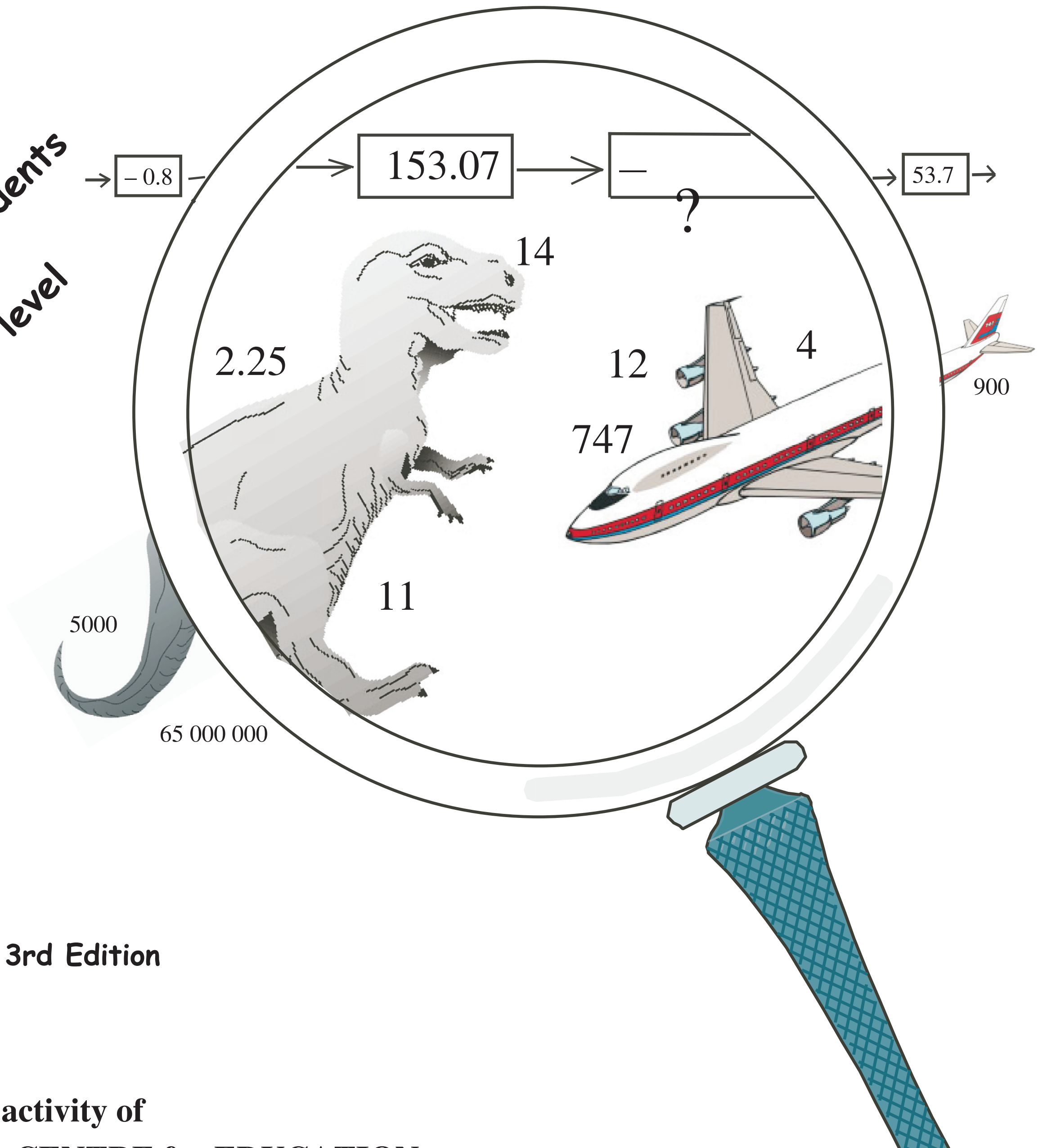
Invitations to Mathematics

Investigations in Number Sense and Estimation

"All Kinds of Numbers"

Suggested for students at the Grade 4 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
Faculty of Mathematics, University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the “Extensions” and related activities included with individual activities/projects, provide ample scope for all students’ interests and ability levels. Related “Family Activities” can be used to involve the students’ parents/care givers.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

“Investigations in Number Sense and Estimation” is comprised of activities which explore the properties, estimation, and uses of whole numbers and fractions in mathematical and everyday settings. A reasonably level of numeracy is essential to navigating the complexities of the highly technical world in which we live. The activities in this unit develop many facets of number sense and apply them to a wide variety of practical situations.

Acknowledgements

Contributing Teachers

Nancy Dykstra (Waterloo County Board of Education)
Jo-Anne Judge (Waterloo County Board of Education)
Ron Sauer (Waterloo County Board of Education - retired)

Authors/Co-editors

Bev Marshman (University of Waterloo)
Lorna Morrow (Mathematics Consultant)

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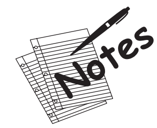
Overview



Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and number sense and estimation specifically. Some of these beliefs are described below.

Numeracy involves an intuitive sense of the meanings of numbers and their various uses and interpretations. It is acquired slowly over a long period of time, and is fundamental, both to mathematics, and to the sciences which provide a quantitative understanding of the world around us.

While facility with number facts and algorithms is clearly important, the focus here is on developing students' thinking and reasoning abilities. This is achieved through investigation and sharing of ideas during group activities involving properties of numbers (whole numbers, decimals, and fractions), comparing and ordering, how numbers are used, and determining reasonable estimates. Problems using a variety of mechanisms (number lines, geometric quantities, mental manipulation, stories, games, etc) encourage flexibility in methods of solution. Similarly, a variety of estimation situations increases students' awareness of the pervasive need for estimates in real life, and their ability to devise estimates competently. Students are encouraged not only to calculate in different ways but also to assess the reasonableness of their answers. In addition, by eliminating the need for boring computations, calculators can be used to permit students to focus on the process of obtaining solutions, and on their interpretation.

Throughout these activities, as they attempt to justify their conclusions using mathematical language, students deepen their insight into and understanding of how numbers relate to each other and to the world around them.

Essential Content

The activities herein explore numbers both in the abstract and in their connection to measures of real quantities, with the goal of developing students' ability to think and work flexibly with different kinds of numbers in a variety of contexts. In addition, there are Marginal Problems, Extensions in Mathematics, Cross-Curricular Activities, and Family Activities, which can be used prior to, during the activity, or following the activity. They are intended to suggest topics for extending the activity, assist integration with other subjects, and involve the family in the learning process.

During this unit, the student will:

- compare sizes of whole numbers (to millions), fractions, and decimals (to hundredths);
- explore properties of number (e.g., $a+b=b+a$, but $a-b \neq b-a$);
- estimate products and sums; identify compatible numbers;
- explore patterns on a hundred chart, including the effects of 'opposite' operations such as '+1' and '–1';
- identify reasonable and unreasonable uses of numbers;
- practice skills in game situations;
- use mathematical language to express their results;
- work together to achieve success.

Overview

Curriculum Expectations

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Using Numbers**	• selecting appropriate numbers for given situations from the real world	• solve problems involving whole numbers and decimals • identify and appreciate the use of numbers
Activity 2 **Comparing and Ordering**	• selecting the greater (or least) of two or three numbers (0.01 to 1000) and adding them • ordering and adding numbers from 0.1 to 10 • estimating answers for addition and multiplication of whole numbers	• recognize and read numbers from 0.01 to 10 000 • compare and order whole numbers and decimals from 0.1 to 10 000 • determine whether results are reasonable
Activity 3 **Number Properties**	• addition and subtraction patterns on a hundred chart • exploring commutative and associative properties of addition and multiplication • identifying place value in addition and subtraction flow charts	• represent and explain number concepts and procedures • represent the place value of whole numbers and decimals from 0.01 to 10 000
Activity 4 **Fractions**	• using segments of circles to represent and compare fractions • determining whether fractions are close to 0, $\frac{1}{2}$, or 1 • using geopaper to represent fractional areas	• represent, compare, and order fractions using concrete materials
Activity 5 **Estimation**	• adding numbers by grouping in 10s • forming 2-digit numbers from four given digits so as to achieve a specified sum, difference, or product • assessing the reasonableness of statements related to real situations	• add and subtract numbers mentally • explain their thinking when solving problems involving whole numbers

Overview

Notes

Prerequisites

Although students should be able to deal with the activities in this book with an understanding of the previous grade's curriculum, it would help if they are familiar with the following:

- place value from tenths to ten thousands;
- rounding to the nearest ten or hundred;
- the meaning of fractions (e.g., $\frac{1}{2}$ means 1 out of 2 equal parts)
- the nature and use of estimates (i.e., what an estimate is and when an estimate is appropriate).

Logos

The following logos, which are located in the margins, identify segments related to, respectively:

Problem Solving Communication Assessment Use of Technology

Marginal Problems

Throughout the booklet you will see problems in the margin (see example to the right). These "Marginal Problems" may be used as warm-ups to a lesson, as quick 'tests' or reviews, as 'problems-of-the-day' or in any other way your experience tells you could be useful. Some 'Marginal Problems' deal with the same topic as the activity and some with other topics in "Number Sense and Estimation". Discussion of individual problems can be found at the beginning of "Solutions and Notes".

Give two fractions between 0 and $\frac{1}{2}$.

Overview

Materials

ACTIVITY	MATERIALS
Activity 1 **Using Numbers**	• Copies of BLMs 1 and 2 • Copies of BLMs 3 and 22 (optional)
Activity 2 **Comparing and Ordering**	• Copies of BLMs 4, and 5 • A calculator for each pair/group • Scissors for each pair/group • Copies of BLM 6 (optional)
Activity 3 **Number Properties**	• Copies of BLMs 7, 8, and 9 • Acetate copies of BLM 7 • Copies of BLMs 10 and 11 (optional)
Activity 4 **Fractions**	• Copies of BLMs 12, 13, and 14 • Scissors and crayons/markers • Copies of BLMs 15 and 16 (optional)
Activity 5 **Estimation**	• Copies of BLMs 17, 18, and 19 • A calculator for each pair/group • Copies of BLMs 20 and 21 (optional) • 10 - 12 game markers for each player (optional)

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week or so, students in our classroom will be participating in a unit titled “All Kinds of Numbers”. The classroom activities will focus on expanding students’ understanding of numbers and estimation while exploring how numbers are related and how they are used. The emphasis will be on developing skill with mental manipulation, estimation, and computation.

You can assist your child in understanding the relevant concepts and acquiring useful skills by working together to perform number-related tasks (e.g., comparing prices when shopping, estimating the total cost, calculating mileage for the family vechicle), and by helping to explore everyday ways numbers are used.

Various family activities have been included for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with measurement in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

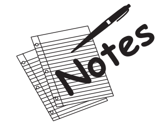
Sincerely,

Teacher’s Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Activity 1: Using Numbers



Focus of Activity:

- Identification of ways numbers are used

What to Assess:

- Reasonableness of answers
- Use of mathematical language
- Ability to justify answers

Preparation:

- Make copies of BLMs 1 and 2.
- Make copies of BLMs 3 and 22 (optional).

'BLM' refers to the Black Line Masters, which follow the Activity Notes.

Activity:

To introduce the activity, ask students questions like the following, and have them justify their answers.

> If you were born in 1997, would you finish high school by 2010?
>
> About how many hours of daylight are there in January in Canada: 7 or 12?
>
> About how many hours of daylight would there be in Mexico City in January: 7 or 12?
>
> If Guido saves $2 each week, about how much would he save in a year: $50 or $100 or $200?

Communication

Distribute copies of BLM 1 (What's A Good Fit? - 1) and read problem #1 together. Ask students to look at the numbers and select any one that they are sure of for any blank. Most should realize that '5' is the only reasonable choice for the number of passengers. Since '215' is too large for the capacity of the gas tank, and '9' is too small, the tank holds 55 L.

If students think '611' is too great for the total distance travelled on a full tank of gas, ask if any of them have been on a long trip with their families, how far it was, and how often they had to stop for gas. If students are still unconvinced, have them check this out with family members and come back with an answer tomorrow.

Read problem #2 together. Since many students will have had some experience with a minivan, they should be confident that '4' and '2' are, respectively, reasonable for the first two blanks. Ask why '20' or '100' would not make sense for either of the first two blanks. [Ans: because cars/vans couldn't possibly have that many doors, given the size of the doors and the size of the van.] Some students may think '6' a reasonable answer for the first blank, and if they can justify their choice, this answer should be accepted.

Notes

See page 3 for a description of "Marginal Problems", an example of which is given here.

If the product of two numbers ends in zero, what can you say about the two numbers?

For more on the new definition of 'planet' see BLM 22. This BLM also contains current data on planet measurements.

Problem Solving

Activity 1: Using Numbers

Ask what number is reasonable for the third blank (number of cylinders). Either '6' or '4' is reasonable, though '6' is more likely. Students should realize that more than one answer may be correct, but only if the answer can be justified.

Students should be familiar with the speed limit in a school zone since there is probably a sign posted outside near the school. This leaves only '100' (km/h) for the speed limit on the highway. In fact, these two blanks (speed limits) may be the easiest to complete, and students may suggest these answers before completing the first three blanks.

If students seem to understand how to tackle the problems, assign the rest of BLM 1 and have students work in pairs/groups to place the numbers in the blanks, discussing each problem as it is completed. Otherwise, continue with a whole class discussion as suggested below.

Read Problem 3. A 10-speed bicycle is common and students should be able to select 35 kg as a reasonable weight for Kim, and 2.1 m as the only possible length of a bicycle. Fifteen kilograms should be seen as too great a weight for a bicycle, so the weight must be 5 kg and Kim's speed 15 km/h.

Read Problem 4. Some of the answers for problem 4 may be difficult without further data, but 65 000 000 is an obvious choice for the first blank. The height cannot be 5000 m (too great) or 2.25 m (too little) so it must be either 14 m or 11 m, although 11 m is the actual value. The last two numbers will be more difficult to place. The actual values are age 14 and 2.25 kg.

Read Problem 5. Recently, (Dec. '06) "National Geographic" reported on a new definition of 'planet' devised by the International Astronomical Union. Among other citeria, a planet must be the only object in its orbit. Since this is not true of Pluto, it is no longer considered a planet, so the first blank in problem 5 should contain the number '8'. Students should have some idea of earth's diameter (12 423 km). Alternatively, a little mental arithmetic should indicate that 12 x 12 000 is about 144 000, so Earth's diameter must be 12 000 km and Jupiter's 141 900 km. The numbers will fit in no other way.

BLM 2 (What's A Good Fit? - 2) gives 5 more problems of the same type as on BLM 1. For each of these problems, the place for at least one number in each is obvious, and often for more than one. If students have difficulty getting started on any of the problems, provide hints based on the following.

In #6, '– 88' must be the temperature in Antarctica, and '58' the temperature in North Africa. Ask "Which temperature is the coldest? Where do you think that would occur?"

Activity 1: Using Numbers

In #7, a little arithmetic will show where the different numbers belong. (25.6 m/base x 4 bases = 102.4 m, and $38\text{ cm} \times 38\text{ cm} = 1444\text{ cm}^2$). Encourage students to use their estimation skills. For example, students will know that 4 quarters equal one dollar, so 4×25.6 is about 100.

In #8, the only reasonable number for the number of floors is 12.

In #9, Gwen could have bought only 1 set of markers, since 'set' is singular.

Paper is sold in packages of 500 sheets. The number '$\frac{3}{4}$' can refer only to the distance she walked.

In #10, some students may know that a Boeing 747 is a commonly used plane for large numbers of passengers. They may also recognize that 4 washrooms would be insufficient and 500 far too many so there must be 12 washrooms.

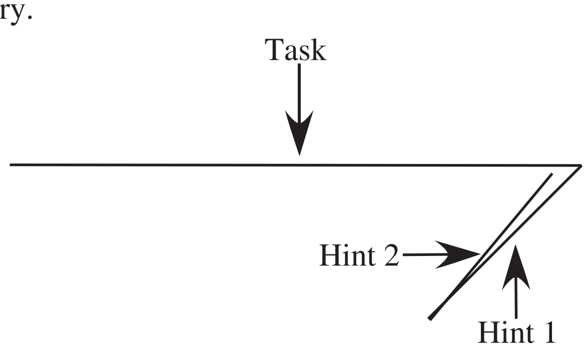
Assessment

Observing and listening to students as they are working through these problems is a good way of assessing both their number sense and their problem solving skills.

Extensions in Mathematics:

Problem Solving

1. Distribute copies of BLM 3: (Growing a Tiger). You may wish to fold the hints out of sight before distributing the copies of the task. The page is designed so that you can fold Hint 2 back first, and then Hint 1. Thus the only part visible to the students is the task itself. (See diagram below showing an edge view of folded BLM.) Instruct students to try solving the problem without looking at the hints, but when they get stuck, they should unfold Hint 1 and try the problem again. This allows students who resist hints because "I want to do it myself" the opportunity to do so, while other students can use the hints as they feel necessary.



You may wish to read through the story of TJ the Tiger Cub with the students and have them discuss, in groups, which number or numbers make sense for each blank. Give groups time to complete the work. In the discussion that follows, students should be asked to justify their choices. If some students opted to read the hints, ask whether or not the hints helped, and why. Ask students to develop a useful hint or two of their own. They may find it can be difficult to give a hint without giving away part of the solution.

If $25 \times 25 = 625$, is 24×25 greater or less than 625? Why?

Notes

If $1600 \div 4 = 400$, is $1621 \div 4$ greater or less than 400? Why?

Activity 1: Using Numbers

Cross-curricular Activities

1. Research other animals to find how their sizes change from birth to adulthood.

Family Activities:

1. Have students ask parents/guardians how much they weighed, and how tall they were at birth (and at various times in their lives, if such information is available). Have them make a table and compare their parents' birth weight and height with their own. Collect all data from the class and see whether there is a 'trend' from one generation to the next.

Other Resources:

For additional ideas, see annotated "Other Resources" list on page 72, numbered as below.

2. "Developing Sense About Numbers"

4. "TJ the Tiger Cub"

Activity 2: Comparing & Ordering

Focus of Activity:

- Identifying the greater of two numbers, either whole numbers or decimal numbers
- Ordering a set of three numbers (whole or decimal)

What to Assess:

- Ability to identify greatest or least of a set of decimal numbers
- Ability to complete a sequence of decimal numbers in order of size

Preparation:

- Make copies of BLMs 4 and 5.
- Provide a calculator for each pair/group.
- Make an acetate copy of BLM 5 for each pair/group.
- Provide scissors for each pair/group.
- Make copies of BLM 6 (optional).

'BLM' refers to the Black Line Masters, which follow the Activity Notes.

Activity:

Which Is More?

Give students pairs of numbers either orally, on the blackboard, or using an overhead projector. Have them record only the greater value. Start with numbers with a considerable difference (e.g., 40 and 50), and gradually narrow the difference (e.g., 45 and 47). Begin with whole numbers and then introduce some decimals (e.g., 4.5 and 4.7, 4.5 and 5.4, 4.5 and 4.05).

BLM 4 (Which Is More?) gives students practice in identifying the relative sizes of numbers. The inclusion of a "Check Number" makes this a self-checking exercise.

Assign BLM 4 (Which Is More?) giving a calculator to each pair or group. As students finish it, ask which questions they found easy, which were more difficult and why.

Assessment

You may wish to refer to the cartoon characters, and ask, "Who is correct". [Jill is correct; Bill is wrong: Will is half right since he is 'bigger' than Bill, but not 'bigger' than Jill].

Note: The terms 'large' and 'small' are relative; whether a number is 'large' or 'small' often depends on the context. For example, when discussing distances between planets, the diameter of Earth is 'small', but when discussing distances between towns in Ontario, the diameter of Earth would be considered 'large'.

To be accurate, we should speak of numbers as being 'greater' or 'lesser' than one another, since 'large' and 'small' refer more to physical size than to value. For example, in the line below, the '2' could be said to be 'larger' than the '7'.

2 7

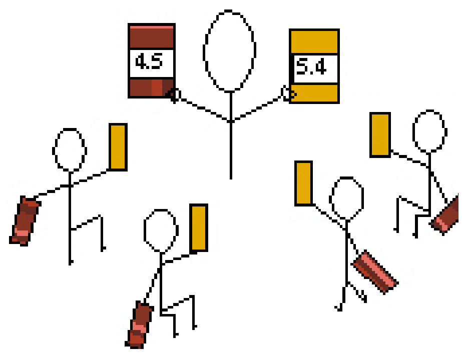
Notes

Activity 2: Comparing & Ordering

Assessment

A no-talking 'comparison-of-numbers' game for the whole class can be made with a bit of preparation. Cut a deck of about 30 cards from each of two colours of paper (blue and orange, for example). On each card write a different decimal number such as the ones on BLM 4 (e.g., 4.5, 5.4, 3.2, 4.9, 6.81, 8.16*). Give each student a scrap of each colour and give the following rules of the game. You (or even one of the students) will hold up a card from each deck, that is, one orange card and one blue card. Students hold up the scrap in the colour of the greater (or lesser) of the two numbers displayed. Once the rules are explained, there is no need for any talking. (This is great for days when you have a headache.) Furthermore, you can easily identify students who are having difficulty or who like to check their classmates' choices before displaying their own (lack of self-confidence?). The cards could also be made with written names of numbers (e.g., forty-five point eight one, forty-five point one eight).*

The "Which is greater?" Game:



Ordering Decimals

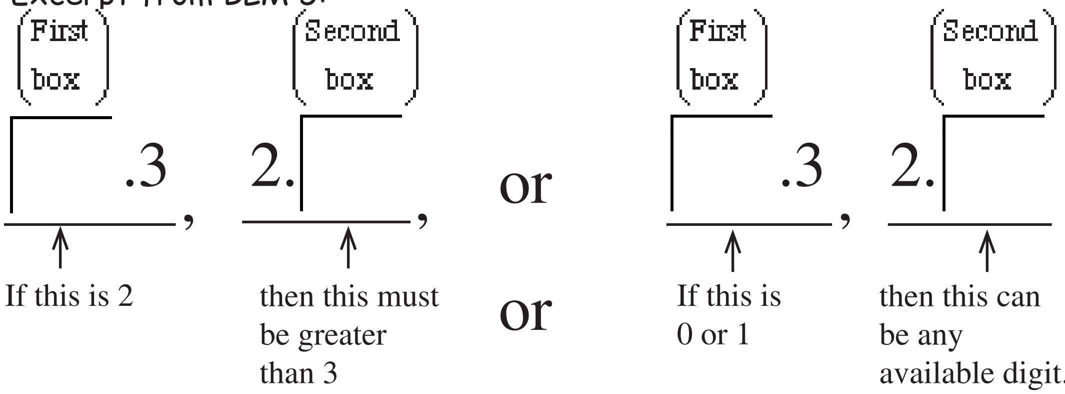
Problem Solving

Using an acetate copy of BLM 5 (Ordering Decimals) on the overhead, explain to the students that each box in question 1 must contain one of the digits from 0 to 9, and each of these digits must be used only once. Students should cut apart the number tiles at the bottom of BLM 5. Using these, students will feel free to adjust their answers as they work through the problems. Without the number tiles, there will likely be a great deal of erasing of 'wrong' answers or early 'guesses', and probably frustration as well. It is less threatening to try 'temporary' answers using the cards, which can be moved about.

Ask what digits could be placed in the first box. Students should see that only 0, 1 and 2 are possible, since the second decimal number in the question has a whole number component of 2. If the 2 is placed in the first box, then the second box must contain a digit greater than 3. However, if the 1 or 0 is used in the first box, then the second box could contain any of the remaining digits.

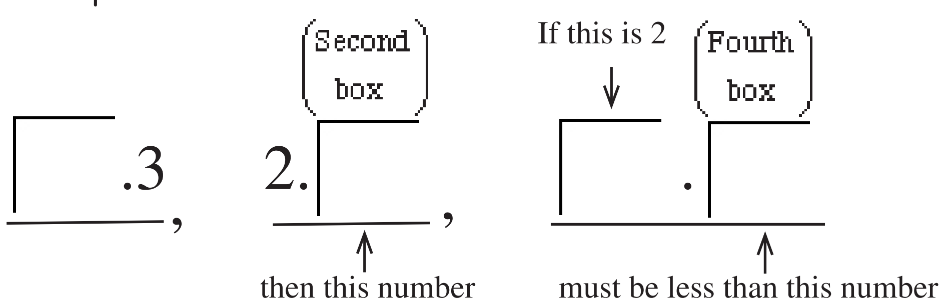
Activity 2: Comparing & Ordering

Excerpt from BLM 5:



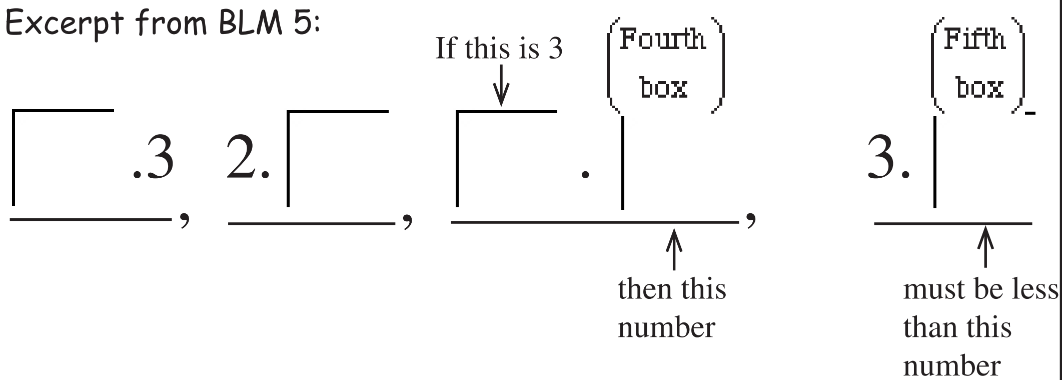
Ask, "If we use '2' in the third box, what do we now know about the digits that must go in the second and fourth boxes?" [The digit in the second box must be less than the digit in the fourth box.]

Excerpt from BLM 5:



"If we put '3' in the third box, what do we now know about the digits in the fourth and fifth boxes?" [The digit in the fourth box must be less than the digit in the fifth].

Excerpt from BLM 5:



Students should realize that any choice they make for any box may affect their choices for other boxes, and that they need not complete the boxes in order.

Have students complete problem #1 together and then assign #2 and #3. Tell students that there *may* be more than one solution for each problem. Have groups write their solutions on their acetate copies of the BLM using markers. Show these on the overhead projector and have the class check them.

Notes

Problem Solving

Write two addition questions whose answers are 49.3

Activity 2: Comparing & Ordering

See "Solutions and Notes" for further discussion on these BLMs.

Questing and Questioning

Communication

Play "Twenty Questions" with numbers. Record a number on a piece of paper hidden from the students. Students are then allowed to ask up to 20 questions to determine the number. Questions must be phrased so they can be answered by "Yes" or "No". For example, if told that the hidden number is a whole number between 0 and 100, students might ask such questions as:

(i) Is the number greater than 50?
(ii) Is it an even number?
(iii) Is it between 20 and 40?
(iv) Is it 87?

Students should see that question (iv) is a poor one since it gives very little information. Questions (i) and (ii) on the other hand, effectively elimate half the potential choices. Have students identify good questions and poor questions and explain why they think so. It is possible that a poor question at the beginning of the game may be reasonable later. For example, "Is it between 40 and 50?" is a poor question in trying to identify a number between 0 and 100, but if the range has been narrowed to, say, mulitples of 3 between 25 and 50, the question is acceptable.

Assessment

As students become better questioners, introduce decimals or fractions. For example, a proper fraction between 0 and 1 might elicit such questions as:

(i) Is it between $\frac{1}{2}$ and 1?

This is a game that can be played by groups of students, allowing the teacher to circulate and evaluate questioning techniques and number sense of various students.

Extensions in Mathematics:

1. For BLM 6 (Imagining Numbers), students first estimate answers. For Question 1, rounding the numbers to the nearest 10 is probably the simplest technique. Students then look at the descriptions below the questions, find a range that includes their estimate, and write the relevant capital letter in that blank.

   For example, 24+ 37 can be rounded to 20 + 40 to give an estimate of 60. It can be seen that the best range for the answer to question "G" is "between 50 and 70". Thus, "G" is written in the blank above this range.

   Ask students why an arrow is used instead of an equal sign. They should realize that the estimated answer is not the exact answer, and that the statement 24 + 37 = 60, for example, is not true.

Give 3 different numbers that will make 4 x $\square$ less than 87.

Activity 2: Comparing & Ordering

Work through another example or two in #1 before assigning the exercise. If students are good at estimating sums and identifying ranges, they should find that the animal in #1 is a DRAGON.

For some parts of Question 2, the simplest way to estimate an answer is to round the two-digit number to the nearest 10 and multiply by the one-digit multiplier. For others, however, especially those with a multiplier of 9, it may be simpler to round the 9 up to 10 and multiply the two-digit number by 10. Unless students are having difficulties, they should be left to discover their own best strategies which can be discussed later. The animal here is the UNICORN.

For Question 3, rounding each number to the nearest 10 is probably the simplest technique to use. If students are having difficulty getting the right number of zeros in the estimate, suggest deleting them and then 're-attaching' them after multiplying.

e.g., $\begin{array}{r} 13 \\ \times 29 \\ \hline \end{array} \xrightarrow{\text{round}} \begin{array}{r} 10 \\ \times\ 30 \\ \hline \end{array} \xrightarrow{\text{detach the zeros}} \begin{array}{r} 1 \\ \times\ 3 \\ \hline 3 \end{array} \xrightarrow{\text{re-attach the zeros}} 300$

The animal in question is the VAMPIRE.

Students might enjoy working in groups to devise another such question to share with their classmates.

Family Activities:

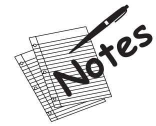
1. Have students and families visit a supermarket, pharmacy, or department store to compare prices. Students should collect a variety of prices for each of at least 5 different products. For each item, they should record the highest price, the lowest price, and 2 or 3 in between. In order to compare prices, students will need to be able to calculate unit prices (e.g., price per gram) or record them from the store's tags on the shelves. Calculators should be allowed for any necessary computation.

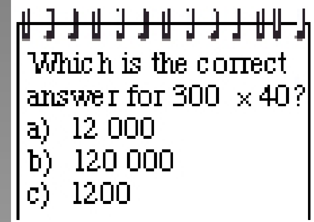
2. Have students record relevant data that might influence a buyer's choice. For example, perfume (in shampoo/deodorant/cleanser/hair products/room fresheners/detergent) or size (will it fit in the location intended for it?) or taste (a preferred brand of cola/bread/juice).

Other Resources:

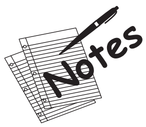
For additional ideas, see annotated "Other Resources" list on page 72, numbered as below.

5. "How Much is a Million?"
6. "How Big is Bill Gates's Fortune?"
7. "A Game Involving Fraction Squares"





Use of Technology



Activity 3: Number Properties

Focus of Activity:
- Exploring patterns on hundred charts

What to Assess:
- Recognition of patterns on hundred charts
- Identification of 'opposite' operations
- Understanding of place value from units to ten thousands
- Addition or subtraction of tens

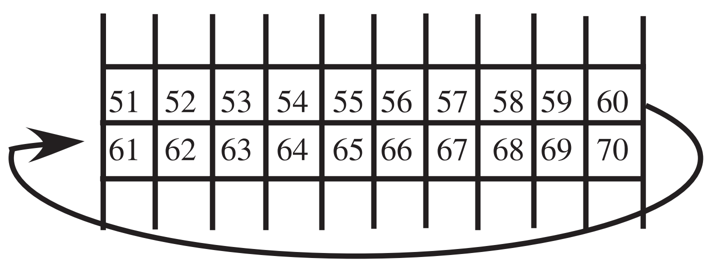
'BLM' refers to the Black Line Masters, which follow the Activity Notes.

Preparation:
- Make copies of BLMs 7, 8, and 9.
- Make 2 acetate copies of BLM 7.
- Make copies of BLMs 10 and 11 (optional).

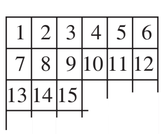
Activity:

This activity explores the nature of the standard hundred chart and the nature of simple algorithms through an exploration of patterns. Some properties of number are introduced informally, and students will have a great deal of practice adding and subtracting tens.

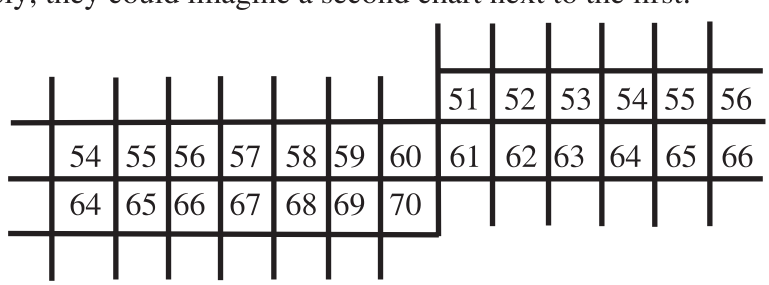
Show the first hundred chart on BLM 7 (Hundred Charts) using the overhead projector. Ask students where they will be if they start at 23 and move one square to the right ... if they start at 1? at 57? at 60? They will see that if they start at 60 and move one square to the right, they will be off the chart. Explain that, in this case, they should 'loop around' to the next line and land at 61.



Hundred charts with different configurations can be found on BLM 3 of Grade 4 "Investigations in Pattern and Algebra". e.g.



Alternatively, they could imagine a second chart next to the first:



Students could explore the meanings of the arrows on these different charts.

Illustrate this with the second copy using the overhead projector.

Show them the second hundred chart on BLM 7, and explain that although hundred charts are usually written with ten numbers in each row, the chart may start at either 0 or 1.

Activity 3: Number Properties

Show students that one way to write the “one-square-to-the-right-move” is $23 \rightarrow$.

Write several such moves on the blackboard and have students tell on which square they will land in each case.

For example, $5 \rightarrow$; $39 \rightarrow$; $74 \rightarrow$; $99 \rightarrow$

[Answers: $5 \rightarrow 6$; $39 \rightarrow 40$; $74 \rightarrow 75$; $99 \rightarrow 100$]

Students should be able to respond without referring to the hundred chart.

Ask students what they think the following might mean:
$5 \rightarrow \rightarrow$; $24 \rightarrow \rightarrow \rightarrow$; $71 \rightarrow \rightarrow \rightarrow \rightarrow$.

[Answers: $5 \rightarrow \rightarrow 7$; $24 \rightarrow \rightarrow \rightarrow 27$; $71 \rightarrow \rightarrow \rightarrow \rightarrow 75$]

Ask, “If $23 \rightarrow$ is the same as 23 + 1, what is another way of writing $5 \rightarrow \rightarrow$ or $24 \rightarrow \rightarrow \rightarrow$?”

Then ask what the following might mean, and what the ‘answers’ are:
$37 \leftarrow$; $43 \leftarrow$; $59 \leftarrow$

Just as ‘37’ is a numeral (i.e. a written sign or code) for a particular number idea (in this case 37-ness), so $37 \leftarrow$ is a numeral for a particular number (i.e. 36-ness). Thus you can ask what number each of the ‘codes’ above illustrates/stands for.

Some students may be more comfortable if the arrow points away from the number, such as $\leftarrow 37$. Explain that the arrow is like an operation (like addition or subtraction) so we start with the ‘starting point’ of the problem (i.e., 37) just as we would for $37 - 1$.

Give the students expressions such as the following to ‘decipher’ or solve:
$25 \rightarrow \rightarrow \rightarrow$; $46 \leftarrow \leftarrow \leftarrow \leftarrow$

Students should realize that the first is analagous to 25 + 1 + 1 + 1 or 25 + 3, (i.e., 25 + 3 x (+1)) and the second to 46 - 1 - 1 - 1 - 1 or 46 - 4 (i.e., 46 - 4 x (-1)).

Ask students what the following represent:
$34 \rightarrow \leftarrow$; $62 \rightarrow \leftarrow$; $12 \rightarrow \leftarrow$; $75 \rightarrow \leftarrow$

Students should see that $\rightarrow$ followed by $\leftarrow$ is analagous to +1 - 1 or +0. This introduces the idea of opposite values that will be essential for later work with negative numbers.

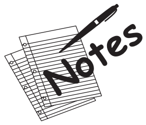
If students are at ease with all of this, present the following:

$19\uparrow$ $19\downarrow$ $57\uparrow$ $57\downarrow$ $74\uparrow$ $74\downarrow$

They should recognize that these new symbols represent +10 ($\downarrow$) and –10 ($\uparrow$).

Notes

A number on the 100 chart is 3 squares away from 45. What could that number be?



Activity 3: Number Properties

Students should now be ready to interpret the ‘codes’ in question 1 of BLM 8 (Follow the Arrows). You may wish to give them copies of BLM 7 as well, or simply leave it on the overhead projector for reference. Observe whether students ‘solve’ these arrow by arrow or if they are abstracting the meaning of a sequence of arrows and finding the answer in one step.

Assessment

Question 2, about the order used, is an informal introduction to the Commutative and Associative Properties which state that the order is not relevant.

The Commutative Property of addition: $a + b = b + a$.

The Associative Property of addition: $(a + b) + c = a + (b + c)$.

Note that neither property holds for subtraction. That is, it is not true that $a - b = b - a$.

If you wish to explore this further with students, have them test multiplication and division. They should find that both properties hold for multiplication but that neither is true for division. See Extensions and Cross-curricular Activities below for further suggestions.

If 47□ x 122 = 57950, what digit must □ represent?

Distribute copies of BLM 9 (A New Slant on Things). Discuss the meaning of the diagonal arrows with the students (e.g., $\searrow$ is equivalent to $\downarrow\rightarrow$ or +11; $\nearrow$ is equivalent to $\uparrow\rightarrow$ or –9).

Assessment

Some students will still need the 100 charts for reference, while others will be able to do the necessary arithmetic mentally. Thus, copies of BLM 7 should be available.

Students might be surprised by some of the results. For example, the pathway in g) ends back at the starting number, 42. Ask students why this happens even though there are no direct opposites among the arrows. The reason is that the combination $\nearrow\leftarrow$ is equivalent to $\uparrow$. Thus $\nearrow\leftarrow$ is the opposite of $\downarrow$ so the combination $\nearrow\leftarrow\downarrow$ is equal to +0. This leaves the combination of $\rightarrow\leftarrow$ which is also equal to +0.

In answering #2, students will spot several opposites (e.g. $\downarrow\uparrow$ or $\rightarrow\leftarrow$) that can be deleted, but they should also be looking for other combinations that can be abbreviated such as $\rightarrow$ for $\uparrow\searrow$, or $\downarrow$ for $\swarrow\rightarrow$.

Extensions in Mathematics:

1. BLM 10 (Order, order) provides further experiences with the relevance of order in arithmetic and everyday activities. (i.e., The Commutative and Associative Properties)

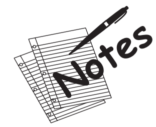
Activity 3: Number Properties

2. BLM 11 (Missing Numbers) involves identifying place values in order to add or subtract appropriate numbers. See example below.

Excerpt from BLM 11

a) 39876
$-\square$ It is necessary to subtract 9000
30876
$-\square$ It is necessary to subract 30 000
876

Notice that each operation in these questions involves one digit other than zero, with the appropriate number of zeros.



How do you know that $\square$46 − 187$\square$ can't equal 160?

Cross-curricular Activities

1. Explore the order of doing things by asking

If you have a sandwich and an apple for lunch, does it matter in which order you eat them?

If you are putting on a jacket and boots to go outside, does it matter in which order you put them on?

For which of the following does the order matter?
- – doing your homework and watching TV
- – putting on your hat and mitts
- – putting on your shoes and socks (*Note that we usually say to a child, "Put on your shoes and socks" when we should be saying, "Put on your socks and shoes." Isn't our use of English odd?*)
- – eating your vegetables and eating your dessert (*Parents may disagree with students' ideas here*.)
- – getting into bed and turning out the light
- – putting money in your bank account and writing a check
- – writing a spelling test and studying for a spelling test

Family Activities:

Problem Solving

1. A place value game to take home: materials needed are one die (or spinner) and paper for each player. Player A rolls the die. Then each player decides whether this number will be tens or ones, and writes the appropriate number. Thus, if Player A rolls 5 and chooses tens, he will write 50. Player B might choose ones; she will write 5. Players take turns rolling the die, but each player records a number for each roll. After 5 rolls, each player adds his/her numbers. The winner is the one closest to 100 but not more than 100. In the example below, Player A wins.

Activity 3: Number Properties

Notes

A number on the 100 chart is farther to the right than 25, farther left than 39, and closer to the top than 43. What could that number be?

Example:

Number rolled	Player A	Player B	Player C	Player D
5	50	5	5	50
6	6	60	6	6
1	10	10	10	1
3	30	30	30	30
2	2	2	20	2
Total	98	107	71	89

Variations:

1. Use 2 dice. Players decide, after each roll, which of the two numbers will be tens. Thus, for a roll of 5 and 2, the player may record either 52 or 25.

2. Let the winner be the one with a total closest to 100, whether this total is less than or greater than 100.

Other Resources:

For additional ideas, see annotated "Other Resources" list on page 72 numbered as below.

8. "Multiplication Games: How We Made and Used Them"

9. "The Influence of Ancient Egypt on Greek and Other Numeration Systems"

10. "'Understanding Aztec and Mayan Numeration Systems"

11."The Chinese Numeration System and Place Value"

Activity 4: Fractions

Focus of Activity:

- Identification and comparison of simple fractions

What to Assess:

- Ability to identify simple fractions.
- Ability to identify fractions equivalent to $\frac{1}{2}$
- Ability to identify the greater of two given fractions

'BLM' refers to the Black Line Masters, which follow the Activity Notes.

Preparation:

- Make copies of BLMs 12, 13, and 14.
- Provide scissors and crayons/markers.
- Make copies of BLMs 15 and 16 (optional).

Activity:

Distribute copies of BLM 12 (Fraction Pieces)

Write two fractions whose sum is $1\frac{1}{2}$.

Direct students' attention to the first diagram.
Ask: What fraction is written in one section?
What does "one–eighth" mean?
How big is each of the other sections?
Have the students label each of the other sections '$\frac{1}{8}$'

Have students label each section of each of the other circles with the correct fraction.

Ask: Which is greater: $\frac{1}{8}$ or $\frac{1}{3}$?

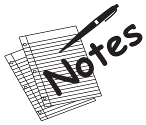
$\frac{1}{10}$ or $\frac{1}{4}$?

$\frac{1}{5}$ or $\frac{1}{6}$?

Students should realize that, if the numerators are "1" in both fractions being compared, the fraction with the greater denominator is the lesser fraction, because it takes more pieces to make up the whole.

Have students cut apart the pieces of each circle carefully, stressing the fact that all the eighths, for example, should be the same size.

You may wish to have students colour each circle in a different colour before cutting the pieces out. In this case, it is useful to have all students use the same set of colours. For example, colour the eighths red, colour the sixths blue, etc. This allows you to refer to the pieces by colour as well as name. For example, "Which is greater, the red eighth or the blue sixth?"



Write two fractions whose sum is less than 1.

Activity 4: Fractions

Have students manipulate the pieces to compare such fractions as

$$\frac{2}{10} \text{ and } \frac{1}{5}$$

$$\frac{2}{8} \text{ and } \frac{1}{4}$$

Have them illustrate one-half in as many ways as possible using the fraction pieces. They should show $\frac{4}{8}, \frac{3}{6}, \frac{5}{10}, \frac{2}{4}$. Have them compare the numerator and denominator of each of these fractions. *BLM 13 (Equal Fractions) provides a template of one-half against which students can compare their fraction pieces, as well as questions to encourage students to think about fractions in different ways.*

Similarly, compare $\frac{1}{3}$ and $\frac{2}{6}$; $\frac{2}{3}$ and $\frac{4}{6}$.

Ask "How many different fractions can you make/illustrate with '4' as the denominator? $\left[\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}\right]$. Students should realize that they can illustrate four different fractions with '4' as the denominator, six different fractions with '6' as the denominator, etc.

Some students may argue that $\frac{4}{4}$ or $\frac{6}{6}$, etc. are not fractions because they are equal to '1'. Mathematically, any number that can be written in the form of $\frac{a}{b}$, where 'a' and 'b' are whole numbers, is considered a fraction -- even such numbers as $\frac{10}{3}$ or $\frac{157}{2}$. Fractions greater than one can be called "improper fractions" if written this way, or "mixed numbers" if written as $3\frac{1}{3}$ or $78\frac{1}{2}$. Fractions less than one are sometimes called "proper fractions".

Some students may suggest that $\frac{0}{4}$ is also a fraction. Technically, this is correct, but we rarely think of zero as a fraction of something. For example, we would not ask "If all the pizza was eaten, what fraction of it was left?" Unless students raise the issue, it is best ignored. Notice that if these fractions are included in the sets they have illustrated, this will mean that there are five different fractions with '4' as denominator, seven different fractions with '6' as denominator, etc.

Distribute copies of BLM 14 (From Zero to One) to each pair or group.

Activity 4: Fractions

Students should have no difficulty completing question 1 using their fraction pieces. Notice that the question asks for fractions "between 0 and 1". This does not include either 0 or 1. However, if students decide, because of earlier discussion, that it is important to include 0 and 1, they should be allowed to do so, writing them as, for example, $\frac{0}{3}$ and $\frac{3}{3}$.

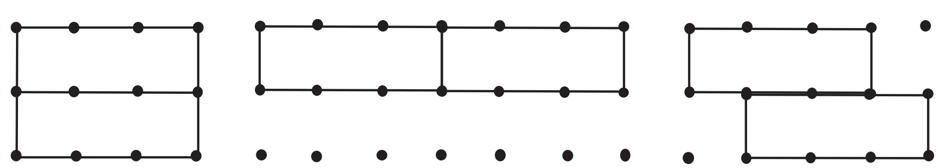
Since the expression "close to" is necessarily vague, students may disagree about such fractions as $\frac{3}{10}$. Some will see it as being close to zero; others will claim it is close to $\frac{1}{2}$. Either should be accepted, especially if the student can justify his/her choice.

Students are not asked if a fraction is closer to 0 than $\frac{1}{2}$. If they were, then $\frac{3}{10}$ would be named as closer to $\frac{1}{2}$. You may wish to introduce this idea, and alter the headings of the columns to "Closer to 0 than to $\frac{1}{2}$", "Closer to $\frac{1}{2}$ than to 0 or 1", and "Closer to 1 than to $\frac{1}{2}$".

Questions 3 and 4 explore some of the properties of fractions close to 0 or $\frac{1}{2}$ or 1. Note that the fractions given in question 5 cannot be illustrated with the fraction pieces from BLM 12. Students should solve these using the generalizations drawn in question 4.

Extensions in Mathematics:

1. All too frequently, students' experiences with fractions involve whole figures (usually squares, rectangles, or circles) which they are then asked to divide to illustrate certain fractions. BLM 15 (Draw the Whole) reverses this. Students are given figures that are a stated fraction of a whole and asked to draw the whole. Note that a wide variety of answers is possible. For example, 1a) could be completed as



Notes

If you wish to explore fractions greater than one, have students pool their fraction pieces.

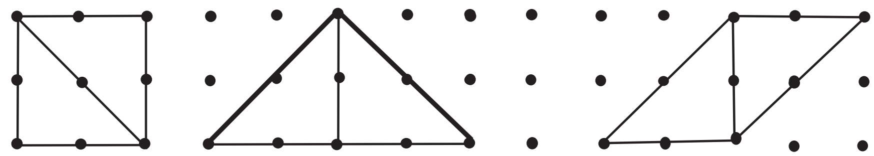
Problem Solving

Write three fractions whose sum is $1\frac{1}{2}$.

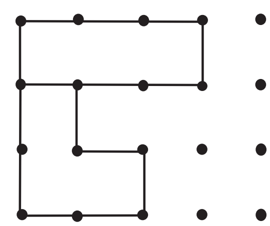
Write a fraction you could add to $\frac{1}{2}$ to give a sum less than 1.

Activity 4: Fractions

and 1 b) as



and so on. Students should realize that the area of the given figure is the important factor, not its shape. For example, 1a) could be answered with the following, since each of its two halves covers the same area.



This will be important in answering question 5.

See "Solutions and Notes" for several possible solutions.

Other Resources:

For additional ideas, see annotated "Other Resources" on page 72 numbered as below.

7. "A Game Involving Fraction Squares""

Activity 5: Estimation

Focus of Activity:

- Developing an estimation/mental math technique based on grouping by tens

What to Assess:

- Facility in finding 'sets of 10s', 'sets of 50s', 'sets of 100s'
- Ability to write pairs of numbers given their sum/difference/product
- Identification of unreasonable answers, and explanations of choices

Preparation:

- Make copies of BLMs 17, 18, and 19.
- Provide a calculator for each pair/group.
- Make copies of BLM 20 (optional).
- Make copies of BLM 21 (optional), and provide 10–12 markers for each player.

'BLM' refers to the Black Line Masters, which follow the Activity Notes.

Activity:

According to the Oxford Dictionary "compatible" means "mutally tolerant" or "able to be used in combination". Explain to students that "compatible" as used with numbers means *numbers with which it is easy to do mental computation,* and the use of compatible numbers is an accepted estimation technique.

Which division problem gives a closer estimate for $356 \div 6$? Why?
$300 \div 6$ or $360 \div 6$

For example, in the table below, the numbers in the first column are not considered compatible, but those in the second or third column are close to the original numbers and are compatible – that is, easy to manipulate mentally.

$24 + 69$	$24 + 70$	$20 + 70$
$37 - 19$	$38 - 20$	$40 - 20$
37×22	40×22	40×20
56 √ 15	60 √ 15	60 √ 10 or 60 √ 20

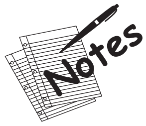
Students who know their addition and multiplication facts particularly well will be able to use compatible numbers such as those in the second column. Students who have more difficulty with arithmetic facts will find it simpler to round to the nearest 10 to give compatible numbers such as those shown in the third column above.

Compatible numbers are more often used with multiplication and division than with addition and subtraction. It is a technique we often use without realizing it.

Distribute copies of BLM 17 (Being Compatible)

Examine question 1. For addition, finding pairs or triples of numbers that have a sum of 10 or 100 is a good mental math technique. Give students a few minutes to find and list appropriate pairs for each part. Discuss with them which pairs were easier to find and why. For example, the single digits will be easier to deal with because they need only basic addition facts. However, finding pairs with sums of 50 or 100 usually involves

Note: There may be numbers in each box that are not used.



Activity 5: Estimation

regrouping. To help with these you might want to suggest the following method (or some students may be using this method already and will be pleased to explain it to others).

To "make 100" with, for example, 45 and 55, first add the units digits which will give 10. Then add the ten digits (e.g., 40 + 50 = 90) and add on the 10 from the units digit. Students should quickly realize that the tens digits of any compatible pair for this question should always have a sum of 90, and the units digits a sum of 10.

Change the dividend so the numbers are compatible.

$27 \div 4 \rightarrow \boxed{} \div 4$

$9\overline{)35} \rightarrow 9\overline{)\boxed{}}$

After a discussion of question 1, assign question 2. Part a) is similar to 1 c), while 2 b) and c) extend the skill. You may wish to have a discussion of question 2 before assigning question 3, which uses compatible numbers to add a column of figures mentally.

Before students begin question 3, ask how many 'sets of 10' they can find in each column. The example shows a *pair* of numbers (4 + 6) and a *set of three* addends (5 + 3 + 2) with sums of 10. In part b) it is even possible to find a set of four addends with a sum of 10 (2 + 4 + 3 + 1).

See "Solutions and Notes" for possible groupings.

Question 4 extends the ideas of questions 1, 2 and 3. Students should be encouraged to find as many combinations as they can of "compatible numbers". Suggest that they look for sets of numbers whose sum is an exact number of tens as in the example. Discuss with students how they can find (or have found) such sets. In each case, the units digits will have a sum of 10.

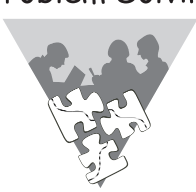
Distribute copies of BLMs 18 and 19.

Technology

Note: Since the focus is on mental math/estimation, the use of a calculator to check possible solutions is recommended.

Have students work in pairs/groups to find solutions. There is more than one combination possible for each part of question 1, and group work is more likely than solitary work to bring this out. If you do not want to raise the issue of more-than-one-right-answer at this point, note that it will come out in question 6 (BLM 19) when students are asked to find other possible answers.

Problem Solving



However, this is not the case for question 2. Only one solution is possible for each part. This may or may not affect answers to question 3 when students are asked to compare the difficulty levels of questions 1 and 2.

Question 4 may be seen as easier than either 1 or 2 simply because students need not use all four given digits. However, other students may see the multiplication as harder, based on their comfort levels with multiplication and division.

Activity 5: Estimation

A point to bring out in discussion of question 4 is that the units digit of the product should indicate which of the given digits should be units in the solution. In the example, we need two factors to give a units digit of 0, so we know they must be 5 and 2 or 5 and 4. The choice of the tens digit for one of the factors should then be made using estimation or mental arithmetic.

Some students may suggest that, once an answer is found, another is possible as was the case in question 1. That is, they may think that if $35\times2=70$, then 32×5 must also be 70. That this isn't true is due to the fact that in 35×2 we are multiplying 30×2, but in 32×5 we are multiplying 30×5.

Observing students as they create problems (question 7) and label them 'easy' or 'harder' will give some indication of their comfort level with arithmetic facts and mental math. Have students exchange questions with each other. The answers should then be given to the question posers for checking.

Notes

Assessment

Communication

How do you know that ☐75 + 887 ▯ can't equal 952?

Extensions in Mathematics:

1. Distribute copies of BLM 20 (Making Sense) to each group. Read #1 together. Ask students if they think the underlined sentence makes sense and why. Rounding the costs to the nearest dollar should convince them that Jamie's dad spent about $6\times\$3$ or \$18, while Marisa's mom spent only about $7\times\$2$ or \$14. Have groups discuss how they would change the story to have it make sense. Some may change the underlined sentence (e.g., "Marisa's mom spent less than Jamie's dad") while others may change other parts of the problem (e.g., Marisa's mom bought 10 wrenches").

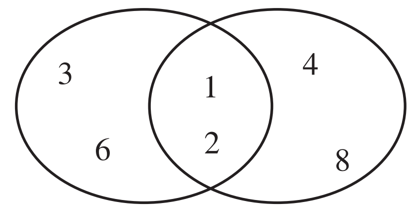
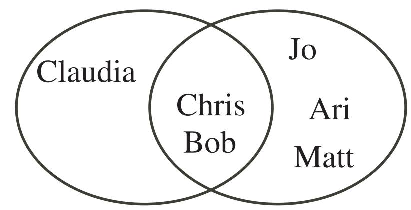
   Different answers are acceptable if the students can justify their ideas, and explain to the class why the change they made was valid.

   For some of the 'stories' one can't tell if the story makes sense of not. For example, in question 2 we need to know the sizes of the pizzas. In question 5, some students could have eaten both a hot dog and a hamburger.

   Note: The word "or", as used here, does not necessarily mean one or the other, although students may interpret it that way. It could mean both at the same time. Mathematicians are familiar with this inclusive "or". A simple example using a Venn diagram can help to make this clear.

Notes

Activity 5: Estimation

The factors of 6 are : 1, 2, 3, 6 The factors of 8 are: 1, 2, 4, 8 The factors of 6 or 8 are 1, 2, 3, 4, 6, 8. That is, 1 is a factor of either 6 or 8; 2 is a factor of 6 or 8; 3 is a factor of 6 or 8; 4 is a factor of 6 or 8; and so on.	Factors of 6 Factors of 8 
A similar diagram of party guests shows that half the 6 guests ate hot dogs and five-sixths of the 6 guests ate hamburgers. Yet each of the six guests could be said to have eaten either a hot dog or a hamburger. That is, we could say, "Chris ate a hot dog" and that would be perfectly true, while "Chris ate a hamburger" would also be true.	Hot Dog Eaters Hamburger Eaters 

Having said all this, we also realize that not all 9-year-olds are going to accept this logic. Some will be firmly convinced that the guests had to make a choice of one or the other, in which case the underlined statement does not make sense. Such students should be allowed to rewrite the 'story' in their own ways so it makes sense to them.

2. The game (Sum Bingo) on BLM 21 gives students further practice in estimating sums. Each group needs a playing board, a calculator, and a few markers, different for each player. A simple way to provide these is to have students cut a piece of paper into small squares. Each student can then write his/her initials on a few, and use these for markers.

   The game can be played with up to six players but is probably best with no more than 4.

   On his/her turn, a player selects two numbers (addends) from the Addends box, trying to select two numbers whose sum is on the Playing Board.

   For example, a student might think "$57+75$ is about $60+70$ or 130", then look at the Board and identify 127 as the probable sum. Unfortunately for this student, checking with the calculator will show that this is incorrect. Thus, this player is not allowed to place a marker on this turn.

   Another student might look at the Board, see 100 in a square and think " $57+43$ is about $60+40$ or 100 and I know that the answer will end in zero because $7+3=10$."

Problem Solving

Activity 5: Estimation

Obviously, this second student has more estimation techniques at his/her finger- tips than does the first student. As students play the game and give their reasoning aloud, they may acquire additional estimation techniques.

Not all sums of pairs of the given addends are on the Board (for example, the sum of $23+32$). Students could be asked to determine which of all the possible sums are on the Board.

Occasionally, a sum on the Board may represent two different pairs of addends. For example, $23+84=107$ and $32+75=107$.

All possible pairs of addends for each sum on the board can be found in "Solutions and Notes".

Notes

Communication

Cross-curricular Activities:

1. Explore the use if the word "compatible" in other situations. For example, which of the following would you consider compatible:

   toast and peanut butter
   shoes and socks
   shoes and gloves
   hot and cold
   you and your best friend

   Make a list of things you consider compatible and explain why. Make a list of things you do not consider compatible and explain why.

Family Activities:

1. Extend Cross-Curricular Activity #1 above by asking students to take home their lists of compatible items and see whether or not the adults in their home agree with them. For example, students may consider listening to loud music and doing homework to be compatible, but their parents/guardians might disagree.

2. The Sum Bingo game on BLM 20 can be taken home to play with family members.

Other Resources:

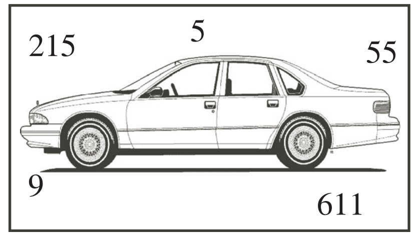
For additional ideas, see annotated "Other Resources" on page 72 numbered as below.

12. "Mental Computation in the Middle Grades".

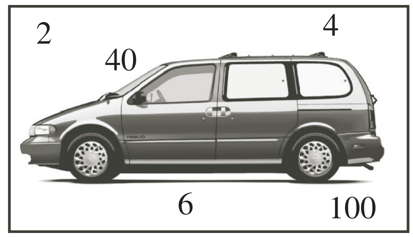
BLM 1: What's A Good Fit? —1

For each problem, select an appropriate number from the box to complete each blank. When you have finished, read each story to be sure it makes sense. Each number should be used once.

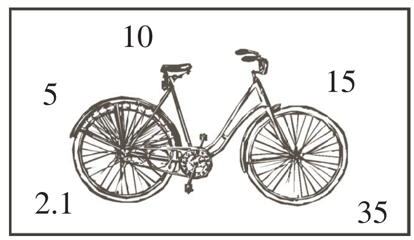
1. Laura's teacher recently bought a new car. It could seat ______ people in comfort. The engine had ______ horsepower. The gas tank held ______ litres of gasoline and took ______ litres to drive 100 km. With a full tank, she could drive ______ km.



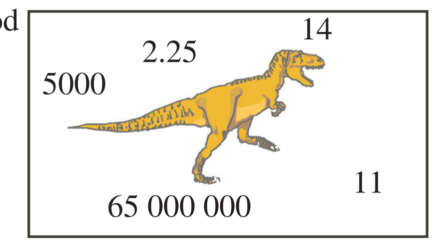
2. Ms. Otto's minivan has ______ doors, ______ of which are sliding doors. The van has a ______ cylinder engine. Ms. Otto drives at the speed limit of ______ km/h on the highway or ______ km/h in a school zone.



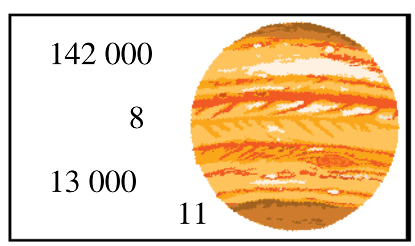
3. Kim's ______ speed bicycle weighs ______ kg, and is ______ m long. Kim weighs ______ kg, and can pedal at a speed of ______ km/h.



4. Tyrannosaurus Rex lived more than ______ years ago. This dinosaur stood up to ______ m tall, and weighed about ______ kg. It gained most of its weight over a four year period, starting around age ______. Each T. Rex gained about ______ kg per day during this time.



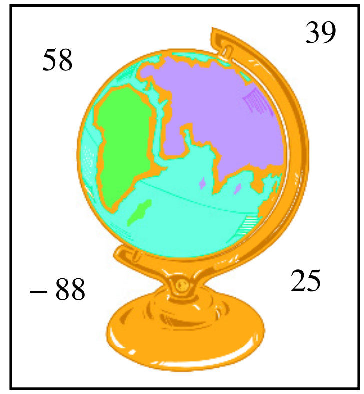
5. Jupiter is the largest of the ______ planets that circle our sun, with a diameter of ______ km. Jupiter's diameter is about ______ times the Earth's diameter, which is about ______ km.



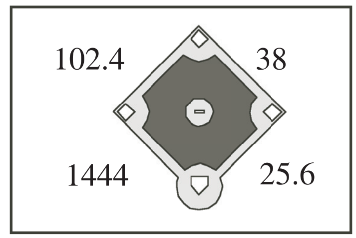
BLM 2: What's A Good Fit? — 2

For each problem, select an appropriate number from the box to complete each blank. When you have finished, read each story to be sure it makes sense. Each number should be used once.

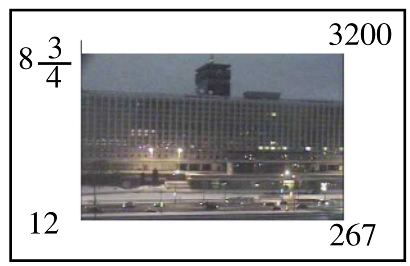
6. Your body is happiest when the temperature around you is about ______ ^{0}C, although if you are sick your body temperature may be as high as ______ ^{0}C. In Antarctica, the temperature can be as low as ______ ^{0}C. In northern Africa, it may be as hot as ______ ^{0}C.



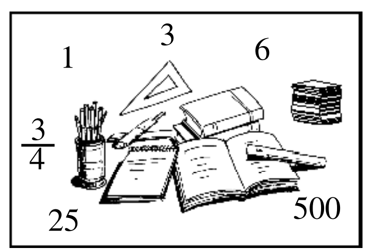
7. The distance between the bases on a baseball field is ______ m, so to make a home run, a player must run a total of ______ m. Each base is a square measuring ______ cm on each side. The area of one base is ______ cm^{2}.



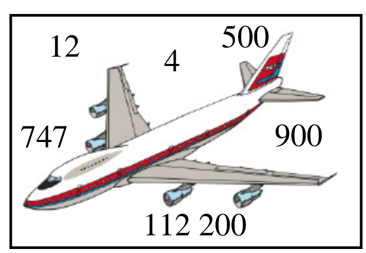
8. One of the largest hotels in the world is the Hotel Rossiya in Moscow, just across the road from the Kremlin. The hotel has ______ floors and ______ rooms, for an average of about ______ rooms per floor. It would take you ______ years to spend one night in each room.



9. On a trip to the mall to buy school supplies, Gwen went to ______ stores. She bought ______ notebooks, ______ set of markers, and ______ sheets of paper. Altogether she spent over $______ and walked ______ km.



10. A Boeing ______ jet can travel at ______ km/h. It has ______ jet engines, and can carry ______ L of fuel. For the convenience of its ______ passengers, it has ______ washrooms.



BLM 3: Growing A Tiger

1. Fold your paper under on the two dotted lines so that you cannot see the hints that are written there.
2. Choose one of the numbers in the box for each blank. You do not need to complete the blanks in the order in which they are given. Read the whole story first.
3. If you get stuck or want to check your choices, unfold one of the hints.

1.4	6	14	140	225
4.5	8.5	114	160	500

The Siberian tiger is very rare. Only about 500 exist in the world, so when a Siberian tiger cub was born at a zoo in Colorado, the zoo keepers were very happy. However, the cub was very small and weighed only _______ kg. His mother weighed _______ kg and his father weighed _______ kg.

The baby tiger, named T.J., was weighed again at _______ weeks old, when he weighed _______ kg. At 10 weeks old, he weighed almost 6 kg. Then his mother died suddenly, and T.J. had to be fed by hand. At first he ate hardly anything and by _______ weeks old, he weighed only _______ kg. When he finally began eating, he gained weight rapidly.

When he was 2 years old, he moved to a zoo in Montana. By then he weighed _______ kg. Two years later he weighed _______ kg. He now weighed more than his father!

Check your school and local libraries to see if they have a book called "Tiger Math" by Ann Whitehead Nagda and Cindy Bickel. This book tells the story of T.J., the tiger cub.

Hint #1: *The number of weeks is always a whole number.*

Hint #2: *Grown-up tiger's weights are given to the nearest whole kilogram.*

BLM 4: Which is More?

1. For each of the following, loop the greater number in each pair. Add these numbers using a calculator. Your answer should match the Check Number.

a) 62 or 65
24 or 74
92 or 52
67 or 77
Check Number : 308

b) 98 or 89
73 or 43
29 or 81
37 or 72
Check Number: 324

c) 455 or 545
201 or 120
136 or 162
109 or 019
Check Number: 1017

d) 45 or 4.5
3.7 or 3.17
98 or 89
5.06 or 5.6
Check Number: 152.3

e) 2.19 or 2.91
3.04 or 4.03
2.5 or 2.05
9.7 or 9.17
Check Number: 19.14

f) 6.2 or 6.5
2.4 or 4.2
0.92 or 0.29
6.7 or 7.7
Check Number: 19.32

2. For the following, write the three numbers in order of size, least to greatest. Loop and add the middle values. For example, in the first line of a), the numbers in order of size would be 5.02, 5.20, 5.21 and you would loop 5.20.

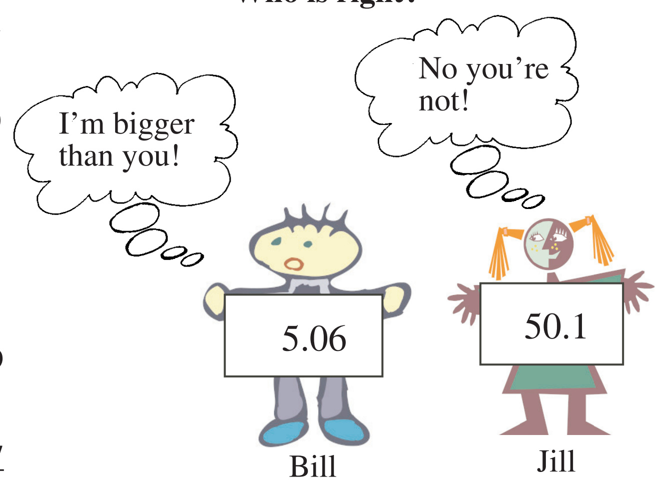
a) 5.21 or 5.02 or 5.20
37 or 3.7 or 7.3
1.9 or 1.99 or 1.09
3.8 or 3.83 or 3.38
Check Number: 18.2

b) 4.32 or 4.23 or 3.42
9.8 or 9.81 or 9.18
0.05 or 0.55 or 0.50
9.1 or 9.01 or 9.18
Check Number: 23.63

c) 7.3 or 0.73 or 3.7
4.0 or 4.06 or 4.16
9.5 or 5.9 or 0.95
23.1 or 21.3 or 31.2
Check Number: 36.76

d) 3.62 or 3.26 or 2.63
1.09 or 1.90 or 1.19
2.14 or 4.22 or 2.44
8.79 or 7.89 or 9.87
Check Number: 15.68

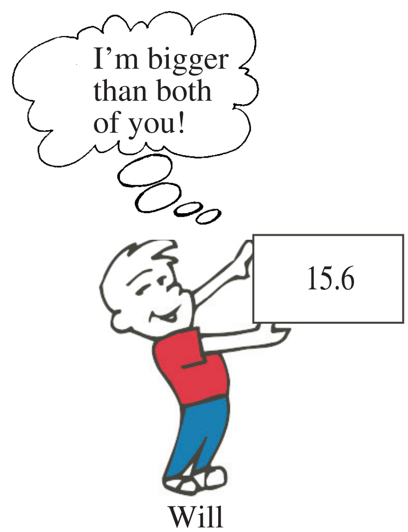
Who is right?



3. For these questions, loop and add the lowest values.

a) 1.13 or 1.11 or 1.31
7.34 or 4.73 or 3.74
8.10 or 8.01 or 8.11
2.31 or 2.13 or 2.33
Check Number: 14.99

b) 2.06 or 2.60 or 2.26
5.09 or 5.59 or 5.95
6.95 or 6.59 or 6.55
0.09 or 0.90 or 0.99
Check Number: 13.79



BLM 5: Ordering Decimals

Putting numbers in order is called 'ordering'. For these problems, you will need the set of number tiles for 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 at the bottom of this page.

1. Use all ten tiles to complete this set of decimals in order, from least to greatest.

$\square.3$, $2.\square$, $\square.\square$, $3.\square$, $\square.\square$, $4.\square$, $\square.\square$

2. Use all ten tiles to complete the following decimals in order, from least to greatest.

$\square.\square$, 0.8, $1.\square$, $\square.5$, $2.\square$, $2.\square$, $3.\square$, $3.\square$, $4.\square$, $4.\square$

3. Use all ten tiles to complete the following additions correctly.

a) $2.\square + \square.5 = 6.3$

b) $3.\square + \square.4 = 8.1$

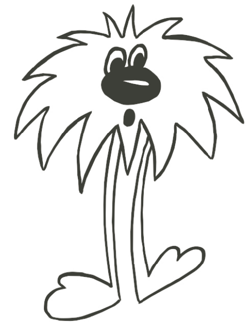
c) $6.\square - \square.2 = 1.\square$

d) $7.\square - \square.3 = \square.6$

Cut out these number tiles to use in solving the problems.

0	1	2	3	4	5	6	7	8	9

BLM 6: Imaginary Numbers?



For each question below, estimate an answer to each question.
There is a capital letter beside each question.
Write the letter in the blank above the best description of your estimate.
If your estimates are reasonable, the letters will spell the name of an imaginary animal.
Each set of problems has an example.
Not all letters will be used e.g., notice that "D" in #2 is not used).

1. Estimate an answer for each addition question.

G) $24+37 \rightarrow$ 60 A) $18+19 \rightarrow$ N) $89+12 \rightarrow$ D) $69+87 \rightarrow$

T) $99+14 \rightarrow$ O) $48+28 \rightarrow$ E) $66+47 \rightarrow$ R) $95+41 \rightarrow$

			G		
more than 150	between 130 and 150	less than 50	between 50 and 70	between 70 and 90	about 100

2. Estimate an answer for each multiplication queston. One letter will be used twice.

D) $29\times8 \rightarrow$ 240 I) $18\times9 \rightarrow$ R) $41\times8 \rightarrow$ N) $92\times5 \rightarrow$

A) $26\times9 \rightarrow$ O) $38\times3 \rightarrow$ C) $84\times8 \rightarrow$ U) $67\times8 \rightarrow$

between 500 and 600	between 420 and 480	between 150 and 200	more than 600	less than 150	between 300 and 350	between 400 and 500

3. Estimate an answer for each multiplication question.

A) $13\times29 \rightarrow$ 300 R) $19\times84 \rightarrow$ I) $32\times40 \rightarrow$ T) $61\times96 \rightarrow$

V) $43\times56 \rightarrow$ M) $36\times87 \rightarrow$ P) $26\times17 \rightarrow$ E) $93\times73 \rightarrow$

	A					
between 2000 and 3000	less than 500	between 3000 and 4000	between 500 and 1000	between 1000 and 1500	between 1500 and 2000	greater than 6200

BLM 7: Hundred Charts

Chart 1:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

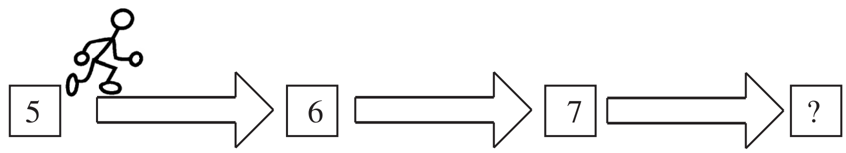
Chart 2:

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

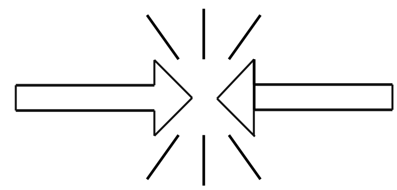
BLM 8: Follow the Arrows

1. Use the 100 chart to find the number at the end of each path.

a) 5 → → →	b) 5 → ↓ →	c) 5 ← ↓ ← ↑
d) 47 ↑ ↑ ↑	e) 79 ← ↓ → ↓	f) 66 ← ↓ ↓ → ↑
g) 35 ↑ ← ↓ → →	h) 28 → → ↓ ↓	i) 6 ↓ → ↑ ←
j) 32 ← ↓ → ↑	k) 63 ← ↑ ↑ ↑ ←	l) 100 ↑ ↑ ← ←



2. a) What is the result of 52 ↑ ↓? of 52 → ←? of 52 → ↓ ↑ ←?



   b) Why does this happen? Does it matter where you start? Does it matter in what order you do the motions?

3. a) Suppose we call a path on the chart 'closed' if the result is the same as the starting number. Which of the paths in #1 are closed paths?

   b) Design four more closed paths, (i) one with two motions or arrows, (ii) one with four motions, (iii) one with six motions and (iv) one with eight motions.

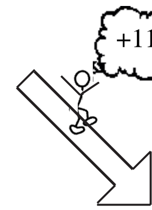
   c) Test your closed paths on the chart. Does it matter where you start?

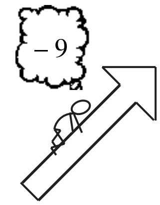
   d) Could you make a closed path with 3 motions? Explain.

4. a) You can use closed paths to make shorter paths. For example, 7 → ← → is the same as 7 →. Why?

   b) Decide which of the paths in #1 can be made shorter and write the shortest possible path for each choice.

BLM 9: A New Slant on Things





1. If 6 ↘ means the same as 6 ↓→, then what does each of the following mean?

   a) 6 ↙ b) 6 ↖ c) 6 ↗

2. Use the 100 chart to find the number at the end of each of the following paths.

   a) 5 ↘↑↘ b) 13 ↙↙→

   c) 70 ↖↓→ d) 91 ↗↑→↖

   e) 33 ↑↘↙↑ f) 56 →↗↗→

   g) 42 ↗→↓←← h) 51 ↘↘↓↗→

   i) 77 ↙↘↗↖ j) 28 ↘←↗↗→→

   k) 10 ↙↓↓↘←← l) 63 ↖←↑↑↘↘

3. a) Which of the paths in #2 can be made shorter? Write a shorter path in each case.

   b) Which of the paths in #2 are closed paths?

4. For each pair of numbers, write the shortest path that takes you from the first number to the second.

   a) 62 ________ 41 b) 33 ________ 10

   c) 76 ________ 43 d) 15 ________ 32

   e) 62 ________ 76 f) 85 ________ 57

BLM 10: Order, Order

1. Which of the following are true? Try to determine this without calculating answers.

a) $3+\frac{1}{2}=\frac{1}{2}+3$

b) $3-\frac{1}{2}=\frac{1}{2}-3$

c) $1\frac{1}{2}+\frac{1}{2}=\frac{1}{2}+1\frac{1}{2}$

d) $2\frac{1}{2}-1=1-2\frac{1}{2}$

e) $\frac{3}{4}-1=1-\frac{3}{4}$

f) $4\frac{1}{2}-4=4-4\frac{1}{2}$

2. Which of the following are true? Try to determine this without calculating answers.

a) $3\times4=4\times3$

b) $15\div5=5\div15$

c) $(7\times2)\times3=7\times(2\times3)$

d) $12\times\frac{1}{2}=\frac{1}{2}\times12$

e) $(12\div6)\div2=12\div(6\div2)$

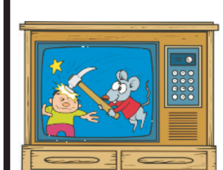
f) $2.5\times1=1\times2.5$

3. a) From the list of activities in the box below, select three pairs of activities that can be done in either order. One such pair might be washing your face and brushing your teeth. Does it matter which you do first?

b) Now select three pairs of activities that should probably be done in certain order. One such pair might be eating your lunch and making your lunch. Which one should come first? Why?

Activities:

washing your face	making your lunch
brushing your teeth	packing your backpack for school
drying your hair	eating your lunch
getting up	putting on your pajamas
getting dressed	putting toothpaste on your toothbrush
watching a DVD	turning on the TV
eating your breakfast	washing your hair



BLM 11: Missing Numbers

1. Write numbers in the boxes so all the subtractions and additions are true.

a)	b)	c)
39876	90876	90281
−	−	−
30876	90806	281
−	−	−
876	90006	201
+	+	+
50876	90106	3201
−	−	−
50076	106	3001

2. Fill in the missing numbers on the path:

a) 84573 (Start) → – 500 → 84073 → + → 84173

↓

− ← 80103 ← − ← 80173 ← −

↓

80003 → + → 89003 → − → 9003 (End)

b) 153.87 (Start) → – 0.8 → 153.07 → − → 53.07 → +

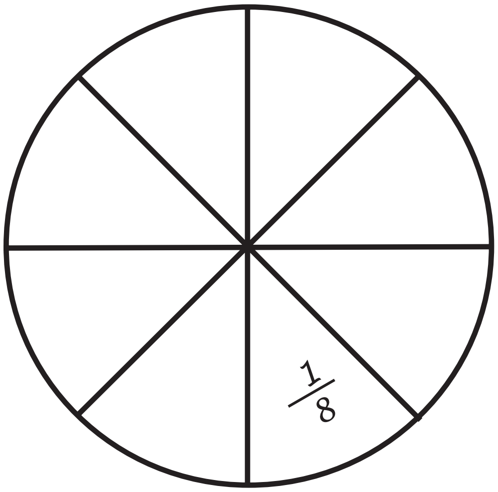
↓

− ← 953.07 ← − ← 1953.07 ← − ← 2053.07

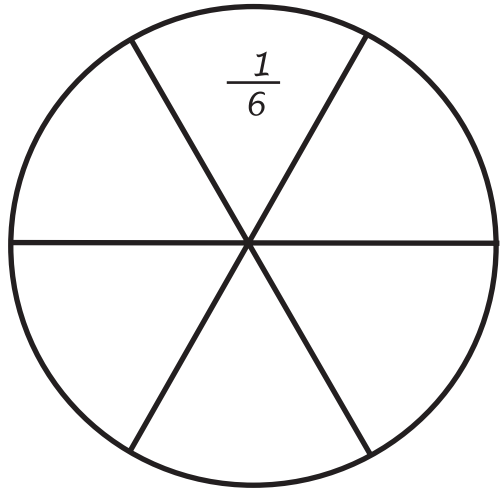
↓

903.07 → + → 905.07 → + → 1305.07 (End)

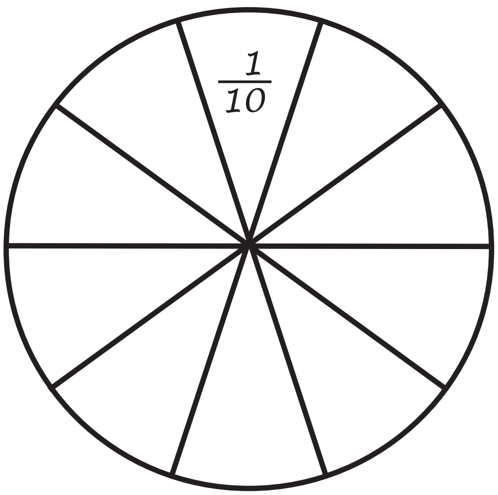
BLM 12: Fraction Pieces



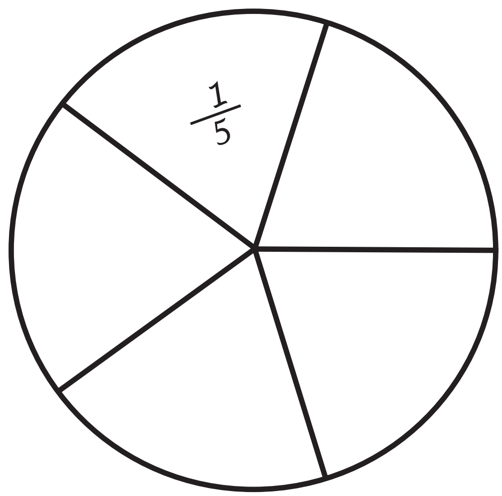
Eighths



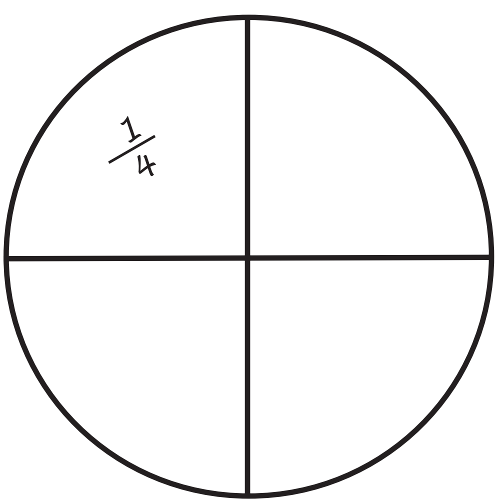
Sixths



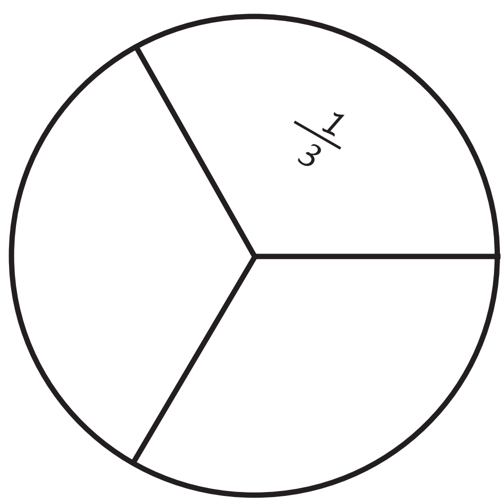
Tenths



Fifths

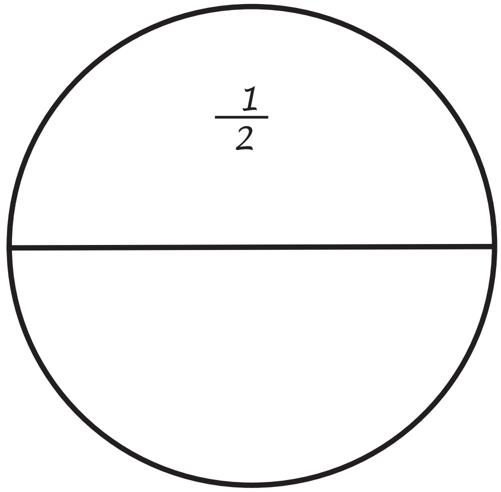


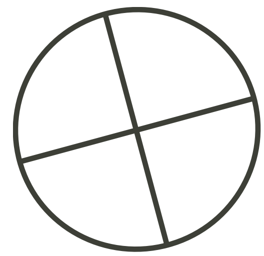
Quarters



Thirds

BLM 13: Equal Fractions

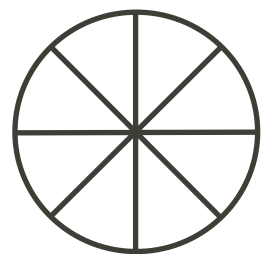




1. Write as many ways as you can to show $\frac{1}{2}$ with your fraction pieces.

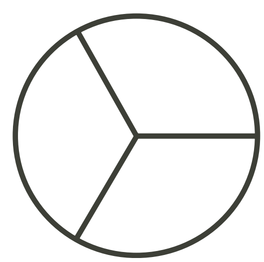
$\frac{1}{2}$ = ____ = ____ = ____ = ____

2. Marty answered question 1 by writing $\frac{1}{2} = \frac{1}{4} + \frac{2}{8}$. Was this correct? Explain.

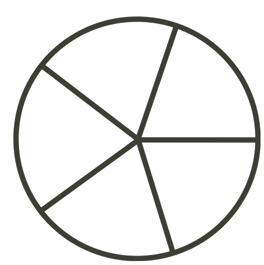


Challenges:

3. Ravi said that it took one-and-one-half thirds to make $\frac{1}{2}$. Was this correct? Explain.

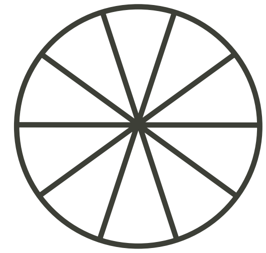


4. How many fifths equal $\frac{1}{2}$?



BLM 14: From Zero to One

1. List all the fractions between 0 and 1 for each of the following denominators:

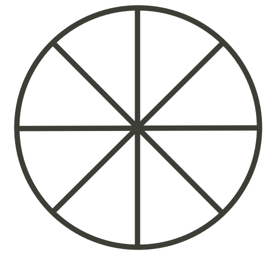


Tenths ______________________________________

Eighths ______________________________________

Sixths ______________________________________

Fifths ______________________________________

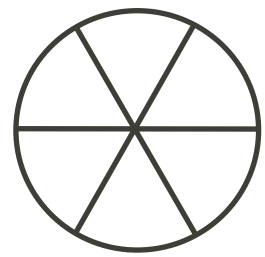


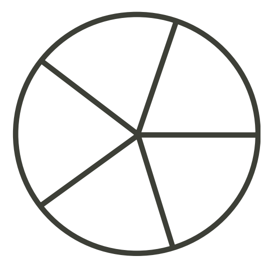
Quarters ______________________________________

Thirds ______________________________________

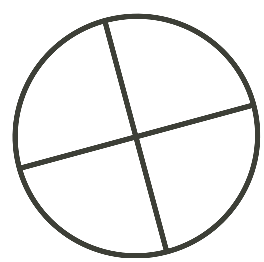
2. Write each fraction from problem 1 in the proper column in the chart below.

Denominator	Close to 0	Close to $\frac{1}{2}$	Close to 1
Tenths			
Eighths			
Sixths			
Fifths			
Quarters			
Thirds			





3. Which denominators have a fraction equal to $\frac{1}{2}$? Why?

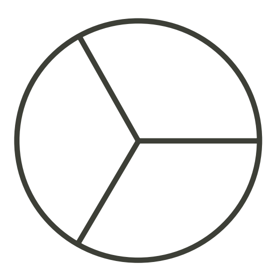


4. True or false: a) The fraction closest to 1 for each denominator has a numerator that is 1 less than the denominator.
   b) The fraction closest to 0 for each denominator has a numerator of 1.

Challenges:

5. Which of the following fractions is closest to 1? How do you know?

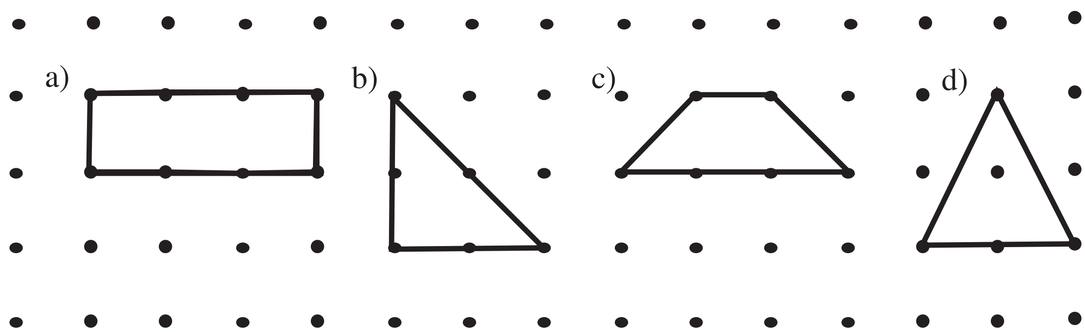
a) $\frac{3}{7}$ b) $\frac{8}{9}$ c) $\frac{1}{11}$ d) $\frac{4}{7}$ e) $\frac{17}{18}$ f) $\frac{1}{9}$



6. Which of the fractions in question 5 is closest to 0? How do you know?

BLM 15: Drawing the Whole

1. Each of the polygons below is $\frac{1}{2}$ of the whole figure. Draw the whole figure for each.



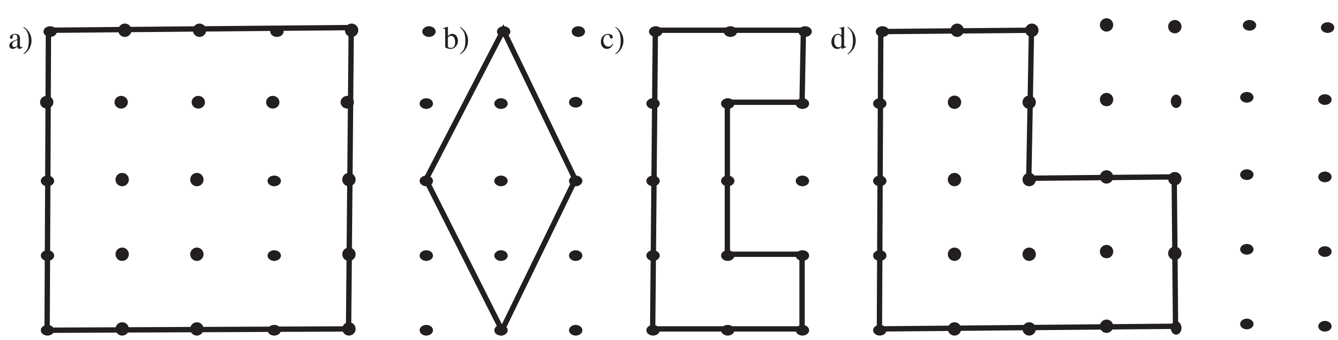
2. Copy each of the diagrams above onto dot paper and draw a different whole figure for each.

3. Copy each of the diagrams in #1 onto dot paper again. For this problem, think of each diagram as $\frac{1}{4}$ of the whole figure. Draw the whole figure in each case. Is it possible to draw more than one whole figure for each diagram? Explain.

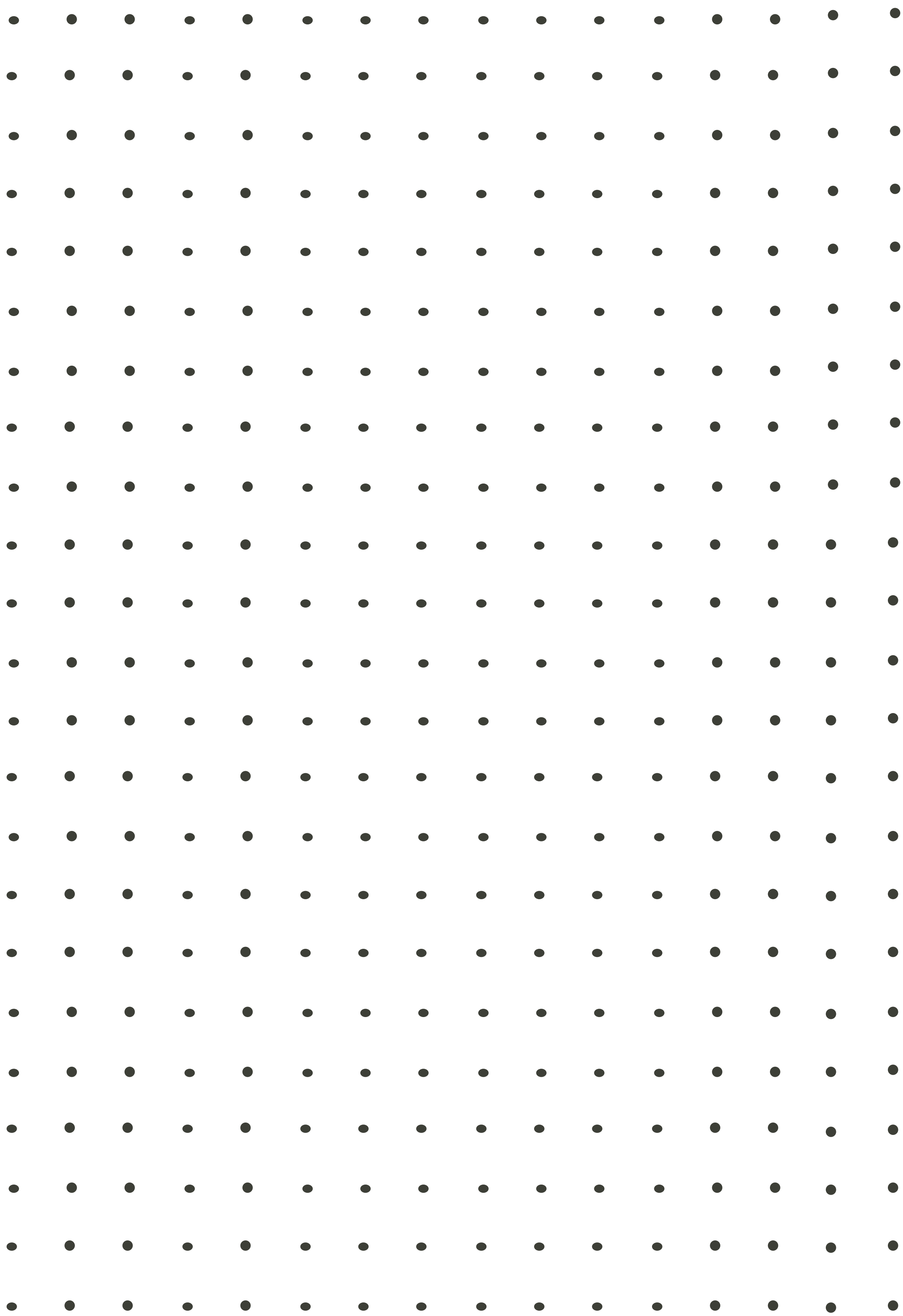
4. Once again, copy each of the diagrams in #1 onto dot paper. Think of each diagram as $\frac{1}{3}$ of the whole figure. Draw at least 2 different whole figures for each.

A Challenge:

5. Each of the following is a whole figure. Divide each into quarters, using lines from dot to dot. Is it possible to do this in more than one way for any of the figures? Explain.



BLM 16: Geo Paper



BLM 17: Being Compatible

1. From each box, select pairs of numbers that have the given sum. Find as many pairs as you can.

a) Sum: 10

6 7 9 5 2 4 8 1 0 3

b) Sum: 100

25 45 15 5 65 55 75 95 35

c) Sum: 100

73 44 18 56 27 7 63 93 82 65

2. From each box, select pairs of numbers that give the given sum. Find as many pairs as you can.

a) Sum: 100

16 26 07 74 19 84 81 25 13

b) Sum: 50

25 45 16 31 44 17 34 19 23

c) Sum: any number that ends in zero.

73 45 18 7 17 45 63 93 62 45

3. When adding a column of one-digit numbers, look for pairs or sets of numbers with a sum of 10.

Example:

Example	a)	b)	c)	d)	e)	f)
5	3	2	5	4	8	7
1	2	4	8	7	6	9
3	9	3	3	5	1	4
2	6	1	2	3	2	5
4	1	5	7	6	9	3
6	2	6	4	4	4	1
21						

(In the example, 5, 1, 3 and 2 are bracketed: 10; 4 and 6 are bracketed: 10.)

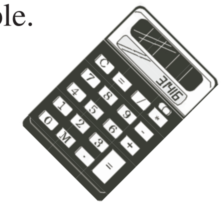
4. Look for compatible pairs of numbers when adding. Mark the pairs as shown in the example. Use these pairs to determine the sum in each case.

Example:

Example	a)	b)	c)	d)	e)	f)
23	34	79	53	17	35	84
15	67	33	19	23	40	33
65	66	55	21	72	35	6
77	82	45	25	64	75	17
29	18	67	25	18	15	54
209						

(In the example, 23 and 77 are bracketed: 100; 15 and 65 are bracketed: 80.)

BLM 18: Mental Manipulation –1



1. Use the four digits given to make 2 two-digit numbers with the given sum, as in the example. Try to do the arithmetic in your head.

	Given Digits	Sum	2 two-digit numbers
Example:	5 0 2 4	92	42 + 50
a)	3 1 5 7	52	
b)	8 9 6 3	179	
c)	4 6 5 7	121	
d)	3 2 4 5	77	

2. Use the four digits given to make 2 two-digit numbers with the given difference, as in the example. Try to do the arithmetic in your head.

	Given Digits	Difference	2 two-digit numbers
Example:	5 1 2 4	9	51 - 42
a)	3 1 2 0	22	
b)	8 9 6 7	13	
c)	4 3 5 4	11	
d)	7 6 5 8	13	

3. Was it easier to find sums or differences? Why?

BLM 19: Mental Manipulation –2

4. Use the four digits given to make a two-digit number and a one-digit number with the given product, as shown in the example. One of the given digits will not be used. Try to do the arithmetic in your head.

	Given Digits	Product	2 two-digit numbers
Example:	2 3 5 4	70	35 x 2
a)	6 4 2 7	182	
b)	8 9 6 4	392	
c)	8 6 1 7	128	
d)	3 0 1 7	210	

5. Was it easier or harder to find products than to find sums? Why?

6. Some problems have more than one right answer. For instance, the example in #1 could be $42+50$ or $52+40$. Write as many different answers as you can for each problem.

7. Make up some questions for your classmates. Make 2 easy ones and 2 harder ones.

BLM 20: Making Sense

Read each story. Tell whether or not the underlined sentence makes sense, and explain why you think so. If the underlined sentence does not make sense, rewrite any part of the story necessary to make sense.

1. Jamie's dad bought 6 wrenches at \$2.95 each. Marisa's mom bought 7 wrenches at \$1.95 each. Marisa's mom spent more than Jamie's dad.

2. Adeela ate $\frac{1}{2}$ the slices of an 8-slice pizza. Amir ate $\frac{1}{2}$ the slices of a 10-slice pizza. Amir ate more than Adeela.

3. One December Monday, Belinda walked 3 km for exercise. On Tuesday she walked 2 km. On Tuesday she walked half the distance that she walked on Monday.

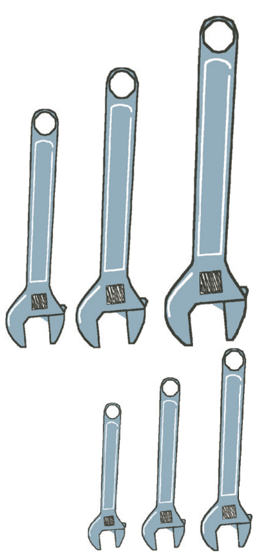
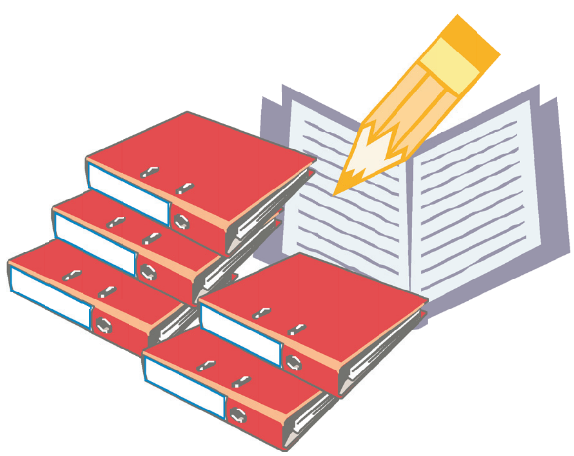
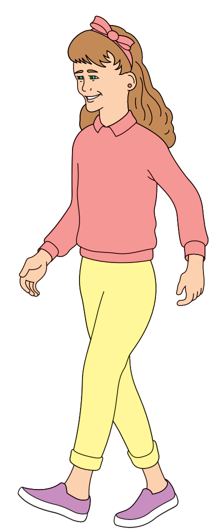
4. Mr. Marshman came home from shopping with 12 bags of groceries. Bev carried in $\frac{1}{2}$ the bags, John carried in $\frac{1}{3}$ of the bags, and Mr. Marshman carried in $\frac{1}{4}$ of the bags.

5. At Michele's party guests ate hot dogs or hamburgers. Half the guests ate hot dogs and $\frac{5}{6}$ ate hamburgers.

6. Luigi and Gianfranco tossed a coin to see who would go first in a game of checkers. Out of 10 tosses, Luigi won 4 times and Gianfranco won $\frac{1}{2}$ the time.

7. Janie bought 6 binders at \$2.45 each. The cashier said, "That will be \$15.35 plus tax".

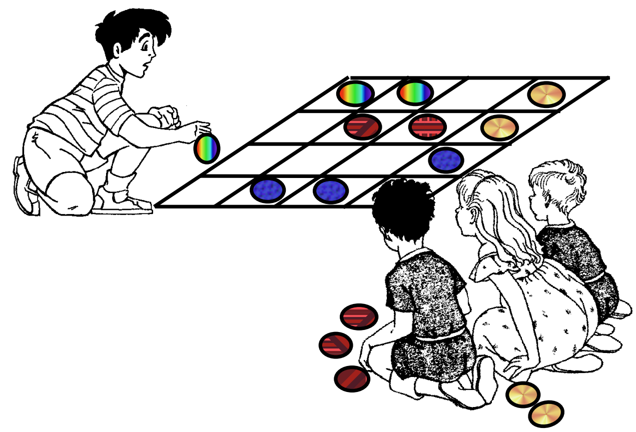
8. Alexei added $58 + 29 + 118$ on his calculator. He read the display as "twenty-five".

BLM 21: Sum Bingo

To play the game each player needs 10 - 12 markers to place on the board according to the rules below.

80	127	132	94
159	42	107	93
98	210	84	156
199	100	179	62



Players take turns.

On your turn, select two addends from the box below that you think will have a sum on the Playing Board. State the addends you have chosen, and the number on the Board that you think is their sum. Check with a calculator.

If you are correct, place one of your markers on the space containing that sum. If you were not correct, you lose that turn. The winner is the first to get four markers in a row in any direction.

Addends: Choose two of these numbers on each turn

23	57	75	43	61
32	95	84	19	115

BLM 22: News on Planets

In its October, 2006 edition, the magazine “The Smithsonian” reported that “The International Astronomical Union, meeting in Prague, in August [of 2006], came up with a new definition of planets .... Many astronomers argued that a planet should have sufficient gravity to sweep up or expel the debris in it orbit, [something] the eight main planets have largely done.”

Pluto, like the other known body in the Kuiper belt, Xena (discovered in 2003), has not done so. Xena (which is larger than Pluto) and Pluto are now to be called “dwarf planets.”

The National Geographic, in November, 2006, worded the definition as follows: “A planet must orbit the sun; it must not be a satellite; it must be massive enough for its own gravity to keep it round, and also big enough to dominate its orbit.”

Planet	Diameter (km)	Average distance from sun (millions of km)	Length of ‘year’ (earth days/years)	Gravitional Equivalent of 45 Earth kg
Mercury	4851	57.4	87.9 (days)	17.1 (kg)
Venus	12 034	107.5	224.7 (days)	41 (kg)
Earth	12 682	148.8	365.2 (days)	45 (kg)
Mars	6755	226.6	686.9 (days)	17.1 (kg)
Jupiter	142 154	773.9	11.9 (years)	112.5 (kg)
Saturn	119 837	1417.4	29.5 (years)	47.7 (kg)
Uranus	50 822	2880	84 (years)	38.7 (kg)
Neptune	49 242	4480	164.8 (years)	49.5 (kg)

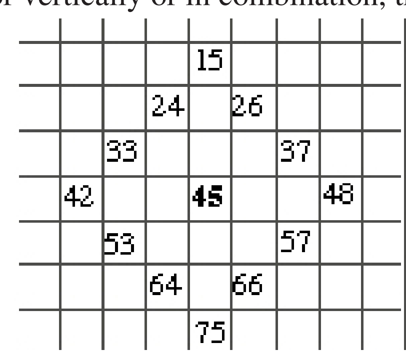
1. On which planets would you weigh less than you do on earth? On which planets would you weigh more? On which planet would you weight almost the same as you do on earth?

2. The length of the ‘year’ is the time it takes for the planet to travel around the sun, and for you to be one year older. If you lived on Mars, would you be older than you are now or younger? What if you lived on Mercury?

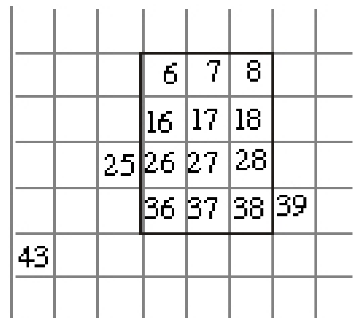
3. What planet is about half as big as Earth? There are two planets that are about 4 times as big as Earth. Which two?

Solutions & Notes

Marginal Problems

Page	Problem	Discussion
3	Give two fractions between 0 and $\frac{1}{2}$.	Any unit fraction (having a numerator of 1) with a denominator greater than 2 is correct $\left(\text{e.g. } \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\right)$. Other fractions are possible and some students may suggest them $\left(\text{e.g. } \frac{2}{11}, \frac{5}{30}, \frac{4}{100}\right)$.
8	If the product of two numbers ends in zero, what can you say about the two numbers?	One of the numbers must be even, and the other must end in '5' OR One of the numbers must end in '0'.
9	If $25\times25=625$, is 24×25 greater or less than 625? Why?	Obviously, twenty-four '25's will be less than twenty-five '25's.
10	If $1600\div4=400$, is 1621 √ 4 greater or less than 400? Why?	Since 1621 is greater than 1600, dividing by 4 will give a greater number. That is, $1621\div4$ is greater than 400.
13	Write two addition questions whose answers are 49.3.	These may vary from the simple $49+0.3$ to examples such as $25+24.3$ or $25.1+24.2$.
14	Give three different numbers that will make $4\times\square$ less than 87.	Any whole number less than 22 OR any fraction less than $21\frac{3}{4}$ OR any negative number.
15	Which is the correct answer for 300×40? (i) 12 000 (ii) 120 000 (iii) 1200	Encourage students to 'count zeros' to determine the correct answers for questions like this. Since 300 has 2 zeros and 40 has one, the answer must be 12 000 with 3 zeros.
17	A number on the 100 chart is 3 squares away from 45. What could that number be?	Using the charts on BLM 7 and counting 3 squares horizontally or vertically or in combination, there are 12 numbers.  If students count squares diagonally, there will be more possibilities.

Solutions & Notes

Page	Problem	Discussion
18	If $47\square \times 122 = 57\,950$, then what digit must $\square$ represent?	$47\square$ must be 475. This problem is related to the one on page 8.
19	How do you know that $\square 46 - 187$ can't equal 160?	$\square 46 - 187 = \_\,\_\,9$. That is, the answer must end in 9.
20	A number on the 100 chart is farther to the right than 25, farther left than 39, and closer to the top than 43. What could that number be?	Using the 100 charts on BLM 7, the number could be any of the numbers in the heavily outlined box below. 
21	Write two fractions whose sum is $1\frac{1}{2}$.	The simplest response is $1+\frac{1}{2}$. Other possibilities are $\frac{3}{4}+\frac{3}{4}$, $\frac{4}{3}+\frac{1}{6}$, etc.
22	Write two fractions whose sum is less than 1.	Students may give responses such as $\frac{1}{4}+\frac{1}{4}$ or $\frac{1}{3}+\frac{1}{3}$ or $\frac{1}{8}+\frac{3}{8}$. Some may use fractions with different denominators, such as $\frac{1}{4}+\frac{1}{8}$ or $\frac{1}{4}+\frac{1}{3}$.
23	Write three fractions whose sum is $1\frac{1}{2}$.	Simple solutions are $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$, $1+\frac{1}{4}+\frac{1}{4}$, $\frac{4}{8}+\frac{4}{8}+\frac{4}{8}$, etc.
24	Write a fraction you could add to $\frac{1}{2}$ to give a sum less than 1.	Any fraction less than $\frac{1}{2}$ is correct. For example, $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$; $\frac{1}{2}+\frac{1}{8}=\frac{5}{8}$.
25	Which division question gives a closer estimate for 350 √ 6? Why? 300 √ 6 or 360 √ 6	Students should see that 360 is closer to 350 than 300 is. Therefore 360 √ 6 gives a closer estimate. This example uses compatible numbers.

Solutions & Notes

Page	Problem	Discussion
26	Change the dividend so the numbers are compatible. $27 \div 4 \rightarrow \square \surd 4$ $9\overline{)35} \rightarrow 9\overline{)\square}$	Keeping the numbers close to the original numbers suggests the following: $27 \div 4 \rightarrow 28 \div 4$ $9\overline{)35} \rightarrow 9\overline{)36}$ The numbers should be chosen so that mental computation is easy.
27	How do you know that $\square 75 + 887$ can't equal 952?	The number 887 is almost 900. The least possible value for $\square$ is 1 and adding 175 to 887 will give over 1000. Alternatively, if students add 75 and 87 they will get 162 with a tens digit of 6, not the 5 that is given in 952.

Activity 1: Using Numbers

BLM 1: What's A Good Fit? – 1

The appropriate numbers to fill in the blanks for each story are given in order for BLMs 1, 2, and 3.

1. 5; 215; 55; 9; 611
2. 4; 2; 6; 100; 40
3. 10; 5; 2.1; 35; 15
4. 65 000 000; 11; 5000; 14; 2.25
5. 8; 142 000; 11; 13 000

BLM 2: What's A Good Fit? – 2

6. 25; 39; –88; 58
7. 25.6; 102.4; 38; 1444
8. 12; 3200; 267; $8\frac{3}{4}$
9. 3; 6; 1; 500; 25; $\frac{3}{4}$
10. 747; 900; 4; 112 200; 500; 12

BLM 3: Growing A Tiger

500; 1.4; 114; 160; 6; 4.5; 14; 8.5; 140; 225

Solutions & Notes

BLM 22: News on Planets

1. You would weigh less on Mercury, Venus, Mars, and Uranus. You would weigh more on Jupiter, Saturn, and Neptune. You would weigh almost the same on Saturn. (Venus and/or Neptune might also be named.)

2. Since Mars takes approximately 687 earth days to travel around the Sun, your age in ‘Mars years’ would be less than in ‘Earth years’. On the other hand, one ‘Mercury year’ is only approximately 88 ‘Earth days’, and so your age in ‘Mercury years’ would be greater than in ‘Earth years’.

3. Mars is about half as big as earth, with diameter approximately 6800 km versus Earth’s 13 000 km. The two planets which are about 4 times as big as Earth are Uranus (diameter approximately 51 000 km) and Neptune (diameter approximately 49 000 km).

Activity 2: Comparing and Ordering

BLM 5: Ordering Decimals

1. Answers will vary. Here are two possibilities.

i) $\boxed{0}.3$, $2.\boxed{8}$, $\boxed{2}.\boxed{9}$, $3.\boxed{1}$,

$\boxed{3}.\boxed{4}$, $4.\boxed{7}$, $\boxed{5}.\boxed{6}$

ii) $\boxed{1}.3$, $2.\boxed{2}$, $\boxed{3}.\boxed{0}$, $3.\boxed{8}$,

$\boxed{4}.\boxed{7}$, $4.\boxed{9}$, $\boxed{6}.\boxed{5}$

2. Answers will vary. Here are two possibilities. Note that the digit in the first box must be a 0 in all cases.

i) $\boxed{0}.\boxed{7}$, 0.8, $1.\boxed{2}$, $\boxed{1}.5$, $2.\boxed{3}$, $2.\boxed{4}$,

$3.\boxed{5}$, $3.\boxed{6}$, $4.\boxed{8}$, $4.\boxed{9}$

ii) $\boxed{0}.\boxed{1}$, 0.8, $1.\boxed{7}$, $\boxed{2}.5$, $2.\boxed{6}$, $2.\boxed{8}$,

$3.\boxed{3}$, $3.\boxed{5}$, $4.\boxed{4}$, $4.\boxed{9}$

Solutions & Notes

3. a) Examine digits in the tenths place: $\boxed{}$ + 5 ends in 3. Thus the first blank must be 8. We now have 2.8 + $\boxed{}$. 5 = 6.3. The blank must be 3. The other problems can be solved in a similar manner.

b) $2.\boxed{8} + \boxed{3}.5 = 6.3$ c) $3.\boxed{7} + \boxed{4}.4 = 8.1$

d) $6.\boxed{2} - \boxed{5}.2 = 1.\boxed{0}$ e) $7.\boxed{9} - \boxed{6}.3 = \boxed{1}.6$

BLM 6: Imaginary Numbers

1. (G) 24 + 37 → 60 (A) 18 + 19 → 40 (N) 89 + 12 → 100 (D) 69 + 87 → 160

   (T) 99 + 14 → 110 (O) 48 + 28 → 80 (E) 66 + 47 → 120 (R) 95 + 41 → 140

   Thus the solution is D R A G O N

2. (D) 29 × 8 → 240 (I) 18 × 9 → 180 (R) 41 × 8 → 320 (N) 92 × 5 → 450

   (A) 26 × 9 → 270 (O) 38 × 3 → 120 (C) 84 × 8 → 640 (U) 67 × 8 → 560

   Thus the solution is U N I C O R N

3. (A) 13 × 29 → 300 (R) 19 × 84 → 1600 (I) 32 × 40 → 1200 (T) 61 × 96 → 6000

   (V) 43 × 56 → 2400 (M) 36 × 86 → 3600 (P) 26 × 17 → 600 (E) 93 × 73 → 6300

   Thus the solution is V A M P I R E

Activity 3: Number Properties

BLM 8: Follow the Arrows

1. a) 8 b) 17 c) 3 d) 17 e) 99 f) 76 g) 36 h) 50 i) 6 j) 32 k) 31 l) 78

2. a) 52 in all three cases

   b) ↑ cancels ↓ and → cancels ←, and vice versa.

   The order of the motions does not matter.

3. a) parts i) and j) in problem 1 are closed paths

   b) (i) ← → or ↓ ↑ ii) ↓ → ↑ ← or → → ← ←, etc.

   (iii) → ↑ ↑ ↓ ← ↓ or ↑ → ↓ ← ↑ ↓, etc (iv) → ↑ ← ↓ ↓ → ↑ ←

   c) It does not matter where you start, unless the motions take you below 1 or above 100.

   d) You cannot make a closed path with 3 motions because the motions must occur in pairs which cancel one another, i.e., ↑ ↓ or → ←. Thus no path consisting of an odd number of motions is closed.

Solutions & Notes

4. a) You can eliminate any pair of form $\uparrow\downarrow$ or $\downarrow\uparrow$ or $\leftarrow\rightarrow$ or $\rightarrow\leftarrow$, even if they are not consecutive (e.g., $\rightarrow\uparrow\rightarrow\downarrow = \rightarrow\rightarrow$).

   b) 1c) $\leftarrow\leftarrow$ 1e) $\downarrow\downarrow$ 1f) $\downarrow$ 1g) $\rightarrow$

BLM 9: A New Slant on Things: [Solutions are based on Chart 1 on BLM 7. Some answers will be different if Chart 2 is used.]

1. a) 6 $\leftarrow\downarrow$ or 6 $\downarrow\leftarrow$ b) 6 $\uparrow\leftarrow$ or 6 $\leftarrow\uparrow$ c) 6 $\rightarrow\uparrow$ or 6 $\uparrow\rightarrow$

2. a) 17 b) 32 c) 70 d) 62 e) 33 f) 40 g) 42 h) 75 i) 77 j) 22 k) 48 l) 53

3. a) 2 a) $\rightarrow\searrow$ 2 b) $\swarrow\downarrow$ 2 c) closed 2 d) $\nearrow\uparrow\uparrow$ or $\nearrow\nearrow\nwarrow$ 2 e) closed 2 f) cannot be made shorter
   2 g) closed 2 h) $\searrow\searrow\rightarrow\rightarrow$ or $\searrow\searrow\searrow\nearrow$ 2 i) closed 2 j) $\searrow\nearrow\nearrow\rightarrow$ or $\nearrow\rightarrow\rightarrow\rightarrow$ 2 k) $\swarrow\swarrow\swarrow\searrow$ or $\swarrow\swarrow\downarrow\downarrow$ 2 l) $\uparrow$

   b) 2 c), e), g), i) are closed

4. a) 62 $\uparrow\nwarrow$ = 41 b) 33 $\nwarrow\nwarrow\leftarrow$ = 10 c) 76 $\nwarrow\nwarrow\nwarrow$ = 43 d) 15 $\swarrow\swarrow\leftarrow$ = 32
   e) 62 $\searrow\rightarrow\rightarrow\rightarrow$ = 76 f) 85 $\nearrow\nearrow\uparrow$ = 57

BLM 10: Order, Order

1. a) T b) F c) T d) F e) F f) F

2. a) T b) F c) T d) T e) F f) T

3. Answers will vary. Here are some possiblilities:
   a) Washing your face and brushing your teeth, getting dressed and eating your breakfast, washing your hair and brushing your teeth (others are possible)
   Order does not matter.
   b) Making your lunch and eating your lunch, putting toothpaste on your toothbrush and brushing your teeth, turning on the TV and watching a DVD.

BLM 11: Missing Numbers

Missing numbers are given in order for these problems.

1. a) – 9000; – 30 000; + 50 000; – 800 b) – 70; – 800; + 100; – 90 000 c) – 90 000; – 80; + 3000; – 200

2. a) + 100; – 4000; – 70; – 100; + 9000; – 80 000 b) – 100; + 2000; – 100; – 1000; – 50; + 2; + 400

Solutions & Notes

Activity 4: Fractions

BLM 13: Equal Fractions

1. $\frac{1}{2}=\frac{3}{6}=\frac{5}{10}=\frac{2}{4}=\frac{4}{8}$. These are easily illustrated using the cut-out pieces of the circles on BLM 12.

2. $\frac{1}{4}+\frac{2}{8}=\frac{2}{8}+\frac{2}{8}=\frac{4}{8}=\frac{1}{2}$. So Marty was correct.

Challenges:

3. $1\frac{1}{2}\times\frac{1}{3}=\frac{3}{2}\times\frac{1}{3}=\frac{1}{2}\times\frac{3}{3}=\frac{1}{2}$. So Ravi was correct. OR $\left(1+\frac{1}{2}\right)\times\frac{1}{3}=\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$.

   Students could fold a '$\frac{1}{3}$' circle in half and show that this 'half of a third' plus a third fills half a circle.

4. Since 5 '$\frac{1}{5}$' circles equal 1 whole circle, and $\frac{1}{2}$ of 5 = $2\frac{1}{2}$, there must be $2\frac{1}{2}$ '$\frac{1}{5}$' circles in half a circle,

   i.e., $2\frac{1}{2}\times\frac{1}{5}=\frac{1}{2}$.

BLM 14: From Zero to One

1. $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$ $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$

   $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$ $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ $\frac{1}{3}, \frac{2}{3}$

2.

Denominator	Close to 0	Close to $\frac{1}{2}$	Close to 1
Tenths	$\frac{1}{10}, \frac{2}{10},$	$\frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}$	$\frac{8}{10}, \frac{9}{10}$
Eighths	$\frac{1}{8}, \frac{2}{8}$	$\frac{3}{8}, \frac{4}{8}, \frac{5}{8}$	$\frac{6}{8}, \frac{7}{8}$
Sixths	$\frac{1}{6},$	$\frac{2}{6}, \frac{3}{6}, \frac{4}{6}$	$\frac{5}{6}$
Fifths	$\frac{1}{5}$	$\frac{2}{5}, \frac{3}{5}$	$\frac{4}{5}$
Quarters	$\frac{1}{4}$	$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$	$\frac{3}{4}$
Thirds		$\frac{1}{3}, \frac{2}{3}$	

Note: If you have opted for "closer to $\frac{1}{2}$ than to 0" (see Activity Notes, page 23) then fractions should be placed as in the chart above. With quarters, however, $\frac{1}{4}$ is as close to 0 as to $\frac{1}{2}$ and $\frac{3}{4}$ is as close to 1 as to $\frac{1}{2}$. Hence these fractions are shown in both columns.

Solutions & Notes

3. Only the denominators which are even numbers have a fraction equal to $\frac{1}{2}$, because the numerator must be a whole number equal to half the denominator and this is not possible for an odd denominator.

4. a) TRUE b) TRUE

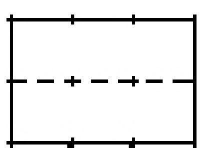
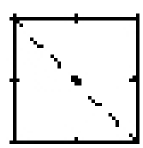
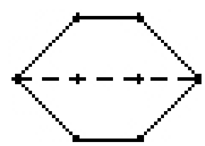
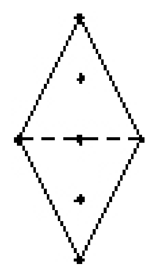
Challenges:

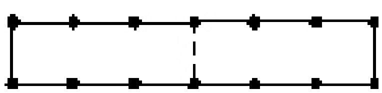
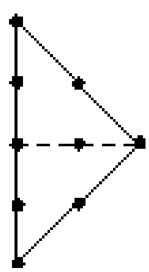
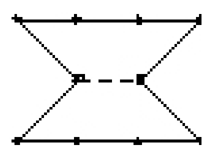
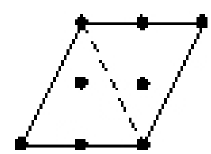
5. The fraction e) $\frac{17}{18}$ is closest to 1 because the numerator is 1 less than the denominator. $\frac{17}{18}$ is only $\frac{1}{18}$ distant from 1, whereas $\frac{8}{9}$, the next greatest fraction, is $\frac{1}{9}$ distant from 1.

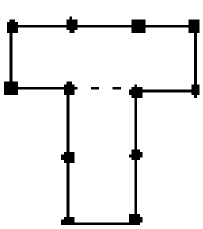
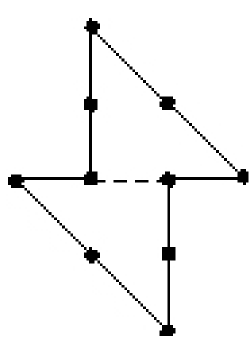
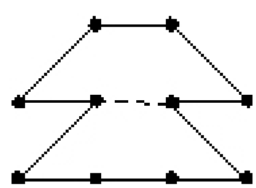
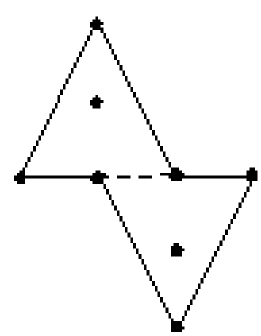
6. The fraction $\frac{1}{11}$ is closest to 0, as it has numerator 1, and is only $\frac{1}{11}$ distant from 0, whereas $\frac{1}{9}$ is $\frac{1}{9}$ distant from 0.

BLM 15: Drawing the Whole

1, 2. There are many possiblilities. Here are some.

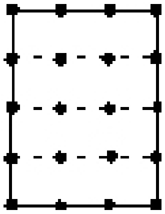
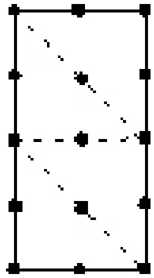
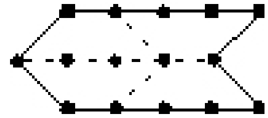
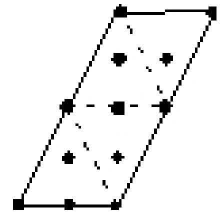
a)  b)  c)  d) 

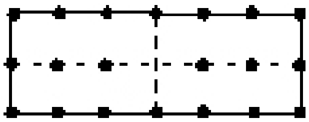
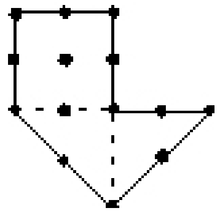
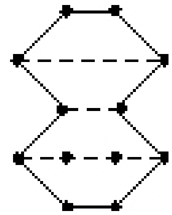
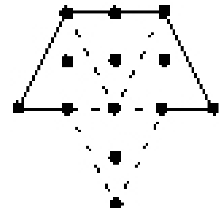
a)  b)  c)  d) 

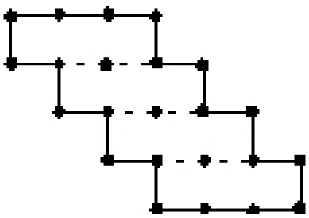
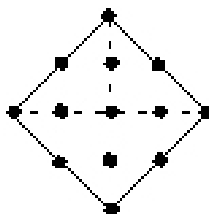
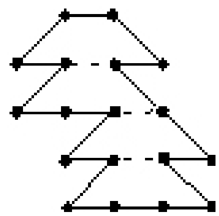
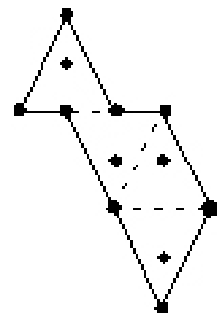
a)  b)  c)  d) 

Solutions & Notes

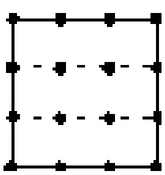
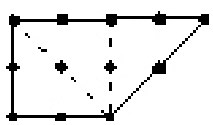
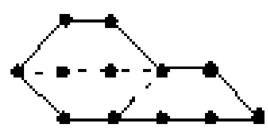
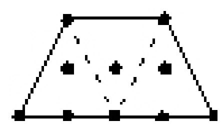
3. There are many possibilities. Here are three for each of a), b), c), and d).

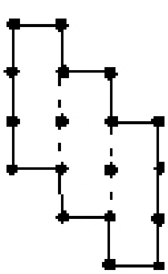
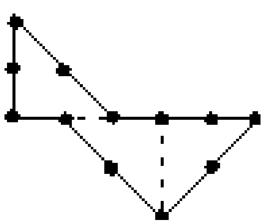
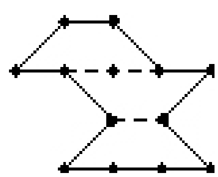
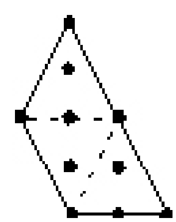
a) 
b) 
c) 
d) 

a) 
b) 
c) 
d) 

a) 
b) 
c) 
d) 

4. There are many possibilities. Here are two for each of a), b), c), and d).

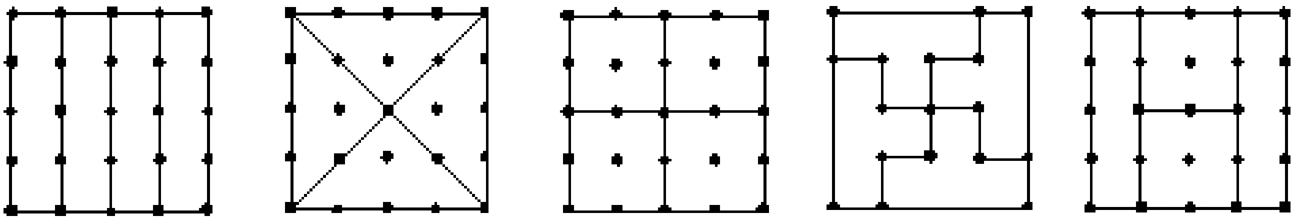
a) 
b) 
c) 
d) 

a) 
b) 
c) 
d) 

Solutions & Notes

Challenge:

5. a)



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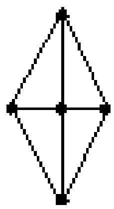
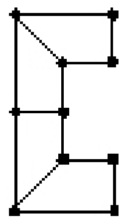
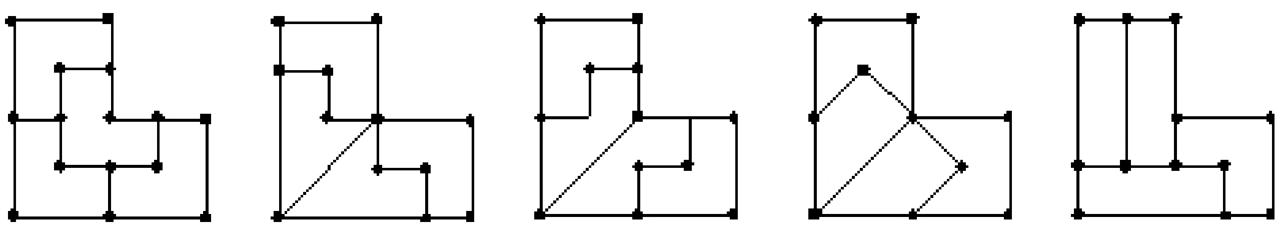


c)



d)



Comment: Students may find other solutions to b) and c) if they draw lines NOT joining dots. Note also that solutions to a) and d) include non-congruent quarters.

Activity 5: Estimation

BLM 17: Being Compatible

1. a) $10 = 7 + 3 = 6 + 4 = 8 + 2 = 9 + 1$

   b) $100 = 25 + 75 = 45 + 55 = 35 + 65 = 5 + 95$

   c) $100 = 27 + 73 = 18 + 82 = 7 + 93 = 44 + 56$

2. a) $100 = 16 + 84 = 19 + 81 = 26 + 74$

   b) $50 = 16 + 34 = 19 + 31 = 23 + 27$

   c) $70 = 63 + 7$; $80 = 73 + 7$; $80 = 18 + 62$; $80 = 63 + 17$; $80 = 45 + 35$;

   $90 = 73 + 17$; $90 = 45 + 45$; $100 = 93 + 7$; $110 = 93 + 17$

Solutions & Notes

3. Possible answers:

a) $$\begin{array}{rrl} & 3 & \\ & 2 & \\ & 9 & \\ 10 & 6 & 10 \\ & 1 & \\ & 2 & \\ \hline & 23 & \end{array}$$

b) $$\begin{array}{rrl} & 2 & \\ & 4 & \\ & 3 & \\ 10 & 1 & 10 \\ & 5 & \\ & 6 & \\ \hline & 21 & \end{array}$$

c) $$\begin{array}{rrl} & 5 & \\ & 8 & \\ & 3 & 10 \\ 10 & 2 & \\ & 7 & \\ & 4 & \\ \hline & 29 & \end{array}$$

d) $$\begin{array}{rrl} & 4 & \\ & 7 & \\ & 5 & 10 \\ & 3 & \\ & 6 & 10 \\ & 4 & \\ \hline & 29 & \end{array}$$

e) $$\begin{array}{rrl} & 8 & \\ 10 & 6 & \\ & 1 & 10 \\ & 2 & 10 \\ & 9 & \\ & 4 & \\ \hline & 30 & \end{array}$$

f) $$\begin{array}{rrl} & 7 & \\ 10 & 9 & \\ & 4 & \\ & 5 & 10 \\ & 3 & \\ & 1 & \\ \hline & 29 & \end{array}$$

4. Possible answers:

a) $$\begin{array}{rrl} & 34 & \\ & 67 & 100 \\ & 66 & \\ & 82 & 100 \\ & 18 & \\ \hline & 267 & \end{array}$$

b) $$\begin{array}{rrl} & 79 & \\ & 33 & \\ 100 & 55 & 100 \\ & 45 & \\ & 67 & \\ \hline & 279 & \end{array}$$

c) $$\begin{array}{rrl} & 53 & \\ & 19 & 40 \\ & 21 & \\ & 25 & 50 \\ & 25 & \\ \hline & 143 & \end{array}$$

d) $$\begin{array}{rrl} & 17 & 40 \\ & 23 & \\ & 72 & \\ & 64 & 90 \\ & 18 & \\ \hline & 194 & \end{array}$$

e) $$\begin{array}{rrl} & 35 & \\ & 40 & 70 \\ & 35 & \\ & 75 & 90 \\ & 15 & \\ \hline & 200 & \end{array}$$

f) $$\begin{array}{rrl} & 84 & \\ 90 & 33 & \\ & 6 & 50 \\ & 17 & \\ & 54 & \\ \hline & 194 & \end{array}$$

BLM 18: Mental Manipulation – 1

1. a) 35 + 17 or 15 + 37 b) 86 + 93 or 96 + 83
 c) 67 + 54 or 57 + 64 d) 32+45 or 42+35 or 54 + 23 or 24 + 53

2. a) 32 – 10 b) 89 – 76 c) 54 – 43 d) 78 – 65

3. Answers will vary.

BLM 19: Mental Manipulation – 2

4. a) 26×7 b) 98×4 c) 16×8 d) 30×7

5. Answers will vary.

6. See answers to #1 above.

7. Answers will vary.

Solutions & Notes

BLM 20: Making Sense

Possible answers:

1. Jamie's dad spent about $6 \times \$3 = \$18$.
   Marisa's mom spent about $7 \times \$2 = \$14$.
   Change the underlined sentence to "Marisa's mom spent less than Jamie's dad".

2. It is impossible to tell who ate more without knowing the sizes of the pizzas. If they were the same size, then each person ate $\frac{1}{2}$ his/her pizza, so they ate the same amount.

3. Half of 3 is not 2, so the underlined sentence is wrong.
   Change the first sentence to "One December Monday, Belinda walked 4 km".

4. $\frac{1}{2}$ of 12 is 6
   $\frac{1}{3}$ of 12 is 4
   $\frac{1}{4}$ of 12 is 3
   Total 13

   Change the last sentence to "Mr. M. carried in 2 bags", or "Mr. M. carried in $\frac{1}{6}$ of the bags."

5. See notes for Activity 5, pages 27 and 28.

6. $\frac{1}{2}$ of 10 is 5, so Gianfranco won 5 times, and Luigi won 4 times. This does not give a total of 10 tosses.
   Change the number of times Luigi won to 5.

7. The binders cost less than \$2.50 each, so 6 binders would cost less than \$15.00. The cashier was wrong. She should have said "\$14.70 plus tax". Students may suggest that cashiers are seldom wrong, and may change the underlined sentence to something like "Jamie thought the binders would cost about \$15.00".

Aternative Solution:

Since $6 \times \$2.45$ must end in zero, the underlined sentence does not make sense. Students can determine the value of $6 \times \$2.45$ using a calculator and change the underlined sentence accordingly.

8. $58 + 29 + 118 = 205$. Obviously, Alexei needs some help with place value. He should have read the answer as "two hundred five".

Solutions & Notes

BLM 21: Sum Bingo

The addends that will give each sum on the board are listed below.

80 $= 57 + 23$ $= 61 + 19$	127 $= 95 + 32$ $= 43 + 84$	132 $= 75 + 57$	94 $= 19 + 75$
159 $= 84 + 75$	42 $= 19 + 23$	107 $= 75 + 32$ $= 84 + 23$	93 $= 61 + 32$
98 $= 75 + 23$	210 $= 115 + 95$	84 $= 61 + 23$	156 $= 61 + 95$
199 $= 115 + 61$	100 $= 43 + 57$	179 $= 95 + 84$	62 $= 19 + 43$

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

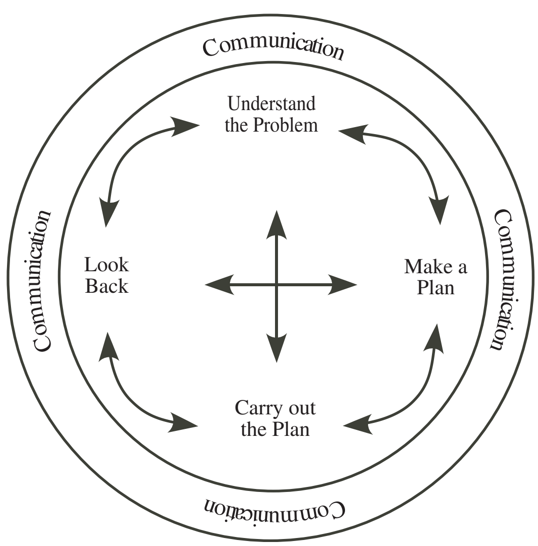
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R's"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



Suggested Assessment Strategies

There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;

Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.

Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
STRATEGIES	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

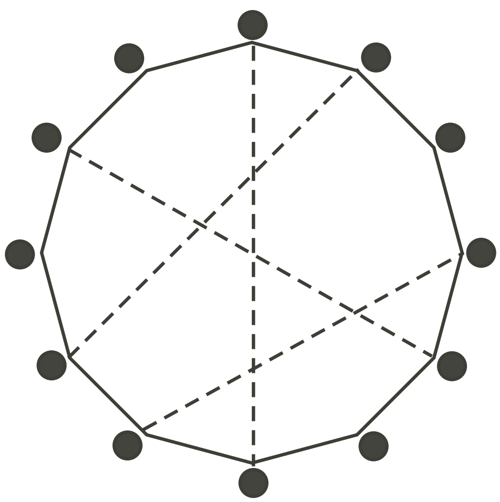
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a “what if...?” problem, such as “What if the question were reversed?”, “What if we had other data?”, “What if we were to show the data on a different type of graph?”.

Suggested Assessment Strategies

Applying the Rubric

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described" by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

Activity 2, BLM 5

Understanding: Do students realize that all ten number tiles must be used for each problem? Do they understand that tiles may be moved from initial placements?

strategies and reasoning: Do students consider the whole problem and look first for number tile positions for which there may be only one or two choices?

reflection: Can students explain the steps they followed and why each number tile was placed in each position?

For example,

- The "Limited" student may try to use one or more digits twice within a problem or may resist moving a tile once it is placed.
- The "Acceptable" student may try to complete the boxes in order and may need to be reminded that tiles can be moved, but will eventually place most or all of the tiles completely.
- The "Capable" student feels comfortable moving tiles from one position to another.
- The "Proficient" student will need to move tiles less often.

Activity 3, BLMs 8 and 9

Understanding: Do students realize that each arrow represents a different arithmetic operation? Do they understand that consecutive operations can be "collapsed" (e.g., $\rightarrow\downarrow$ is equivalent to $\searrow$)?

strategies and reasoning: Do students need to follow each arrow on a chart, or can they interpret the arrows mentally? Do students locate pairs of opposite arrows to shorten the work for #1 on BLM 8 , and #2 on BLM 9?

- The "Limited" student relies on the chart, interpreting the arrows one at a time.
- The "Acceptable" student recognizes the arithmetic operation represented by each arrow, but may still interpret the arrows one at a time.
- The "Capable" student will group the arrows in some cases (e.g., recognizes that $\rightarrow\rightarrow\rightarrow$ represents +3).

Suggested Assessment Strategies

Activity 5, BLM 18

Understanding: Do students realize that all four given digits must be used? Do they realize that the two 2-digit numbers formed must have the given sum?

strategies and reasoning: How random are students' attempts at solutions?

For example,

- The "Limited" student tries different possibilities without reference to the sum;
  or may use a digit more than once, and fail to use all 4 digits (e.g., 1 a) 51 + 1 =52);
  or may make errors in computation (e.g., 1 c) 76 + 55 = 121).
- The "Acceptable" student may use trial and error but does attempt to reach the given sum.
- The "Capable" uses number sense (e.g., for 1 a) the units digits must be 5 and 7 to reach a sum of 52).
- The "Proficient" student uses estimation skills and number sense (e.g., for 1 a), front end estimation indicates that to reach a sum of 121, the two numbers should be either 6__ + 5__ or 4__ + 7__).

Resources for Assessment

1. The Ontario Curriculum, Grades 1-8: Mathematics.

2. *Linking Assessment and Instruction in Mathematics: Junior Years*, Ontario Association of Mathematics Educators/OMCA/OAJE, Moore et al., 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level.

3. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, by Jean Kerr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels.

4. *How to Evaluate Progress in Problem Solving*, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Other Resources

1. *"Number Sense and Operations*, *Addenda Series,"* *Grades K-6,* ed. by Grace Burton, 1992, National Council of Teachers of Mathematics (NCTM), Reston, VA (1-800-235-7566)
   The booklet contains suggested lessons for each grade dealing with numbers. Activities deal with multiplication patterns, exploring large numbers through counting blades of grass, and estimating.

2. "Developing Sense About Numbers", by Carole Greenes, Linda Schulman and Rika Spungin, *Arithmetic Teacher*, pp 279-284, January 1993, NCTM.
   This article lists seven number-sense skills, such as "recognizing the various uses of numbers", "estimating results of computations", and "understanding phrases that establish mathematical relationships". Several "Fit the Facts" activities (similar to BLMs 1, 2, and 3 in Grade 4 "Investigations in Number Sense and Estimation") are included.

3. "The Revolution in Arithmetic" , by William A. Brownell, *Mathematics Teaching in the Middle School*, pp 27-30, August 2006, NCTM.
   This is a reprint of an article first printed in 1954. It describes changes in content, teaching methods, and text books during the first half of the 20th century. It is worth considering whether or not 21st century content, teaching methods, and text books have continued to evolve/improve.

4. *"*Tiger Math: Learning to Graph from a Baby Tiger,*"* by Ann Shitehead Nagda and Cindy Bickel: Henry Holt and Co., New York, 2000.
   This book tells the story of an orphaned cub and the attempts of the Denver Zoo staff to feed him. Several graphs of different types (picto-, circle, line, and bar) chronical the baby tiger's growth.

5. "How Much is a Million?", by David M. Schwartz and Steven Kellogg, William Morris and Co., New York, 1985.
   This book helps students come to grips with one million (1 000 000), one billion (1 000 000 000), and one trillion (1 000 000 000 000) through the answers to such questions as "If a million kids climbed onto one another's shoulders", how tall would they be? The answers may surprise you.

6. "How Big is Bill Gates's Fortune?", by Hamp Sherard, *Mathematics Teaching in the Middle School*, pp 250-252, December 2000, NCTM.
   Students explored large numbers by determining the weight or size of Gates' fortune in $100 dollar bills. One student calculated that the fortune was 62.5 miles taller than Mount Everest if the bills were stacked one on top of the other. Recently (summer of 2007) Gates' fortune was estimated at 100 billion dollars. (A $100 bill is approximately 6.6 cm wide, 15.6 cm long, 1 mm thick and 0.9 g mass).

7. "A Game Involving Fraction Squares" by Enrique Ortez, *Teaching Children Mathematics*, pp 218-222, December 2000, NCTM.
   This article describes a game similar to the idea on BLM 5 (Ordering Decimals) in this book, but with the stress on equivalent fractions.

Other Resources

8. "Multiplication Games: How We Made and Used Them", by Constance Kamii, and Catherine Anderson, *Teaching Children Mathematics*, pp 135-141, November 2003, NCTM.

   Several easily-made multiplication games are described, from those that use one multiplication table at a time (e.g., x 4) to those that use three or more factors. Using these games throughout the year, from the simplest to the more difficult, students showed considerable improvement in their speed and accuracy.

9. "The Influence of Ancient Egypt on Greek and Other Numeration Systems" by Claudia Zaslavsky, *Mathematics Teaching in the Middle School,* pp 174-178, November 2003, NCTM.

   The article describes the Egyptian hieroglyphic numbers and how they influenced the Greeks who used letters of the alphabet as numerals. Two Black Line Masters for students are given.

10. "'Understanding Aztec and Mayan Numeration Systems", by Angela L. E. Walmsley, *Mathematics Teaching in the Middle School,* pp 55-59, August 2006, NCTM.

    A comparison of the Aztec system (lacking a zero) and the Mayan system (including a zero) to the Hindu-Arabic system brings out many properties of numbers and emphasizes place value. Several examples of all systems are given. A source for more information on the Mayan system is given as "www.ancientscripts.com/aztec.html".

11. "The Chinese Numeration System and Place Value", by Frederick Uy, *Teaching Children Mathematics*, pp 243-247, January 2003, NCTM.

    The article describes how Grade 4 students were introduced to the Chinese number system. While translating Arabic numerals to Chinese numerals, students were forced to think carefully about the nature of place value.

12. "Mental Computation in the Middle Grades: The Importance of Thinking Strategies", by Alistair McIntosh, Robert E. Reys, and Barbara J. Reys, *Mathematics Teaching in the Middle School,* pp 322-327, March - April 1997, NCTM.

    The article discusses the nature of thinking strategies, number sense, mental computation, and estimation. Students' opinions and thinking strategies are given. Conclusions include the following:
    - Students success with computation is much higher when students see the problem as opposed to when the problem is read to them.
    - The more confident a student is, the more likely he/she is to develop alternate strategies.
    - Students' perceptions of what is meant by mental computation differ greatly.