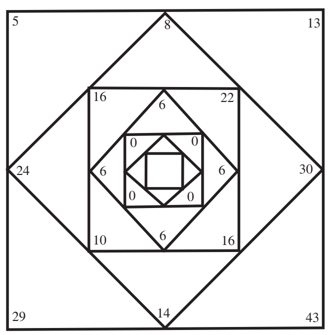
BLMS 12 and 13: Devilish Differences 1 and 2

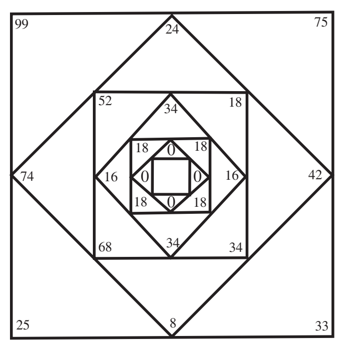
1. 

2. 

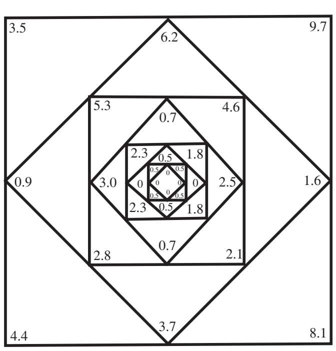
3. 

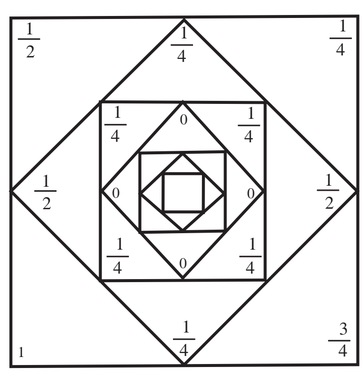
4. a) 

b) 

c) 

See page 19 for discussion of this.

5. a) 

b) 

Comment: In some cases, the order in which the numbers are assigned to the vertices will affect the number of squares needed. For example, in 4 c), if the numbers 99, 75, 33, 25 are entered in the order 25, 75, 33, 99, the solution takes seven square instead of six.

Solutions & Notes

BLM 14: Flowing Division

1. a) 6 b) 4 c) 6 d) 3 e) 6

2. a) 6 b) 4 c) 6 d) 3 e) 6

3. The sequential divisions in #1 are equivalent to division by the product of the divisors, as done in #2.

4. a) $\boxed{\div 8}$ b) $\boxed{\div 8}$ c) $\boxed{\div 9}$ d) $\boxed{\div 25}$ e) $\boxed{\div 15}$ f) $\boxed{\div 18}$

5. Several answers are possible; here are some samples.

a) $\boxed{\div 2} \rightarrow \boxed{\div 6}$ or $\boxed{\div 3} \rightarrow \boxed{\div 4}$

b) $\boxed{\div 4} \rightarrow \boxed{\div 10}$ or $\boxed{\div 8} \rightarrow \boxed{\div 5}$

c) $\boxed{\div 3} \rightarrow \boxed{\div 3} \rightarrow \boxed{\div 2}$ or $\boxed{\div 9} \rightarrow \boxed{\div 2}$

d) $\boxed{\div 5} \rightarrow \boxed{\div 5}$

e) $\boxed{\div 4} \rightarrow \boxed{\div 3} \rightarrow \boxed{\sqrt{3}}$ or $\boxed{\div 6} \rightarrow \boxed{\div 6}$

f) $\boxed{\div 6} \rightarrow \boxed{\div 2} \rightarrow \boxed{\div 4}$ or $\boxed{\div 8} \rightarrow \boxed{\div 6}$

Challenge:

6. a) 5 b) 9 c) 3 d) 9 e) 5 f) 3

Activity 4: Fractions

BLM 15: About Halves

1. a) more b) less c) less d) more e) less f), g), h), i) more

2, 3. Answers will vary.

BLM 16: More Halves

1, 2. Answers will vary.

BLM 17: Estimating Fractions

1. Answers will vary; here are some possibilities.

a) $\frac{1}{4}$ or $\frac{1}{8}$ or $\frac{3}{8}$

b) $\frac{1}{4}+\frac{2}{4}$ or $\frac{1}{4}+\frac{5}{8}$

c) $\frac{7}{8}+\frac{3}{8}$ or $\frac{1}{2}+\frac{7}{8}$

d) $\frac{1}{4}-\frac{1}{8}$ or $\frac{2}{4}-\frac{3}{8}$

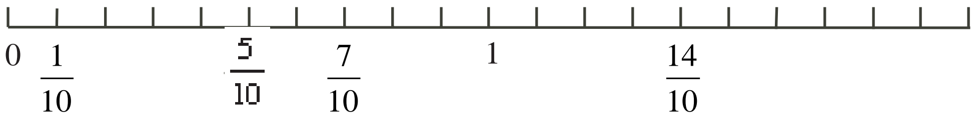
e) $\frac{1}{2}\times\frac{7}{8}$ or $\frac{5}{8}\times\frac{3}{4}$

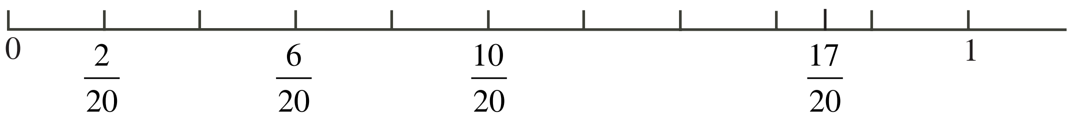
f) $\frac{1}{4}+\frac{1}{8}+\frac{3}{8}$

2. All but f) unless the same fraction is used twice, as in $\frac{1}{4}+\frac{1}{4}+\frac{1}{8}$.

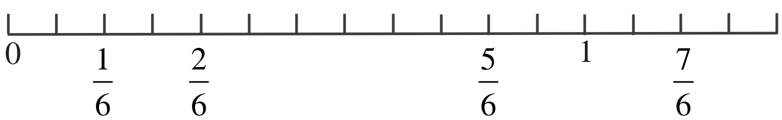
Solutions & Notes

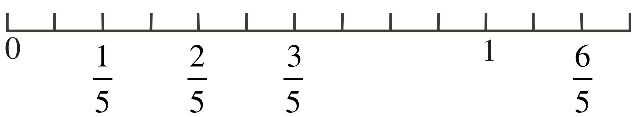
BLM 18: Sizing Fractions

1. a) 

b) 

c) 

d) 

e) 

2. Recall that not all the given letters will be used in the decoding.

   a) MATH b) EQUALS c) ONE HALF d) FRACTION e) FOURTHS

Activity 5: Estimation

BLM 19: About or Exact

1. a) Probably an estimate.
   b) Probably exact.
   c) Probably an estimate.
   d) Exact if based on gate receipts.
   e) Probably exact.
   f) Exact.
   g) Could be either – a good discussion question (e.g., are station breaks included?).

2. The blanks are filled in order:
   about 7:30 a.m.; about 30 minutes; exactly 8:15 a.m.; about (or exactly) 8:00 o'clock; about quarter to nine; exactly 24; exactly 26; exactly 21; exactly 34; exactly 3 buses; exactly 3 parents; about 9:30 a.m.; about 45 minutes; exactly \$5.00; exactly \$9.50; about $1\frac{1}{2}$ hours; about 1:30 p.m.

Solutions & Notes

BLM 20: Rough Estimates

1.

Actual Costs ($)	Front End Estimate ($)	Adjustment (¢)	Final Estimate ($)	Enough?
a) 3.50 + 2.95 + 1.99 + 4.49	3 + 2 + 1 + 4 = 10	50 + 95 + 99 + 49 about 300	$13.00	Yes
b) 3.49 + 3.76 + 2.15 + 4.98	3 + 3 + 2 + 4 = 12	49 + 76 + 15 + 98 about 250	$14.50	Yes
c) 3.98 + 3.44 + 2.19 + 6.50	3 + 3 + 2 + 6 = 14	98 + 44 + 19 + 50 about 200	$16.00	No
d) 9.33 + 2.98 + 1.59	9 + 2 + 1 = 12	33 + 98 +59 about 200	$14.00	Yes
e) 2.44 + 4.32 + 4.39 + 0.98	2 + 4 + 4 + 0 = 10	44 + 32 + 39 + 98 about 200	$12.00	Yes

2. a) Tax: about $13\times10¢+\frac{1}{2}(13\times10¢)=130¢+65¢=\$1.95$. Thus total cost is about $\$13.00+\$1.95=\$14.95$, still less than $15.

   b) Tax: about $14.5\times10¢+\frac{1}{2}(14.5\times10¢)=145¢+72.5¢$ which is about 218¢ = $2.18. Thus total cost is about $\$14.50+\$2.18=\$16.68$, more than $15.

   c) Since $16 > $14, this case will also be more than $15 once the tax is included.

   d) Tax: about $14\times10¢+\frac{1}{2}(14\times10¢)=140¢+70¢=\$2.10$. Thus total cost is about $\$14+\$2.10=\$16.10$, more than $15.

   e) See the given example on BLM 20.

BLM 21: Over and Under

1. a) UNDER
   b) OVER
   c) HARD TO TELL (actually UNDER)
   d) UNDER
   e) HARD TO TELL (actually UNDER)
   f) HARD TO TELL (actually OVER)

2. Students may have selected HARD TO TELL because the given estimate was obtained by rounding up for one factor and down for the other factor.

Solutions & Notes

3. a) HARD TO TELL (actually UNDER)
   b) HARD TO TELL (actually UNDER)
   c) HARD TO TELL (actually OVER)
   d) OVER
   e) OVER

4. Students may have selected HARD TO TELL because the given estimate was obtained by rounding up for one addend and down for the other addend.

5. Generally, students find addition easier.

BLM 22: A Range of Estimates

1. There may be more than the one solution given below.
   a) 63, 28 b) 36, 46 c) 56, 68 d) 82, 12 e) 87, 16 f) 77, 51

2. Answers will vary.

BLM 23: Product Bingo

Each product on the board has at least 2 pairs of factors available, as shown below. There may be others.

720 $= 24 \times 30$ $= 48 \times 15$ $= 16 \times 45$	1080 $= 60 \times 18$ $= 30 \times 36$ $= 90 \times 12$	432 $= 6 \times 72$ $= 12 \times 36$	900 $= 45 \times 20$ $= 15 \times 60$
144 $= 8 \times 18$ $= 9 \times 16$	648 $= 18 \times 36$ $= 54 \times 12$ $= 9 \times 72$	1152 $= 24 \times 48$ $= 72 \times 16$	360 $= 15 \times 24$ $= 30 \times 12$ $= 6 \times 60$
384 $= 8 \times 48$ $= 16 \times 24$	288 $= 12 \times 24$ $= 36 \times 8$ $= 6 \times 48$	1440 $= 16 \times 90$ $= 48 \times 30$	540 $= 15 \times 36$ $= 30 \times 18$ $= 60 \times 9$
2700 $= 60 \times 45$ $= 30 \times 90$	810 $= 15 \times 54$ $= 45 \times 18$	405 $= 9 \times 45$ $= 27 \times 15$	324 $= 6 \times 54$ $= 9 \times 36$

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

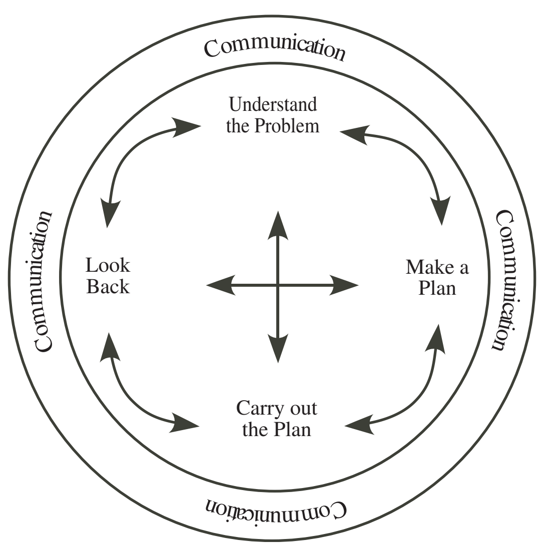
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R''s"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



Suggested Assessment Strategies

There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;

Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.

Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
STRATEGIES	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

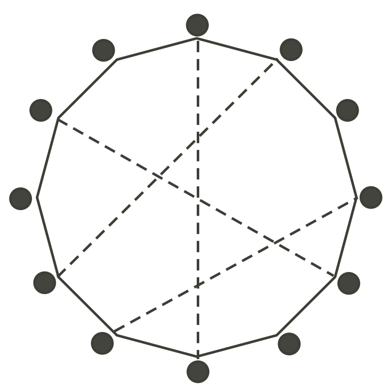
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a "what if...?" problem, such as "What if the question were reversed?", "What if we had other data?", "What if we were to show the data on a different type of graph?".

Suggested Assessment Strategies

Adapting the Rubric

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described" by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

Activity 2, BLM 4

Understanding: Do students realize that all ten number tiles must be used for each problem? Do they understand that tiles may be moved from initial placements?

Strategies and Reasoning: Do students consider the whole problem and look first for number tile positions for which there may be only one or two choices?

Reflection: Can students explain the steps they followed and why each number tile was placed in each position?

For example,

- The "Limited" student may try to use one or more digits more than once and may resist moving a tile once it is placed.
- The "Acceptable" student may try to complete the boxes in order and may need to be reminded that tiles can be moved, but will eventually place most or all of the tiles completely.
- The "Capable" student feels comfortable moving tiles from one position to another.
- The "Proficient" student will need to move the tiles less ofter.

Activity 4, BLM 17

Strategies and Reasoning: Are students able to identify fractions that are close to $\frac{1}{2}$? Can students write equivalent fractions (e.g., convert all fractions to eighths)?

For example,

- The "Limited" student uses manipulatives or drawings such as the Fraction Lines on BLM 6 it identify fractions less than, or greater than $\frac{1}{2}$, or to write equivalent fractions.
- The "Acceptable" student may refer to manipulatives or Fraction Lines but is able to make such statement as, "$\frac{1}{4}$ is less than $\frac{1}{2}$".

Suggested Assessment Strategies

Activity 5, BLM 21

Understanding: Does the student understand the effect on a product of rounding one (or both) of the factors up (down)?

Strategies and Reasoning: To what extent does the student attempt to estimate by comparing the rounded numbers to the given numbers?

For example,

- The "Limited" student who selects "hard to tell" on most questions may not make the connection between the original numbers and the rounded numbers.
- The "Acceptable" student may compute the exact answer to determine if the given estimate is an over- or under-estimate.
- The "Capable" student is able to explain his/her strategy by such statements as "I know 40×25 gives an under-estimate because 40 is less than 43," but may have trouble with examples where one factor is rounded up and one is rounded down.
- The "Proficient" student is able to explain why, for example, 70×50 gives an under-estimate for 73×49 by stating that 73 is rounded down by more than 49 is rounded up.

Resources for Assessment

1. The Ontario Curriculum, Grades 1-8: Mathematics.

2. *Linking Assessment and Instruction in Mathematics: Junior Years*, Ontario Association of Mathematics Educators/OMCA/OAJE, Moore et al., 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level.

3. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, by Jean Kerr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels.

4. *How to Evaluate Progress in Problem Solving*, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Other Resources

1. *"Number Sense and Operations, Addenda Series," Grades K-6,* ed. by Grace Burton, 1992, National Council of Teachers of Mathematics (NCTM), Reston, VA (1-800-235-7566)
   The booklet contains suggested lessons for each grade dealing with numbers. Activities deal with multiplication patterns, exploring large numbers through counting blades of grass, and estimating.

2. *"Developing Sense About Numbers"*, by Carole Greenes, Linda Schulman and Rika Spungin, *Arithmetic Teacher*, pp 279-284, January 1993, NCTM.
   This article lists seven number-sense skills, such as "recognizing the various uses of numbers", "estimating results of computations", and "understanding phrases that establish mathematical relationships". Several "Fit the Facts" activities (similar to BLMs 1, 2, and 3 in Grade 4 "Investigations in Number Sense and Estimation") are included.

3. "The Revolution in Arithmetic", by William A. Brownell, *Mathematics Teaching in the Middle School*, pp 27-30, August 2006, NCTM.
   This is a reprint of an article first printed in 1954. It describes changes in content, teaching methods, and text books during the first half of the 20th century. It is worth considering whether or not 21st century content, teaching methods, and text books have continued to evolve/improve.

4. "How much is a Million?", by David M. Schwartz and Steven Kellogg, William Morris and Co., New York, 1985.
   This book helps students come to grips with one million (1 000 000), one billion (1 000 000 000), and one trillion (1 000 000 000 000) through the answers to such questions as "If a million kids climbed onto one another's shoulders", how tall would they be? The answers may surprise you.

5. "How Big is Bill Gates's Fortune?", by Hamp Sherard, *Mathematics Teaching in the Middle School*, pp 250-252, December 2000, NCTM.
   Students explored large numbers by determining the weight or size of Gates' fortune in $100 dollar bills. One student calculated that the fortune was 62.5 miles taller than Mount Everest if the bills were stacked one on top of the other. Recently (summer of 2007) Gates' fortune was estimated at 100 billion dollars. (A $100 bill is approximately 6.6 cm wide, 15.6 cm long, 1 mm thick and 0.9 g mass).

6. "A Game Involving Fraction Squares", by Enrique Ortez, *Teaching Children Mathematics*, pp 218-222, December 2000, NCTM.
   This article describes a game similar to the idea on BLM 4 (Decimal Tiles) in this book, but with the stress on equivalent fractions.

7. "Multiplication Games: How We Made and Used Them", by Constance Kamii, and Catherine Anderson, *Teaching Children Mathematics*, pp 135-141, November 2003, NCTM.
   Several easily-made multiplication games are described, from those that use one multiplication table at a time (e.g., x 4) to those that use three or more factors. Using these games throughout the year, from the simplest to the more difficult, students showed considerable improvement in their speed and accuracy.

Other Resources

8. "The Influence of Ancient Egypt on Greek and Other Nueration Systems", by Claudia Zaslavsky, *Mathematics Teaching in the Middle School*, pp 174-178, November 2003, NCTM.
   The article describes the Egyptian hieroglyphic numbers and how they influenced the Greeks who used letters of the alphabet as numerals. Two Black Line Masters for students are given.

9. "Building Percent Dolls: Connecting Linear Measurement to Learning Ratio and Proportion", by Joan Moss and Beverly Caswell, *Mathematics Teaching in the Middle School*, pp 68-74, September 2004, NCTM.
   Grade 5 and 6 students determined what percentage of their height various measurements were (e.g., length of hand, shoulder width, arm span). Students then constructed figures using these proportions, and whatever materials they felt were appropriate (e.g., balloons, wooden dowels, modelling clay).

10. "Understanding Aztec and Mayan Numeration Systems", by Angela L. E. Walmsley, *Mathematics Teaching in the Middle School*, pp 55-59, August 2006, NCTM.
    A comparison of the Aztec system (lacking a zero) and the Mayan system (including a zero) to the Hindu-Arabic system brings out many properties of numbers and emphasizes place value. Several examples of all systems are given. A source for more information on the Mayan system is given as "www.ancientscripts.com/aztec.html".

11. "The Thinking of Students: A Fruitful Crop", by Gladis Kersaint, *Mathematics Teaching in the Middle School*, pp 95-99, September 2004, NCTM.
    This column appears in every issue providing a problem for students in grades 5 - 8. This one also includes responses from students describing how they solved the problem. This issue's problem is

    Of 6000 apples harvested, every third apple was too small (S), every fourth apple was too green (G), and every tenth apple was bruised (B). The remaining apples were perfect (P). How many perfect apples were harvested? [Answer: 4800]

12. "Science Is ...", (2nd ed.), by Susan V. Bosak et al, Scholastic Canada Ltd., 1991
    This practical book has an extensive collection of activities/experiments encompassing environmental studies and mathematics, developed with input from students, teachers and parents. While directed at children ages 6 - 14, these hands-on activities provide fun and ample learning opportunities for all. Permission to copy for classroom use is granted.

13. "Mental Computation in the Middle Grades: The Importance of Thinking Strategies", by Alistair McIntosh, Robert E. Reys, an Barbara J. Reys, *Mathematics Teaching in the Middle School*, pp 322-327, March - April 1997, NCTM.
    The article discusses the nature of thinking strategies, number sense, mental computation, and estimation. Students' opinions and thinking strategies are given. Conclusions include the following:
    - Students success with computation is much higher when students see the problem as opposed to when the problem is read to them.
    - The more confident a student is, the more likely he/she is to develop alternate strategies.
    - Students' perceptions of what is meant by mental computation differ greatly.