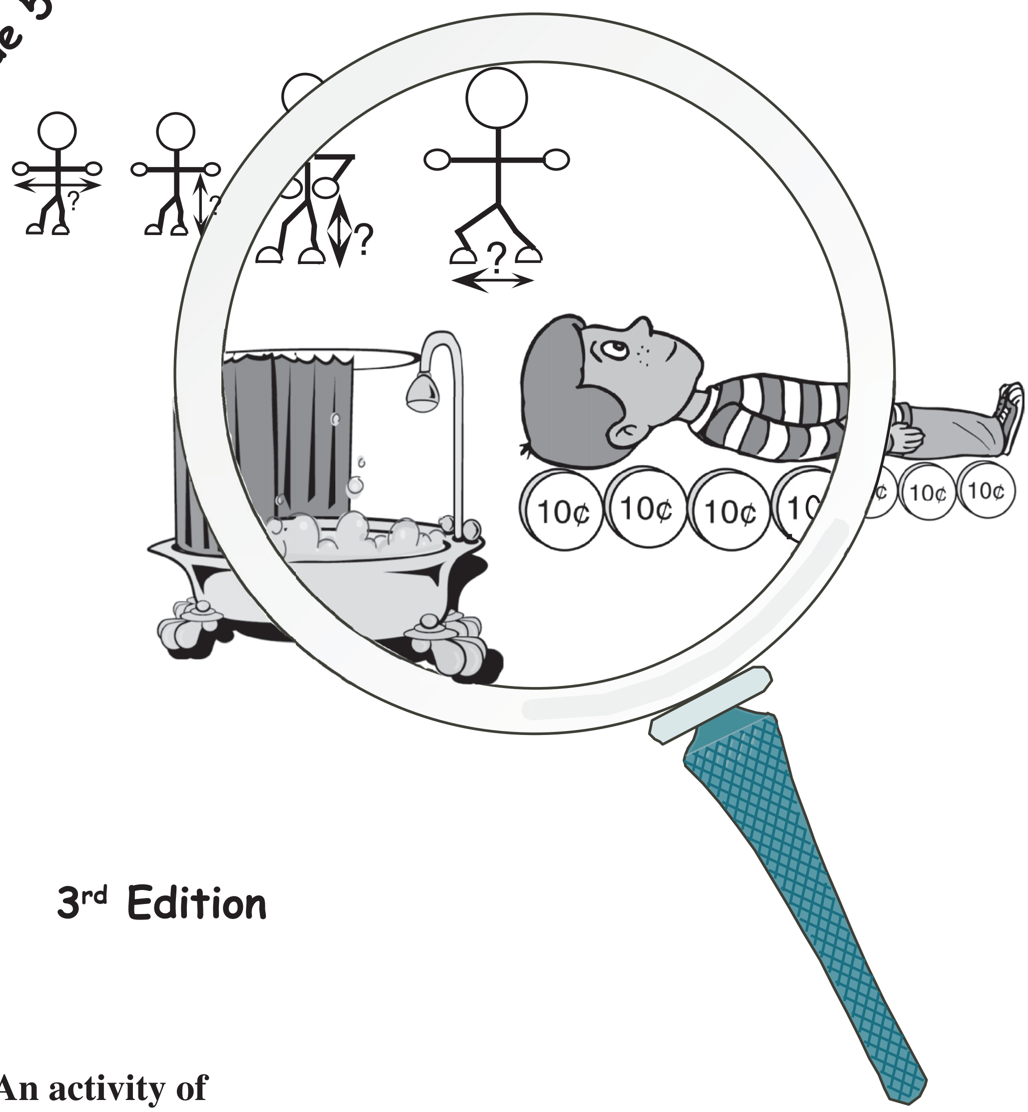
Invitations to Mathematics

Investigations in Measurement

"Inside and Out"

Suggested for students at the Grade 5 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
Faculty of Mathematics, University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the extensions and related activities included with individual activities/projects, provide ample scope for all students' interests and ability levels. Related "Family Activities" can be used to involve the students' parents/care givers.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

"Investigations in Measurement" is comprised of activities which explore estimation and measurement, and the selection of appropriate tools and units. Measurement provides the means to describe and analyse the everyday world in concrete terms, from grocery shopping through car assembly to building a space module. The activities involve making and testing hypotheses, and other forms of problem solving, as well as connecting strands of mathematics to each other and to other curriculum areas.

Acknowledgements

Contributing Teachers

Nancy Dykstra (Waterloo County Board of Education)
Jo-Anne Judge (Waterloo County Board of Education)
Kelly Lantink (Waterloo County Board of Education)
Ron Sauer (Waterloo County Board of Education - retired)
Barry Scully (York Region Board of Education/York University)
Jan Scully (York Region Board of Education/OISE/University of Toronto)

Authors/Co-editors

Bev Marshman (University of Waterloo)
Lorna Morrow (Mathematics Consultant)

We wish to acknowledge the support of the **Centre for Education in Mathematics and Computing**, and in particular the efforts of Ron Scoins, Gord Nichols, and Carolyn Jackson. A special thank you goes to Bonnie Findlay for prompt, accurate type-setting and creative diagrams.

Table of Contents

Overview

Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and measurement specifically. Some of these beliefs are described below.

Measurement provides the means for describing and quantifying our everyday lives. As such, it is rich with opportunities for exploring both mathematical concepts and their applications in the ‘real’ world.

Dynamic interaction between students and their environment is essential to developing skill with the processes of measurement and deriving the mathematical possibilities. Concrete, hands-on activities involving estimating, then measuring, in both non-standard and standard units, promote awareness of what units and tools are appropriate for a specific task, why standard units are necessary, and that all measurements are approximations with varying degrees of human and instrument error. Activities with ordinary objects/attributes (e.g. eraser, desk, blackboard, height, heartbeat, gasoline consumption, shadows, packaging, etc.) will provide limitless opportunities for problem-solving, involving concepts such as length, area, volume, time rates, ratio and proportion, similarity and congruence, as well as connecting mathematics directly to the physical world.

Justifying their own reasoning and discovering the patterns and logical connections which lead to mathematical formulas, or to a deeper understanding of their everyday world, increases the students’ ability to reason analytically and to express their thoughts clearly and concisely.

Essential Content

In the activities herein, students will explore the process of measurement and its implications in a variety of contexts, both inside and outside the classroom, with the goal of developing a solid foundation for using instruments and formulas with skill and precision, and analysing the meaning of their measurements. In addition, there are Extensions in Mathematics, Cross-Curricular Activities and Family Activities. These may be used prior to or during the activity as well as following the activity. They are intended to suggest topics for extending the activity, assisting integration with other subjects, and involving the family in the learning process.

During this unit, the student will:
- estimate and measure length, area, mass, capacity, temperature, time;
- relate personal referents (e.g. hand span) to standard units;
- explore methods for measuring bodily statistics such as heart rate;
- explore the capacity of a cubic metre for waste paper;
- calibrate a 1 litre measure and use it to measure container volumes;
- estimate weekly water use and gasoline consumption;
- use mathematical language to express their results;
- work with others to achieve success.

CURRICULUM EXPECTATIONS

"Curriculum Expectations" are based on current Ontario curricula.

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Measuring Me**	• estimating and measuring length, area, mass, capacity, temperature, and time • developing personal referents (such as 'hand span') and relating them to standard units for length and area	• solve problems related to ... perimeter and area ... • develop methods of using grid paper to ... measure the area of ... irregular two-dimensional shapes • identify relationship between the movement of objects and speed
Activity 2 **Inside Me**	• estimating and measuring bodily statistics such as heart rate and lung capacities in a variety of ways	• read an analog clock to the nearest second ... • relate the volume of an irregular three-dimensional figure to its capacity through displacement of a fluid
Activity 3 **Around Me**	• becoming familiar with the cubic metre • estimating classroom paper wastage in cubic metres	• estimate, measure, and record the capacity of containers ... • solve problems related to the calculation of [volume and capacity] • measure containers by volume using standard units
Activity 4 **Me and My Family**	• making a calibrated 1 litre measure • measuring volumes of labelled containers and computing cost per unit • estimating and measuring home water use and gasoline consumption, weekly. • estimating time of travel (walking, biking, driving)	• estimate, measure and record the capacity of containers ... • estimate ... time intervals • identify relationships among measurement concepts [size vs cost]
Activity 5 **Estimating Everywhere**	• estimating and comparing lengths, volumes, weights and times using both non-standard and standard units	• explain the rules used in calculating the perimeter of rectangles • investigate measures of circumference using concrete materials • solve problems related to ... perimeter and area of regular and irregular shapes

Overview

Notes

Prerequisites

Students should be familiar with metric units and measurement terms (e.g., area, perimeter, volume, cm, m, L, kg) as well as the use of various measuring devices such as a thermometer, a clock, and a ruler. The optional "Scavenger Hunt" included in Activity 1 provides the opportunity to identify students who need some review in these basic notions before proceeding with the other Activities.

Logos

The following logos, which are located in the margins, identify segments related to, respectively:

Problem Solving Communication Assessment Use of Technology

Snippets

"Snippets" that appear as small notebook pages in the margins are bits of data somehow related to the measurement tasks the students are being given. Sometimes these snippets will include a problem posed for the students. For others, questions will no doubt come to the teacher's mind even as he/she is sharing the snippet with students. Students themselves may identify related questions that they would be interested in pursuing. It is hoped that students will find these bits of information interesting and will realize how frequently measurements are used in everyday life.

Rules of Thumb

R of T

"Rules of Thumb" are ways to help in estimating. For example, the rule-of-thumb "Two pages written by hand will give one page when typewritten" will give an author some idea of the length of his/her erudite article, so he/she knows when the article has reached a permissible length. Rules of Thumb (R of T) have been placed in margins (on file cards) along-side the Activity notes. They can be used as jumping off points for good problems, or just enjoyed for their (possible) values. A worthwhile activity is trying to decide whether each R of T is valid. All R of T's in this book are gleaned from "Rules of Thumb" and "Rules or Thumb -2" by Tom Parker. See "Other Resources" on page 47 for more detail.

Notes

Materials

ACTIVITY	MATERIALS
Activity 1 **Measuring Us**	• Copies of BLM 1 (optional) • Copies of BLMs 2 and 3 for each pair/group • Scavenger Hunt materials (optional): 30 cm ruler 1-metre tape; 2 copies of BLM 2; a 1-kg mass, two or three 1-litre containers; thermometers; clock/watch which can measure seconds
Activity 2 **Inside Us**	• Copies of BLM 4 for each pair/group • Paper fasteners, or wooden matches and thumbtacks • Paper towel cores and scissors • Balloons, string, bottles, pans, flexible straws (optional) • Copies of BLMs 5 and 6 (optional)
Activity 3 **Around Us**	• Old newspapers and masking tape • Copies of BLM 7 (optional) • A package of 500 sheets of paper (optional)
Activity 4 **Me and My Family**	• Copies of BLM 8 for each group • Measuring containers • Copies of BLM 11 (optional) • Empty bottles showing volume and price • Measurements of a bath tub • Data on gas usage in a home for one week • Copies of BLMs 9 and 10 (optional)
Activity 5 **Estimating Everywhere**	• Copies of BLM 12 for each student • String, roll of paper towels, large cereal box, small container of popcorn, a few nickels and dimes • Copies of BLM 13 (optional) • Clock/watch which shows seconds, calculator (optional)

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week or so, students in our classroom will be participating in a unit titled “Inside and Out”. The classroom activities will focus on estimating and measuring length, area, mass, capacity, temperature and time through a variety of problem-solving situations. The emphasis will be on developing familiarity with basic units, and an understanding of when estimates are acceptable.

You can assist your child in understanding the relevant concepts and acquiring good measurement skills by working together to perform simple tasks (e.g., cooking from a recipe, sewing a tablecloth, building a chest, ...), helping to explore everyday ways measurement is used.

Various family activities have been planned for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with measurement in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher’s Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Notes

Activity 1: Measuring Us

Focus of Activity:
- Review of basic metric units
- Establishing personal referents

What to Assess:
- Familiarity with basic units (e.g., cm, m, kg, L)
- Accuracy in measuring
- Reasonableness of the choice of referents

Preparation:
- Make copies of BLM 1 for each pair/group (optional).
- Make copies of BLMs 2 and 3 for each pair/group.
- Provide materials as indicated under ‘Introduction’ below for Scavenger Hunt (optional).

Activity:

NOTE: You will need a collection of materials for Activities 2 and 4.

Activity 2 needs cardboard cores from rolls of paper towels.

Activity 4 needs
- *clear glass or plastic bottles, to make measuring containers*
- *a selection of empty bottles showing price and quantity of contents in millilitres for BLM 8,*
- *size of bathtub, information about home water usage and prices (BLM 9, 2a and 3a)*
- *gasoline usage (BLM 10)*

Students should start collecting these materials and information immediately. For students who may not have access to such data (e.g., the family travels by public transit, or the washing machines are communal), either have students share with each other, or provide them with data from your own water and gasoline usage.

Note that BLMs 9 and 10 provide an Extension and a Family Activity and are therefore optional.

Comments in italics are explanatory, and need not be conveyed to students.

INTRODUCTION:

In order to determine how familiar students are with measuring techniques and metric units, you may wish to have students go on a “Metric Scavenger Hunt” (BLM 1). Students should work in pairs or small groups and should be provided with the following:
- either a 30-cm ruler or a 1-m tape [for items 1, 2, 3, 7, 9, 14, 15, 18]
- a sheet of cm^2 paper (BLM 2) [for items 13,17]
- access to a 1-kg mass [for items 4, 5]
- access to two or three different 1L containers [for items 6, 8]
- access to thermometers [for items 10, 11]
- access to a clock/watch that measures time in seconds [for items 12, 16]

Note that items 14 and 15 on BLM 1 ask students to determine an average of a set of measurements. You may wish to review/teach this procedure before assigning the “Hunt”.

Notes

Activity 1: Measuring Us

Explain to the students that they are to identify things in the classroom that meet each criterion given on BLM 1. If you lack some materials (e.g., thermometers), have students omit the relevant items. Students should also realize that they need not find the items in the order they are given. For example, if all the litre containers are in use, other groups can skip items 6 and 8 and come back to them when they gain access to a litre container. The area items (13 and 17) can be found by tracing the item and counting squares on cm^2 paper (BLM 2), rather than using a formula. This reinforces the meaning of “area”, rather than its computation.

Communication

Explore with students possible meanings of “best work” (as indicated on BLM 1: “The group that does the best work is the Metric Winner”). Ideas that may come up are:

- ‘best’ means the most variety
- ‘best’ means the most accurate
- ‘best’ means the most examples identified in the time given
- ‘best’ means having good reasons for the choices made
- ‘best’ means giving good explanations (i.e., clear and reasonable)
- ‘best’ means the best we can do (in which case everyone should be a winner)

As in the last example, students may decide that there can be more than one winner. Have students decide on the criteria to be used in judging the results of the Scavenger Hunt. Suggest that students may want to compare their responses with these criteria while pursuing the hunt.

Students may request clarification of some terms. For example,
- is a “footprint” (#13) made with shoes on or off?
- must the alphabet (#16) be the English alphabet or are other alphabets (e.g. Greek, Russian, Hebrew) allowed? How neatly must they be written? (At a later time, you may wish to explore with students which languages do not have alphabets (e.g., Mandarin, Cantonese, Korean, or older languages such as Aztec or ancient Egyptian.)

If such questions arise, discuss them briefly with the class and use the majority opinion as the “correct” interpretation.

Assessment

Some of the criteria given allow for a wide range of answers. For example, almost anything in the room will be warmer than 2°C (#10). Students should realize that they do not need to measure the temperatures of objects, but simply feel them. If students have difficulty with this item, they obviously need some experience with measuring temperatures.

Item 5 also allows for a wide range of answers. Students should be able to find several suitable items even without the availability of a 1-kg mass. Erasers, pencils, pens, mittens, and chalk are all reasonable answers.

Similarly, if students are at all familiar with the litre, they should be able to recognize the class waste basket as a good answer for #6.

When student have completed the hunt, compare answers from the whole class. Ask students which items (if any) they estimated and which they measured more or less accurately. Students should realize that measurements in “real life” are often estimates.

Activity 1: Measuring Us

Decide which group(s) is/are Metric Winners. Refer back to the criteria decided on for "best work". Discuss with students whether or not they want to alter any of the criteria based on what they have discovered about the measurements they made or estimated. Have them assess their own results in light of these criteria, giving their reasons for their rating. You may wish to have students write their reasons, for inclusion in a journal or portfolio.

Personal Measurements:

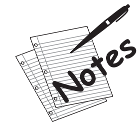
If students have completed the Scavenger Hunt they will have already made some personal measurements. The following activity is intended to help students develop personal referents for various measurements. A referent is a symbol or item representing something else. For example, if a fingernail is found to be 1 cm wide, then the fingernail becomes a referent for 1 cm. If students can identify personal measurements as referents for, say, metres and centimetres, they can then use these referents to estimate lengths when they do not have access to a ruler.

Distribute BLM 3. Explain to students that "hand span" means the width from tip of little finger to tip of thumb when fingers are spread as far apart as possible. Hand width is the width of the hand at its widest point when fingers are together. (See illustration on BLM 3.) Have students measure their handspans and their hand widths to the nearest centimetre and then use either of these measures to determine the width of various items such as a desk, a text book, the width of a door, and so on. Discuss with students which is the more convenient measure. For example, if hand- width is close to 10 cm while hand span is 14 cm, calculations would be easier if the student used hand width.

For problem 4, students should have access to cm^2 paper (BLM 2). They can then trace around their hands on the paper and count the squares covered. It will be necessary for students to decide how they are going to deal with partially covered squares. One way is to count any square that is more than half covered as one square and ignore any square that is less than half covered. Some students may choose to estimate the area of a hand by tracing around the hand with the fingers together while others do this with the fingers spread apart. Although the first is probably easier, students should realize that either one is acceptable.

A discussion of #5 could be extended if students check other family members to find their personal referents.

Discuss with students the various personal referents they have found. Ask why having such referents is useful. Have them identify situations in which estimates of linear measurements or areas are appropriate, and which personal referents will be most useful in those situations. (See Family Activities 1.)

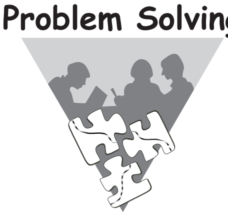


Assessment

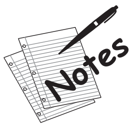
See page 3 for an explanation for "Snippets", an example of which is given here.

The height of a horse is measured in "hands". A "hand" is taken to be 4 inches (10 cm). The horse's height is the distance from the bottom of the front feet to the top of the shoulder bone at the base of the neck.

Problem Solving



Communication



Problem Solving

Activity 1: Measuring Us

Extensions in Mathematics:

1. If you were going to wrap yourself in a box as a present, what size box (either a rectangular prism or a cylinder) would you need? You will need to consider not only your size, but whether or not the box will be comfortable, and which ways your joints bend and ways they do not bend. What other things might you need to consider? Write about how you would determine the best size of box for you. Estimate its size and tell how you made the estimate.

Cross-curricular Activities:

1. (a) A car driving in a school zone should not be going faster than 40 km/h. This is approximately 11 m/s, or we could say the car travels about 100 m in 9 s. Can you run this fast for 100 m? Have someone time you and compare your speed to the car.

   (b) Are there any animals that can run as fast as the car? Are there any that can swim or fly as fast as the car? Where could you find this information? Make a list of animals and the speeds they can run/fly/swim.

   See "Solutions and Notes" for further data.

   Are there any people who can run or cycle or skate as fast as the car passing a school? (Hint: Look for Olympic Records.)

2. Continue a discussion of alphabets as suggested above for Scavenger Hunt item 18. What languages other than English use the "English" alphabet?

Family Activities:

1. Students can discuss with their families when measurements must be accurate and when estimates will do. For example, if a room is to be painted, does the exact area have to be calculated? Why or why not? Students should make a list of examples to share with classmates. This can be a long-term assignment. As students come up with ideas, these can be posted on a bulletin board.

2. If each family member bites into a piece of bread/potato/apple the width of everyone's smile can be easily determined, and the person with the widest smile can be identified. (See Scavenger Hunt item 18). Discuss with students possible differences between "bite" and "smile". For example, a bite into a piece of bread will show imprints of more teeth than are usually shown in a smile. Students may wish to identify family members with the widest bite rather than, or in addition to, the widest smile.

For a short time: A chicken can travel 4 km/h; a pig 18 km/h; a human 42 km/h; a cat 48 km/h. Are you faster than any of these?

A fruit fly flies at a rate of 57 body-lengths per second. How fast can you run in body-lengths per second?

"*See page 3 for a description of Rules of Thumb*."

R of T

When you walk, your leg length is equal to your stride.
Find a Rule of Thumb for running.

Activity 1: Measuring Us

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

3. By the Unit or Square Unit? by B.B. Ferrer.
8. Using a Lifeline to give Rational Numbers a Personal touch, by W. Weidemann, A. Mikovch, and J.B. Hunt.
10. How Big Is Your Foot? by Suzanne Levin Weinberg.
14. Honoring Traditions: Making connections with Mathematics Through Culture, by D. McLean.

Notes

A stride is the distance between successive positions of the same foot when walking (or 'striding').

Activity 2: Inside Us

Focus of Activity:
- Novel ways to measure pulse, heart rate, and lung capacity

What to Assess:
- Reasonableness of measurements

Preparation:
- Make copies of BLM 4 for each pair/group.
- Provide paper fasteners or wooden matches and thumbtacks.
- Provide paper towel cores and scissors.
- Provide balloons, string, bottles, pans, flexible straws (optional).
- Make copies of BLMs 5 and 6 (optional).

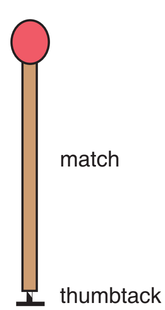
Activity:

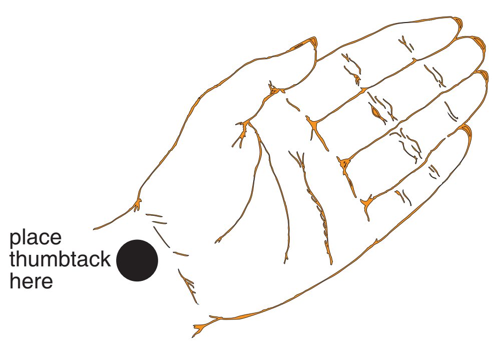
In this activity, students will be measuring things about themselves that they have probably never measured before such as heart rate, pulse, and lung capacity.

Ask students if they have ever had a doctor or nurse take their pulse by feeling their wrists. Some students may even be able to show approximately where on the wrist the pulse can be felt. Tell them that you are going to let them see their pulse.

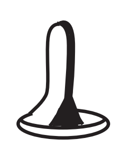
Stick a wooden matchstick onto a thumbtack as shown on left below. (You may wish to used burnt matches for safety reasons.) The matchstick can be fastened to the tack with a tiny bit of plasticene or modelling clay, rather than trying to push the tack into the matchstick. However, too much modelling clay may affect the movement of the matchstick or make the movement difficult to see. *To save time you may wish to prepare one match per group beforehand.*

Hold your hand so the spot shown below on the right is horizontal. This is easier if you lay the back of your hand and forearm on your desk. Locate a pulse point, and place the thumbtack on it so that the matchstick is vertical. If you have the thumbtack in the correct position, you will see the matchstick vibrate/twitch slightly in rhythm with your pulse.





Alternatively, use paper fasteners. Place one upside down on the pulse point, and count the vibrations of the prongs. Because the prongs are shorter than the matchstick, it may not be as easy to see (and count) the vibrations.



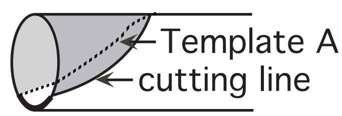
Activity 2: Inside Us

Have students practice finding a pulse point. Give a match and thumbtack to each pair/group. Have students take turns measuring each others' pulse rate by counting the number of vibrations of the match head per minute, or count the number of vibrations in 10 seconds and multiply this by 6.

Check that all pairs/groups are successful in this. Distribute copies of BLM 4 and instruct students to record their pulse rates under "Normal pulse rate."

You may wish to discuss the type of exercise that would be suitable for problem #2. If this can be done in the gym or outside then running or jumping or even turning somersaults might be appropriate exercises. You may wish to extend this idea by asking students to predict an exercise that they think will have only a small effect on pulse rate and an exercise that will have a much greater effect, then test their predictions.

When constructing the stethoscope, students should first cut out the two templates at the bottom of BLM 4. They should be wrapped around the ends of the tube as shown. They will not go all the way around the tube but will leave part of the original end of the tube as shown by the heavy line.



Students should know that the heart is located in the centre of the chest, but the beat is stronger in the left ventricle because it is from that chamber that blood is pumped out to the body. The right ventricle is responsible for pumping blood only to the lungs, a much easier task.

Extensions in Mathematics:

1. Distribute BLM 5, Huffing and Puffing. You may wish to use only one of the two techniques given or to split the class and have some use one technique and some use the other. Discuss with students why 'good' lung capacity is important. Have students suggest people who would have a high lung capacity and why (e.g., swimmers, musicians playing wind instruments). Enrico Caruso, the famous tenor, was able to expand his chest measurement by 9 inches (approx. 23 cm).

Note that the "lung capacity" is measured in centrimetres of air in the bottle. If you wish to have students measure lung capacity in millilitres, postpone this activity until after students have calibrated bottles in Activity 4.

Cross-curricular Activities:

1. Research such questions as:
   How many litres of blood are in your body?
   How often does blood circulate?
   How many times does your heart beat in 24 hours?

   See "Solutions and Notes" for some data and sources of information for these and similar questions.

Notes

During hibernation, the Japanese dormouse lowers its heart rate from 500/min to 50/min.

R of T

A healthy adult should be able to exhale a deep breath completely in three seconds.

For hygienic reasons, each student should have his/her own straw.

Communication

Notes

Use of Technology

R of T

When you're relaxed, you take approximately 12 breaths per minute.

Activity 2: Inside Us

2. Students could attempt to answer some of the following questions, giving reasons for their suggestions. Then they can try to find the answers in the school or local library, or on the Web. What animals have more blood than humans? What animals have less? What animals have a higher pulse rate than people? What animals have a higher lung capacity?

Family Activities:

1. If any of the students have doctors or nurses in the family, those students could collect questions from the class on heart rate and/or lung capacity and bring answers (or a guest speaker) to class.
2. Send home copies of BLM 6, and have students bring results to class and compare with the bottle-and-straw method of BLM 5. For classroom use, note that, for hygienic reasons, each student should have his/her own balloon.
   You may wish to review how to determine averages, if not done earlier for the Scavenger Hunt, BLM 1.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

2. Addenda Series, Grades 5-8: Measurement, 'Heartbeats', page 76.

Activity 3: Around Us

Focus of Activity:

- Measuring paper usage in the classroom
- Becoming familiar with the cubic metre

What to Assess:

- Estimation skills; reasonableness of estimates
- Understanding of the size of one cubic metre

Preparation:

- Provide old newspapers and masking tape.
- Make copies of BLM 7 (optional).
- Provide a package of 500 sheets of paper (optional).

Activity:

Building a cubic metre: A stable cubic metre skeleton can be built using rolled newspaper. Open a page and lay it on the floor. Open another page and overlap the first one as shown in Figure 3.1. Tape the pages to each other. Place a ruler across one corner and begin rolling the newspaper in the direction of the arrow, removing the ruler after you get started. Roll as tightly as possible, taping the last corner down when the roll is complete. (Figure 3.2)

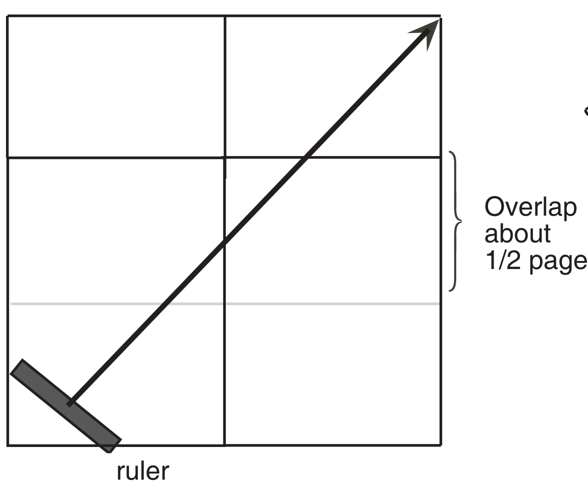


Figure 3.1

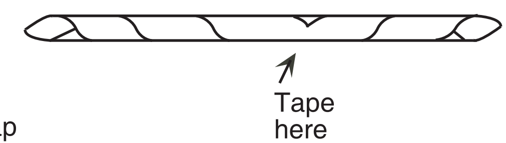


Figure 3.2

Cut a bit off each end of the roll so that the cylinder is slightly longer than 1 metre. This will allow a little extra for taping together the corners of the cube.

You will need 12 of these one-metre rolls as well as some shorter ones to build a cubic metre skeleton. (See Figure 3.3). Tape the corners together. You will probably need to brace some corners (See Figure 3.4) to keep the structure solid. Use the shorter rolls for this.

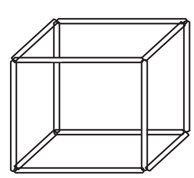


Figure 3.3

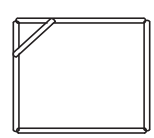
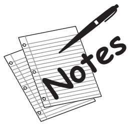


Figure 3.4

If you have room in the classroom for more than one of these, each group of students could construct one.



Activity 3: Around Us

To help students internalize the cubic metre ask such questions as:

How many students do you think would fit inside a cubic metre?
How many teachers do you think would fit inside one?
How many wastebaskets could be emptied into a bin the size of a cubic metre?
How many cubic metres do you think would fit into our classroom?

Have some students get inside the cubic metre. Try to fit as many in as possible. Those outside can help with suggestions (e.g., “There’s room in this corner” or “Mattie’s leg is sticking out”).

Problem Solving

To make a cubic metre waste bin, cover the sides and bottom of one of the skeletons with sheets of newspaper taped together. Have students work together in groups to estimate how many times they could fill the class wastebasket and dump it into this cubic metre bin. Have them write how they arrived at their estimate. Compare ideas from different groups. If any group decides to change their estimate because of this sharing, they should be allowed to do so. No estimates should be called wrong.

> Sheets of paper the size of this page are $8\frac{1}{2}$" × 11" This is approximately 21.5 cm by 28 cm.

Keep this cubic metre in a corner of the classroom and every time the wastebasket is full, a student should empty it into the bin. (Use paper waste only — no food). *If all (or most) paper is put into a recycle bin, read “recycle bin” for “waste basket” for this activity.* You may want to involve the caretakers in this project, too. Have students keep track (in their notebooks or on chart paper or the bulletin board) of the number of times the wastebasket is dumped into the cubic metre. As the week progresses ask students if any of them wish to change their estimates, why, and in what direction (up or down).

You might wish to have students estimate the number of pieces of writing/copy paper (8“ x 11”) that are thrown out each day. This can later be used to calculate the cost of paper used by the class.

When the bin is full, compare the actual number of wastebasketfuls with the estimates. Ask students how long it took to fill the bin, and have them calculate how many cubic metres of waste paper the class would throw out in a year. Have students then bag or box the paper for discarding or recycling.

> A ream of paper contains 500 sheets or 20 quires of paper.

Discuss ways the class could cut down on the waste paper. Have students consider this in groups and then report to the class.

Determine the cost of one ream (500 sheets) of copy paper. If students estimated the number of pages discarded each day, they could use this information to calculate the cost of the paper that the class discards each week or during the whole school year.

Students could also determine how many wastebaskets there are in the school. If every one of them was responsible for the same amount of discarded paper, what would the cost be to the school for the whole year?

Communication

Pose the question: Since some papers can be recycled, would it be a good idea to have two wastebaskets in every class — one for recyclable papers and one for the others? Why? Do you think students would use the correct basket? Why or why not?

Note: What can be recycled will vary from municipality to municipality.

Activity 3: Around Us

Extensions in Mathematics:

1. BLM 7: Pieces of Paper presents a different problem dealing with paper. Students need only a piece of scrap paper and a 30-cm ruler.

   Before any measuring is done, have students read #1 and 2 and suggest ways of determing answers. They should realize that, for #1, they don't need to know the thickness of one piece of paper, but can make a pile of paper 1 cm tall and then count the sheets.

   After this discussion, show students a package of 500 sheets of paper. Ask them how they can use this stack of paper to determine answers to #1 and 2.

   Have a student measure the height of the unwrapped pile of 500 sheets. The height will be ***very*** close to 5 cm. Students should see that this gives the answer to #2 (i.e., 500 sheets) and that the answer to #1 is 100 sheets.

   Students may suggest that papers may by thinner (e.g., newsprint, onion skin tracing paper) or thicker (e.g, construction paper) than the copy paper. Ask how this would affect their answers to #1 and 2. You may wish to provide samples of different papers and have different groups determine answers to the problems on BLM 7 using these different papers. For example, 250 sheets of cover stock give a pile more than 5 cm tall.

   Students could do further research on different kinds and weights of paper by visiting office supply stores in person or via the web.

   Have students work on the other problems on BLM 7, using calculators to assist with the computation.

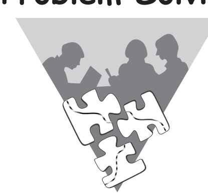
   Note that 5(a) may be interpreted by students in two ways:
   (i) "one tear at a time" can mean to tear only one thickness of paper at one time, or
   (ii) "one tear at a time" can be through several thicknesses if all are torn at the same time.

   Obviously, if students use the second interpretation, they will calculate a much higher stack of paper in (c) and (d) than if they use the first interpretation. You may wish to allow each group to choose which interpretation is preferred (and then compare results) or you may have the class decide as a whole which interpretation will be used.

2. Have students collect litter (wearing plastic gloves) in and around the school yard, and sort it by size of items, weight of items, type of items (e.g., paper, plastic, metal, glass), or some other property. If the technology is available, students can create graphs (bar or circle) and write reports electronically.

Notes

Problem Solving



Use of Technology

For more on graphs see "Investigations in Data Management", Gr. 5. Ordering information can be found on the last page of this booklet

Notes

It takes about 17 trees to make a tonne of newsprint.

Activity 3: Around Us

Cross-curricular Activities:

1. (a) Try to find how much paper is produced from a medium-sized tree that is cut down for paper. This information could be obtained from a company which manufactures paper either by direct contact (telephone or letter) or through their website.
   (b) Determine how many trees the classroom (or the school) uses in a year.

Family Activities:

1. Students could check wastebaskets at home to determine the amount of waste paper generated in a week. They can also determine which papers can be recycled and which cannot.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

11. Power Over Trash, by R. Ronau and K. Karp.
12. The Environmental Math Workbook, by Evannah Sakamoto.
16. Weather Watch, by Valerie Wyatt.

Activity 4: Me and My Family

Focus of Activity:

- Measuring capacity of bottles
- Determining cost per unit

What to Assess:

- Construction of measuring bottle
- Accuracy of measurements of capacity
- Reasonableness of estimates

Preparation:

- Make copies of BLM 8 for each group.
- Make copies of BLM 11 for each group (optional).
- Provide measuring containers for each group, or construct them as described below.
- Display the empty bottles with labels that students have brought to school, showing the volume of contents and the price of the bottle.
- Measure a bathtub, and provide these measurements for students who have not done this (optional).
- Provide data on your gas usage for a week (See BLM 9, 1(a)) for students who do not have such data (optional).
- Make copies of BLMs 9 and 10 (optional).

Activity:

The problems on BLMs 9 and 10 ask students to collect data from home about water usage and prices, about gasoline usage, etc. Students will need to have these data before tackling the problems, either from their own families or from their classmates or from a staff member.

For BLM 8, students need a variety of bottles with labels (in millilitres) and prices. Students will need "measuring containers" for this activity as well. If you do not have enough measuring cups or graduated cylinders for each group, you may wish to have each group construct one using the technique described on BLM 11. Alternatively, you may wish to construct enough yourself so that each group can have some kind of measuring cup or bottle.

If students use water to mark the levels on their measuring bottles, they need to wait till the water has settled before marking the tape. If they are using solid matter, such as rice/sand/aquarium gravel, they should shake the bottle slightly to make the material as level as possible before marking the tape. They may find it easier to pour the material into the bottle and to level the material if they cut the top off the bottle as described on #7 on BLM 11.

If some of the bottles narrow considerably towards the base, have students compare the vertical distance from the bottom of the bottle to the 250 ml level with the vertical distance from the 250 ml level to the 500 ml level. Ask students why the measurements are different.

Notes

The human body is 70% water; milk is 90% water; hot dogs are 56% water; watermelon is 92% water.

Activity 4: Me and My Family

Once the measuring bottles are made, distribute copies of BLM 8 and several bottles to each group. Students can immediately list the containers and the "Volume indicated" on the label. For "Capacity of bottle" students should use their measuring bottles to determine how much there would be in the bottle if the bottle were full. If they are using measuring bottles constructed using instructions on BLM 11, then students may need to estimate capacities of some of the bottles they are testing, since the measuring bottles are marked at every 125 ml only. If you have measuring materials available that are more accurate, students may not need to estimate.

If there are several different examples of one type (e.g., shampoo), compare the unit prices and discuss with students reasons why the prices differ, and why some people prefer to buy the more expensive brands. Item 5 on BLM 8 suggests students visit local supermarkets to compare unit prices. This could become a worthwhile Family Activity or a class project.

Extensions in Mathematics:

1. For BLM 9, students need an estimate of the capacity of a bathtub, the volume of water used for a bath or shower, and the volume of water needed by a washing machine for washing and for rinsing. Students can share each other's data if necessary as noted above.

   Note that #1 on BLM 9 can be done before the data is collected because it asks students how they would solve the problems given in 1(a), (b), (c), and (d), but does not ask them to solve the problems. # 2 and 3 pose problems to be solved. #4 asks for an opinion based on what students have learned about water usage by completing #1, 2, and 3.

Cross-curricular Activities:

1. After discussing #4 on BLM 9, have students try to find out more about water usage (and availability of drinkable water) in other countries. Investigate why other countries are trying to buy water from Canada.

Family Activities:

1. For BLM 10, Students need to collect data for a week (or longer if you wish) before attempting problems 1 and 2. For problem 4(b) they will need to collect data on their own trips back and forth from home to school.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

2. Addenda Series, Grades 5-8: Measurement, 'The Weight of Wetness', page 77.
13. Food for Thought, by G. Cloke, N. Ewing, and D. Stevens

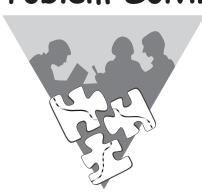
Notes

A dripping tap can waste 9000 L of water a year

Communication

Taking a shower uses 20 L/min; flushing a toilet uses 16 L; running the tap uses 4.5 L/min.

Problem Solving



Cows drink about 160 L of water a day; humans drink about 2.2 L.

For information on suitable types of graphs, see "Investigations in Data Management", Grade 5.

R of T

Soaking in a hot-tub adds about 1 L of perspiration per hour per person to the water.
If 6 people are in a hot-tub for half an hour ...?

Activity 5: Estimating Everywhere

Focus of Activity:
- Estimating lengths, mass, time
- Identifying situations where estimates are acceptable

What to Assess:
- Understanding of the nature of estimates
- Reasonableness of estimates
- Explanations as to whether or not estimates are acceptable in given situations

Preparation:
- Make copies of BLM 12 for each student.
- Make copies of BLM 13 (optional).
- Provide materials for the problems on BLM 12 which include string, a roll of paper towels, a large cereal box, a small container of popcorn, a few nickels and dimes.

Activity:

Distribute copies of BLM 12. Read the instructions with the class. Be sure students know that they do not need to determine answers at this stage. Select 3 or 4 of the problems and ask students whether or not the problem can be answered accurately.

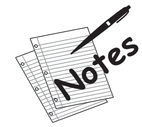
For example, problems 1 and 2 can easily be checked with string and a ruler. Problem 3 might be either an estimate or accurate. If the wrapper is still on the paper towel packet, the length of each towel is given but students will need to estimate or measure the distance around the classroom.

Problem 5 could be answered either way as well. You may wish to discuss with students whether or not an accurate answer is worth determining for this problem. Ask if there are other problems for which an accurate answer is possible but may not be needed. In other words, when is an estimate acceptable?

Allow students time to decide on estimation or accuracy for each problem, then discuss their opinions. (At this time, ignore the instructions in the box at the bottom of the page.) Students should be encouraged to accept differing opinions and to justify their own. They should realize that the context in which a question is asked often determines whether an accurate or estimated answer is satisfactory. Ask students if an estimated answer would be acceptable or not for each problem.

Have students read the instructions at the bottom of the page. You may wish to assign particular problems to different groups of students or to leave the choices up to them. If time permits each group could do more than one problem

When discussing their solutions with students, bring out the idea that rounding measurements is a type of estimating. That is, for problem 8, students might determine the height of 6 nickels to be almost 1 cm, and use that value to answer the problem. Thus, if a student is 120 cm tall, the stack of nickels will have 120 x 6 or 720 nickels in it. This is accurate enough for the purpose of answering the problem. Using the value of 1.01 cm rather than 1 cm will not change the final value substantially, and will make the computation unnecessarily clumsy. Students may be able to suggest reasons why ‘rounding’ to 1 cm is valid. For example, not all nickels will be exactly the same size (due to wear and tear); some nickels might be U.S. currency and some Canadian.



R of T

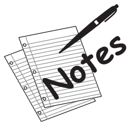
Twice around the thumb = once around the wrist.
Twice around the wrist = once around the neck.
Twice around the neck = once around the waist.
Are these Rules of Thumb true for you?

Problem Solving

Communication

A medium-sized cumulus cloud can weigh 2 300 000 kg to 5 700 000 kg. Is this an estimate or a measurement? Explain.

Activity 5: Estimating Everywhere



The country of Monaco has an area of about 1.3 square kilometres. What is the approximate area of your county?

Students should also realize that all measurements are really estimates. For example, even when using a ruler to measure the width of a rectangle, our measurement will be only as accurate as the ruler is, and will also depend on the thickness of the lines marking the boundary of the rectangle.

Optional

Two more estimation problems are provided on BLM 13. The only equipment needed will be a clock/watch that shows time in seconds, and a calculator for the computation.

Communication

Since each problem will take some time to complete, you may wish to use them on different days. Alternatively, have half the class work on “Racing Hands” and the other half on “Slow Racing”, on the understanding that students will be expected to report to the whole class by outlining the problem, and the method used to solve it. Students who did not work on that particular problem should be encouraged to ask such questions as, “Why did you...?” or “What if you had...?”

If students recorded their arm spans as part of #2 on BLM 3, they can use these measurements to estimate the length of a three-person chain for “Racing Hands”. Others, who are uncomfortable with estimating, may choose to measure the chain. In either case encourage students to round off the total length to the nearest decimetre or the nearest half-metre. This rounding can be seen as another form of estimating.

The three-toed sloth travels through the trees at a speed of 1.8 m/min. Would a sloth do well in Slow Racing?

“Slow Racing” involves estimating short time lengths. Some practice is suggested on the BLM before the students attempt the actual “race”, since time estimating is particularly difficult. We all know that a half-hour in a doctor’s waiting room seems a good deal longer than a half-hour spent doing something enjoyable; ask students if they are familiar with this phenomenon. They will probably be able to provide other situtaions (e.g., 1 h of TV seems faster than 1 h of homework).

Problem Solving

Extensions in Mathematics:

1. Extend #4 on BLM 12 by making a class collection of boxes and having students measure the height and distance around (which can be called either perimeter or circumference) each box. Is there any pattern in the numbers? Is there any relationship between height and circumference?

Family Activities:

1. Have students test family members on estimation of size by giving this problem to as many family members as possible.

    From memory, draw:

    a rectangle the size of a $5 bill;
    a circle the size of a $1 coin;
    a circle the size of a nickel.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

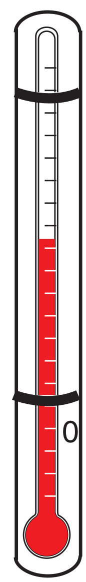
6. What Is A Good Guess?, by F. Kuwahara Lang.
9. How Big Was the Cat?, by C.E. Sakshaug and K.A. Wohlhuter.

BLM 1: Metric Scavenger Hunt

Work with a partner or in small groups.
Find the following items in your classroom. Write the names of the items or people in the blanks.
The group that does the best work is the Metric Winner.

Find:
1. the tallest person in your group ______________________
2. the person whose wrist measurement is closest to 12 cm _____________
3. something that is about 1 m long _____________________

4. something that weighs about 1 kg ___________________
5. something that weighs less than 1 kg ____________________
6. something that holds more than one litre ________________
7. something that is more than 4 m long __________________
8. something that holds between 2 L and 5 L __________________
9. something about 10 cm long _________________________

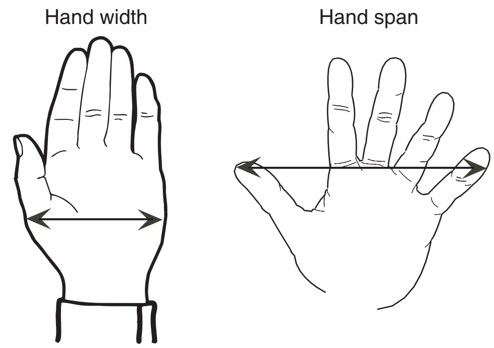


10. someplace in the room warmer than 2° C.__________________
11. the person in your group with the warmest hand _______________
12. the longest anyone in your group can stand on one foot with his/her eyes closed ___________________ seconds
13. the smallest footprint in your group _________________ cm^2
14. the average height of your group _________________ cm
15. the average arm length of your group ________________ cm
16. the fastest someone in your group can write the alphabet _______ seconds
17. something in the classroom that has an area of about 600 cm^2 _________
18. the person in your group with the widest smile __________________

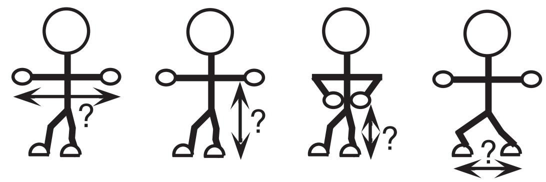
BLM 2: Centimetre Grid

BLM 3: Measuring Me

1. (a) With your partner/group measure each person's hand span and hand width.
   (b) Which of these measurements would be easier to use to estimate:
      (i) the width of your desk? Why?
      (ii) the height of your math text? Why?
      (iii) the width of the classroom door? Why?



2. Find a measurement on you that can be used to estimate 1 m. It might be the distance from hand to hand when your arms are stretched out sideways. It might be the distance from the floor to your waist or your shoulder. It might be the length of two of your usual steps. Describe your own "personal metre".



3. Find some part of your hand that could be used to estimate one centimetre. Describe your "personal centimetre".

4. What is the area of your hand print in square centimetres? How can you use this to estimate the area of your desk top?

5. Which of these personal measures do you think will change over the next 5 years? Why?

BLM 4: My Heart and Lungs

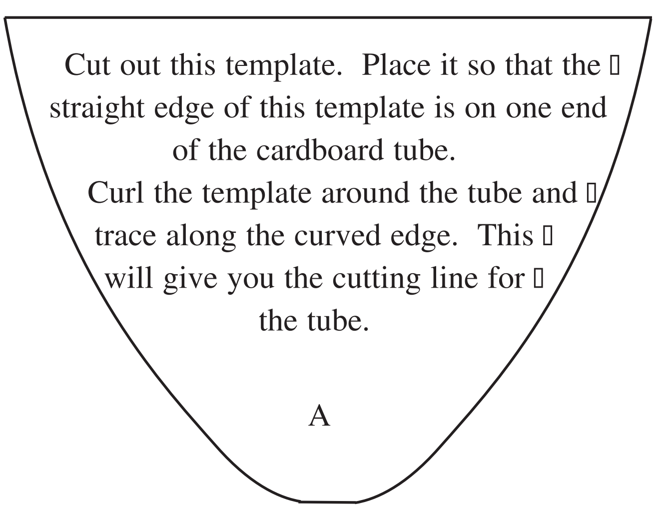
1. Record the pulse rate for each member of the group in the chart below under “Normal pulse rate”.

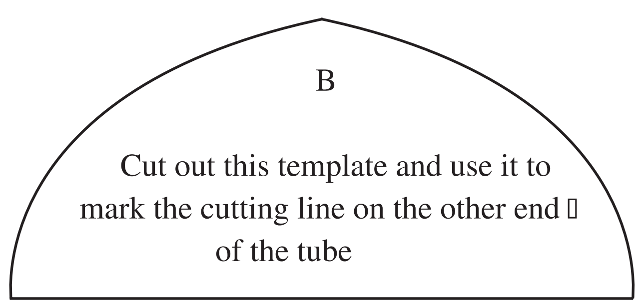
Name	Normal pulse rate	Pulse rate after exercise

2. What do you think will happen to your pulse rate after exercise? Why? Each of you in turn should do one minute of exercise such as bending to touch your toes or running on the spot. Take your pulse again immediately after you stop exercises. Enter these pulse rates in the chart above under “Pulse rate after exercise”. Was your prediction correct?

3. A Tubular Stethoscope: You will need a cardboard roll from paper towels or wax paper or aluminum foil. The roll should be about 27 cm long. You will need to cut a piece out of each end as shown in the diagram below. Use templates “A” and “B” below to mark the cutting lines.

4. Hold the “stethoscope” with end “A” on your chest over your heart and end “B” over your ear. (Don’t try to hear your heart through more than 2 layers of clothing). Move end “A” until you can hear your heart beat. Count the number of heart beats per minute. Compare this with your normal pulse rate in the chart above. Why should these two be the same or nearly the same?





BLM 5: Huffing and Puffing

Here are two ways to get an idea of your lung capacity.

Method 1: Water and Straw

1. You will need a bottle, a bowl, a flexible straw, and water. See Figure 1
   Put some water in the bowl. Do not fill it. Leave it less than half full. Fill the bottle with water. Put a piece of paper or cardboard over the top and quickly turn the bottle upside down with the mouth of the bottle under water in the bowl. Tilt the bottle just a little, allowing in as little air as possible. Slide the short end of a flexible straw up into the bottle.

2. Mark the level of water in the bottle. Blow into the straw. Be careful not to tip the bottle. You should see bubbles in the bottle and the water level in the bottle should drop. See Figure 2. How far can you make it drop with one blow? Measure to the nearest centimetre. Try three times, and record the three measures.

Figure 1

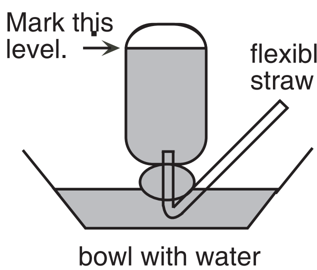
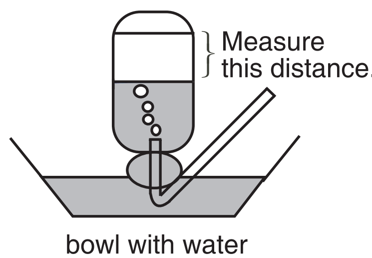


Figure 2



3. Did your 'lung capacity' change during the three trials? If so, how did it change? Why do you think this happened?

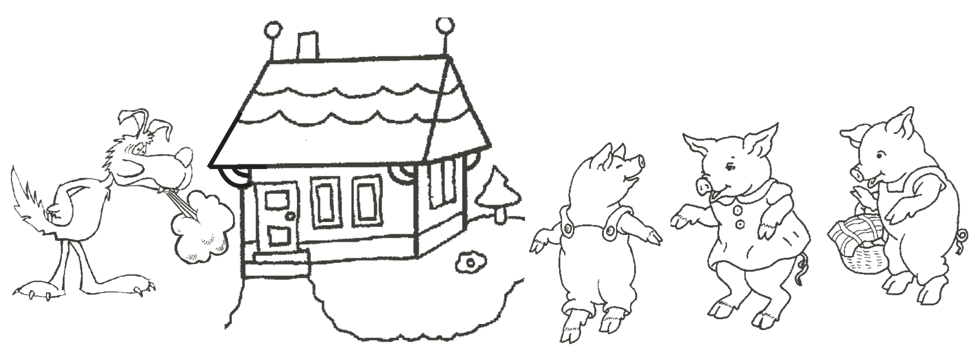
Method 2: Expansion

4. You will need a metre tape or a piece of string and a metre stick or 30-cm ruler.
   Blow as much air out of your lungs as you can. Measure the distance around your chest.

5. Breathe in as much air as you can and hold your breath while you measure your chest again.

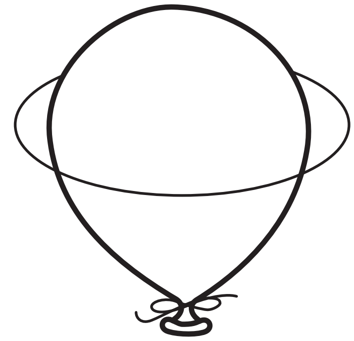
6. Try three times and record the results.

7. Did your 'lung capacity' change during the three trials? If so, how did it change? Why do you think this happened?



BLM 6: My Family and Balloons

1. Blow up a round balloon. How could you measure the distance around it? Decide on one method for your whole family to use for all the trials in this activity.



2. Let the air out of the balloon.

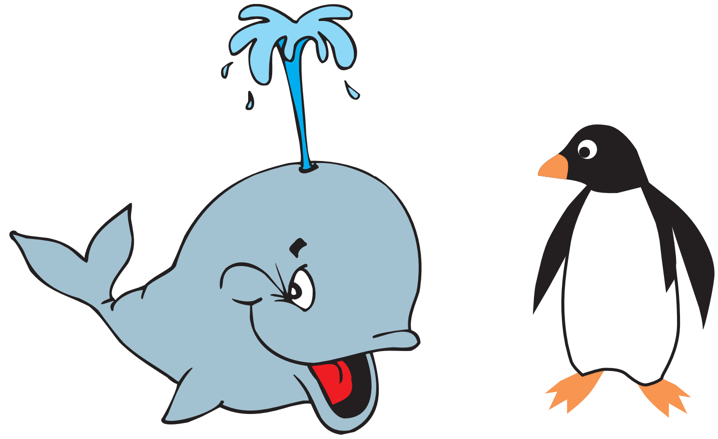
3. How big do you think you can blow up the balloon with one breath? Try it. Measure the distance around the balloon. Have each person try three times and record the results in the chart below.

Person	Trial 1	Trial 2	Trial 3	Average

4. The distance around the balloon is a measure of "lung capacity". Does the lung capacity depend on the size of the person? Explain.

5. Did anyone's 'lung capacity' change during the three trials? If so, how did it change? Why do you think this happened?

6. Do you think these animals need good lung capacity? Why?
   What can you find out about them?



BLM 7: Pieces of Paper

You will need some scrap paper and a ruler that measures in centimetres.

1. How many pieces of paper does it take to make a pile 1 cm thick? Tell how you can find this out.

2. If a stack of paper is 5 cm thick, about how many sheets are in the stack? Tell how you can find this out.

3. How many sheets should there be in a stack 1 m high?

4. How high a stack would a million sheets of paper make?

5. (a) Take a piece of scrap paper. Into how many pieces of paper can you tear it in one minute, making only one tear at a time?

   (b) Put all the pieces into the wastebasket/recycle box before continuing with the problem.

   (c) Calculate how thick a stack you could make if you piled all the little pieces on top of one another.

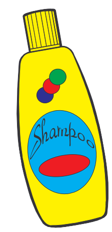
   (d) How long would it take you to tear enough pieces to make a stack 20 cm high? 50 cm high? 1 m high?

BLM 8: A Bevy of Bottles

1. Collect several bottles from home. Make sure that the bottles still have their labels and that their prices are known. Try to find bottles that hold different things — for example, shampoo, ketchup, cleaning fluid, soft drinks.

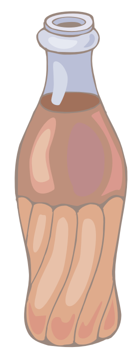
2. Make a list of things the bottles contained and the volume of liquid that is indicated on the label. Then use your measuring container and measure how much water will fit into the bottle and record that in the chart. One example is given.

Container	Volume indicated	Capacity of bottle	Cost	Cost per unit
shampoo	120 ml	125		



3. List the price of each item in the fourth column of the chart. Then, using your calculator, determine the cost per unit to the nearest cent. For the example above, write \$14.95 in the fourth column. In the fifth column write 1495¢ ÷ 120 ml = 12¢ per ml (to the nearest cent).

   Complete columns 4 and 5 for all the other bottles in the chart.



4. (a) Most bottles will not be filled to the top with whatever they contain. Which bottles have the most empty space at the top? Is this related to price in any way? Explain.

   (b) Why would manufacturers not fill bottles to the top? Would it not be cheaper for them to use smaller bottles and fill them right to the top?

   (c) Make a list of things that are sold in boxes or bags that are not filled right to the top. Give reasons why this might be so.

5. Supermarkets have tabs on their shelves giving the price per unit of their goods. Compare some of your prices per unit from column 5 in the chart with the supermarket's prices per unit. How close are the two numbers?

BLM 9: Getting Wet

1. Write how you would solve the following problems. Do not try to solve them yet.

   (a) An average bathtub holds about 200 L of water. About how many litres of water do you think your bathtub holds? Estimate how much water you use when you take a bath.

   (b) Do you take showers instead of baths? Estimate how many litres of water you use for a shower.

   (c) Estimate how many baths/showers you take in a year and how many litres of water you use in a year.

   (d) Estimate how many litres of water your whole family uses in a year, for baths or showers.

2. Solve the following problems. Try to use the methods you described in #1.

   (a) Measure your bathtub at home. Estimate how many litres it would hold if full.

   (b) Next time you have a bath or shower, estimate how many litres of water you used.

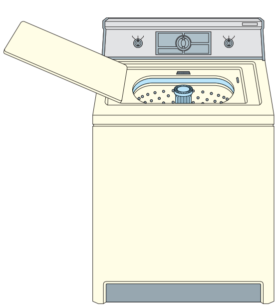
   (c) Using this estimate, calculate the answer to #1(c).

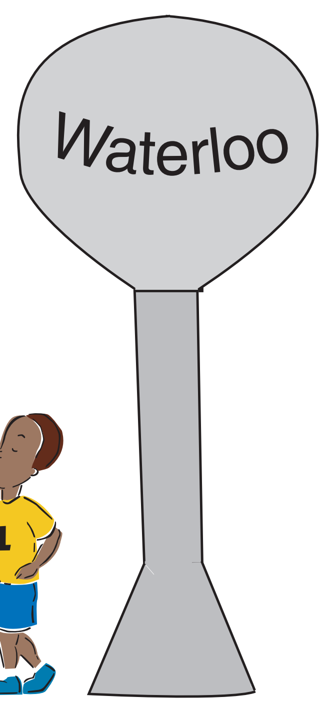
3. (a) About how many litres of water does your washing machine need for a full load including both washing and rinsing? (You can find this in the manual that comes with the machine.)

   (b) About how many litres of water would your family use during the year to do laundry?

4. The average Canadian uses about 390 L of water each day. In parts of Africa the average family of 5 uses about 6 L each day for cooking and washing. What reasons could there be for this difference? How could you save water at home?

5. How much water do you think you drink in a week? How could you find out?





BLM 10: Travelling Time

Let's Drive

1. Keep track of how much gasoline your family buys in one week.
   Record the cost, the number of litres, and the date of the purchase.

2. Compare your data with others in your group and then with the class. Graph the class's data. What kind of graph is most appropriate? Why?

3. Answer the following questions from your graph.

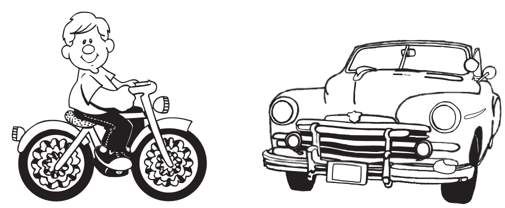
   (a) On which day of the week did most of the families buy gasoline?

   (b) On which day of the week was the least amount bought?

   (c) Suggest reasons for this.

Let's walk

4. (a) Estimate the time it takes you to get to school each day whether you walk, take a bus, ride a bicycle, or get a ride.



   (b) Time yourself each day for a week. Include all "detours" such as stopping to buy something or calling for a friend. Calculate the average time taken. Compare this to your estimate. Was your estimate close? Why or why not?

   (c) Do you think it is easy or difficult to estimate time? Why? Try the following:
   (i) Close your eyes; open them when you think a minute has passed and check your estimate.
   (ii) Work at some problems with your group; when you think 5 minutes have passed, check.
   (iii) At recess time try to predict when recess will be over.

   (d) Which of these was the easiest to estimate? Which was the most difficult? Why?

BLM 11: Making a Measuring Bottle

You will need:

- a large bottle made of clear plastic with sides that are vertical or almost vertical,
- a measuring cup,
- a strip of masking tape, and
- a marker.

When you are identifying water levels, make sure that the bottle and measuring cup are on flat surfaces, and the water has settled.

1. Place the strip of masking tape on the side of the bottle running from top to bottom. (See diagram below.)

2. Measure 125 ml of water in the measuring cup, and pour this into the bottle.

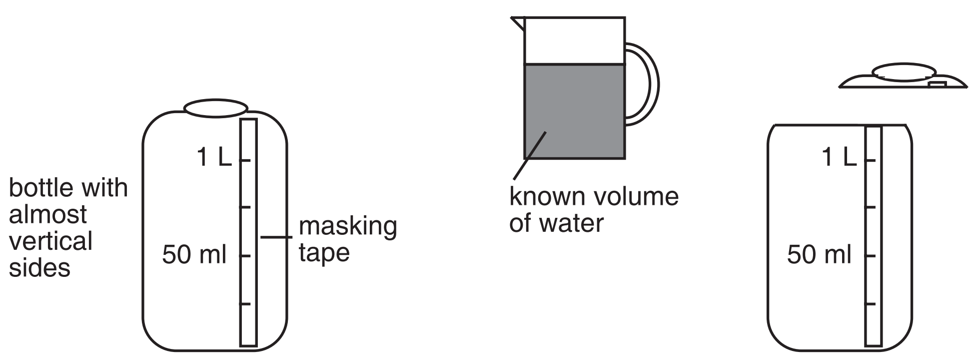
3. When the water has settled, mark the level of the water on the masking tape and label it “125 ml”.

4. Measure another 125 ml of water with the measuring cup, and pour that into the bottle.

5. When the water has settled, mark the level of the water on the masking tape and label it “250 ml”.

6. Repeat to find and mark levels for 375 ml, 500 ml, 625 ml, and so on up to 1000 ml (one litre).
   If your bottle is big enough, you can mark other levels such as 1125 ml, 1250 ml, and so on.

7. If you are going to be using the bottle to measure dry materials such as sand/rice/aquarium gravel, you may with to cut the top off the bottle. This will make it easier to pour in the dry material.



BLM 12: When to Estimate

Read the following problems.
Tell whether or not you could determine an accurate answer. Explain why.

1. If you had a piece of string as tall as you are, how many times do you think you could wrap the string around your wrist?

2. Which do you think is greater — the length of your foot or the distance around your wrist? Explain.

3. Is there enough paper in a roll of paper towels to go all the way around your classroom? Explain.

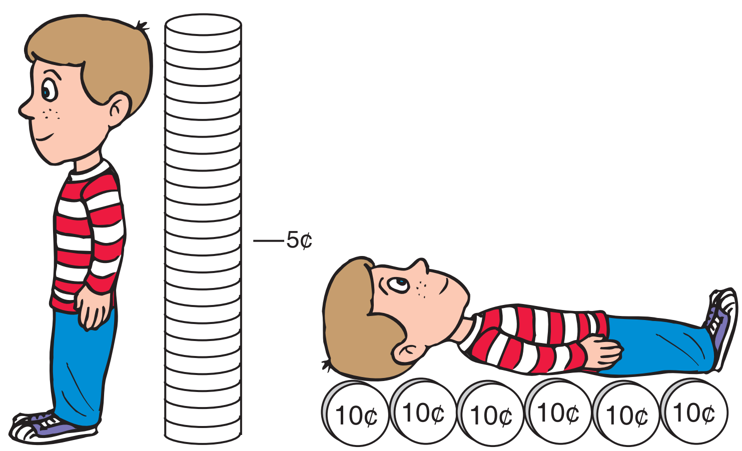
4. Is the height of a large box of cereal greater or less than the distance around the box?

5. How many pieces of popcorn could fit in a wastebasket?

6. Are there more people in school wearing shoes with laces or wearing shoes without laces?

7. If you asked people to name their favourite one-digit number (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9), what do you think would be the most commonly chosen number? Why do you think this?

8. Which do you think would be worth more — a stack of nickels as tall as you or a row of dimes laid edge to edge from your toes to your head as you are lying down?



9. Which would you rather have — your weight in nickels or your weight in quarters?

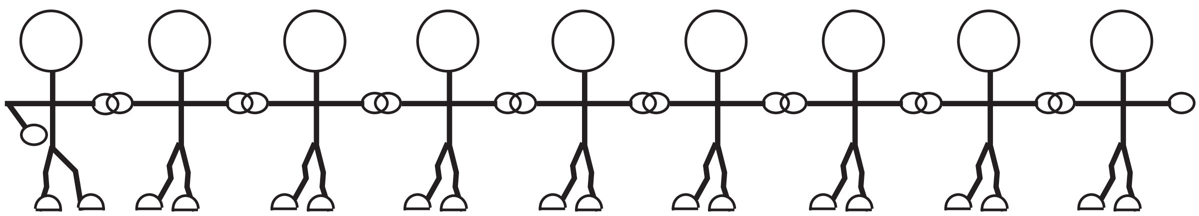
10. If you were to try spending $1 000 000.00 by spending $100.00 every hour, day and night, how long would it take you to spend it all?

Select a problem from 1 to 10 for which you think an accurate answer is possible.
Describe a method you could use to answer it accurately.
Carry out your method if possible. If it is not possible, explain why not.

BLM 13: Estimating Estimates

Racing hands

1. Estimate how long a chain you and two friends can make by joining hands.
2. How long a chain could be made with all the students in the class?
3. How many students would be needed to make a chain 100 m long?
4. Find out how many kilometres it is from your town or city to Toronto.
5. How many students would it take to make a chain that long?



6. Make a chain of five people. Pass a hand squeeze down the line. How long did it take?
7. How long would it take to pass a hand squeeze along the chain from your town to Toronto?

Slow racing

1. To compete in this walking race, you will need to mark a starting point and an end point. Make these 3 to 5 m apart.
2. You also need to be able to estimate time. Practice with this activity:
   Close your eyes. Open them when you think 30 seconds have passed.
   Check your estimate with a clock or watch.
   Repeat this, estimating 20 seconds, and then 10 seconds.
3. Now you are ready for your walking race. Start at the starting point and walk to the end point, without changing your speed, so that your race time is as close as possible to 30 seconds.
4. Take three more tries. Are you getting better or worse? Why?

Start End

Solutions & Notes

Activity 1: Measuring Us

Cross-Curricular Activities

1. (b) Since some of the data will probably come from U.S. publications, students will need to compare miles per hour with kilometres per hour. The following scale should help:

30 km/h	40 km/h	50 km/h	100 km/h
19 mph	25 mph	31 mph	63 mph

The "mph" figures are rounded to the nearest whole number.

The following animals (given with what are considered to be their maximum speeds) can all exceed the car's speed of 40 km/h but only for short distances.

Grizzly	48 km/h	Horse	77 km/h
Cheetah	113 km/h	Elk	72 km/h
Lion	80 km/h	Reindeer	52 km/h

Olympic records are a good source of human speeds and they are given in metric units. For example, in 1976 the USSR's cycling team went 100 km/h in just over 2 hours, about 47 km/h; in 1976 Cathy Priestner of Canada speed-skated 500 m in just over 43 seconds, about 42 km/h. Students could be given the task of locating information on more recent games.

Activity 2: Inside Me

Cross-Curricular Activities

1. An adult has about 5 L of blood, which circulates through the body in about 1 minute. That is, a "round trip" for a blood cell from the heart and back to the heart again takes approximately 1 minute.

   An adult has approximately 100 000 km of blood vessels.

   The adult heart pumps about 70 ml of blood at each beat, and beats about 70 times a minute at rest. During strenuous exercise, the heart can pump six to eight times the amount of blood pumped at rest.

   Some helpful books that might be found in the children's section of your local library are:

   "The Heart and Blood" (1989), by Steve Parker, published by Franklin Watts of Toronto;
   "The Heart and Circulatory System" (1996) by Carol Ballard, published by Wayland Publishers Ltd. of East Sussex
   "Respiration and Circulation" (1998) from Gareth Stevens Publishing of Milwaukee, WI

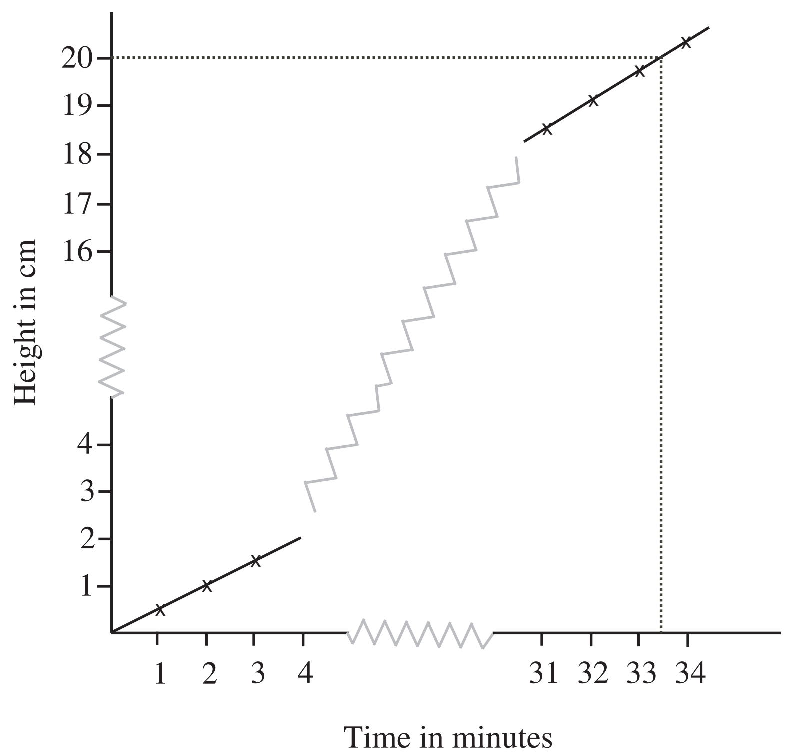
Solutions & Notes

Activity 3: Around Us

BLM 7: Pieces of Paper

Using copy paper, answers should be close to the following:

1. 100 sheets
2. 500 sheets
3. 10 000 sheets
4. 100 m
5. Answers will vary. Suppose the height of the stack after 1 min. of tearing is 6 mm or 0.6 cm. Then after 2 min. of tearing at the same rate, the stack will be 1.2 cm high; after 3 min. 1.8 cm high, etc. If we graph these, we can see that the points lie in a straight line. Extend the line up to 20 cm in height. Draw a horizontal line from the vertical axis to the graphed line. Draw a line perpendicular to the time axis, meeting it at approximately 33.3 min. Thus it would take 33.3 min. to tear enough bits to make a pile 20 cm high.



Activity 4: Me and My Family

BLMs 8, 9, and 10

Answers will vary.

Solutions & Notes

Activity 5: Estimating Everywhere

BLM 12: When to Estimate

It is possible to determine an accurate answer for most of these, but estimates are acceptable for many. Items 5 and 6 suggest sampling, since it would be difficult to determine the type of shoe worn by every student/teacher/custodian/ ... on any given day; the nebulous “people” in #7 makes this impossible to determine accurately.

Estimates are acceptable for most of the others. Accurate answers are reasonable for #1, 2, and 4; and #3 may involve an estimate of the distance around the room, but the length of paper towelling can be calculated fairly accurately from the label on the towels.

BLM 13: Estimating Estimates

Answers will vary.

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

"For additional ideas, see annotated Other Resources list on page 47, numbered as below."

1. The Ontario Curriculum, Grades 1-8: Mathematics.

2. *Linking Assessment and Instruction in Mathematics: Junior Years*, Ontario Association of Mathematics Educators/OMCA/OAJE, Moore et al., 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, by Jean Kerr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels.
   Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. *How to Evaluate Progress in Problem Solving*, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Suggested Assessment Strategies

A GENERAL PROBLEM SOLVING RUBRIC

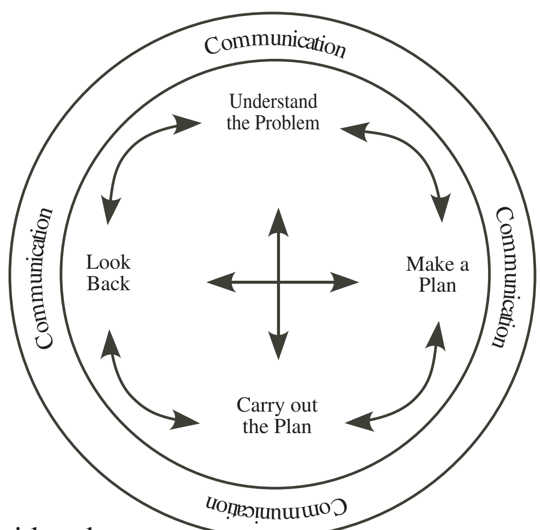
This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R''s"

There are five criteria by which each response is judged:

Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;
Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.
Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.
Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
STRATEGIES	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

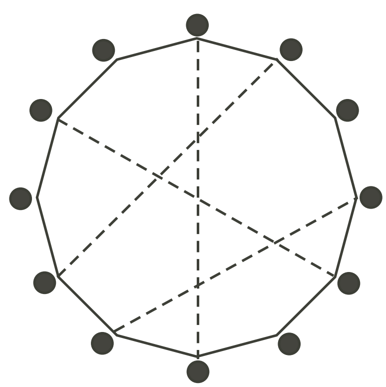
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a “what if...?” problem, such as “What if the question were reversed?”, “What if we had other data?”, “What if we were to show the data on a different type of graph?”.

Suggested Assessment Strategies

Adapting the Rubric

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described' by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

Activity 1, BLM 1

Strategies and Reasoning: What strategies do students use during the Metric Scavenger Hunt? How well can they explain and justify their techniques?

For example,

- The "Limited" student needs to measure almost all items to be confident in his/her choices. He/she gives as a reason for a response, "I measured it."
- The "Acceptable" student shows some confidence in estimating measurements of several items. Reasons given for estimated values may be comments such as "It looks that long."
- The "Capable" student uses estimation confidently, but relies on measurement for such items as the average height or the fastest someone can write the alphabet.
- The "Proficient" student confidently estimates answers for most items, and gives such reasons as "My foot is about 10 cm wide, so I know that pen is about 10 cm long." or "We all held our arms out and took the middle one as close to the average, so we only had to measure one."

Activity 5, BLM 12

Understanding: How well do students understand the nature and use of estimates?

For example,

- The "Acceptable" student will identify most of the items as ones for which an accurate answer is possible, but reasons given may not be valid. For example, "They are math problems so there has to be an answer."
- The "Capable" student will recognize that most can be calculated accurately, but that estimates are suitable for many of those.

Suggested Assessment Strategies

STRATEGIES AND REASONING: Are their strategies feasible?

For example,

- The "Limited" student might suggest solving problem 8 by stacking nickels until they are as high as the student.
- The "Capable" student will understand that a small stack of nickels can be measured, and this used to determine the number of coins in the student's height.
- The "Proficient" student will see that the height of a dime is more than a stack of two nickels, and conclude that the stack of nickels would be a better choice.

Other Resources

1. *Principles and Standards for School Mathematics*, Jean Carpenter and Sheila Gorg (Ed.), 2000, NCTM
   Includes a general discussion of measurement expectations for each grade level, and how to help students achieve them. The Grade 6-8 section has an intriguing suggestion for an activity dealing with relative sizes, based on Shel Silverstein's poem 'One Inch Tall'.

2. *Measurement, Addenda Series, Grades 5-8*, Grances R. Curcio (Ed.), 1994, NCTM
   This booklet contains problems for grades 5-8 dealing with measurement and estimation. Topics include making, and assessing the validity of, estimates of mass, length, area, volume, capacity, selecting appropriate units of measurement, using a variety of measurement tools, graphing data, and developing formulas based on observations. There is a wide variety of excellent applications to real-world problems.

3. "By the Unit or Square Unit?" by B.B. Ferrer, pp 132-137, *Mathematics Teaching in the Middle School*, Nov. 2001
   The article describes teachers' ways of helping students understand perimeter and area and any relationships between them. For example, students were presented with the problem: "For the school carnival, sponsors will pay for advertising their products on signs. How can the school make the most money: by charging the sponsors by the square unit of area or by measuring the perimeter of the signs and charging by the linear unit?" Both regular and irregular shapes (e.g., pentominoes, students' footprints) are measured, using grid paper and string, bringing out some of the less intuitive aspects of how perimeter relates to area.

4. "Rules of Thumb" and "Rules of Thumb 2", Tom Parker, 1983 and 1987, Houghton Mifflin Company, Boston, ISBN 0-395-34642-8 (pbk.) and 0-395-42955-2 (pbk.)
   These two books contain, between them, 1826 "rules-of-thumb" on almost any topic you can think of (e.g., body measurements, children, distance, travel, and weather). The index helps one locate any one of these topics. The rules-of-thumb are presented as they were submitted and sometimes contradict each other. Parker makes no claims for their validity. One good problem for students would be to select a rule-of-thumb and have students devise an experiment to test its validity.

5. "Responses to the Patterns in Squares Problem", by Tim Boerst, pp 169-172, *Teaching Children Mathematics*, Nov. 2001
   A problem of drawing squares on geopaper was given to Grade 5 students. The article explores ways the students proved that they could draw squares with areas of 2 square units or 5 square units as well as 1, 4, 9, 16, ... square units. (One square unit is the smallest square that can be drawn with corners of the square on the dots.)

6. "What Is A "Good Guess" Anyway?", by Frances Kuwahara Lang, pp 462-466, *Teaching Children Mathematics*, April 2001, NCTM
   This theoretical discussion analyzes the strategies children use in estimating and gives examples of how teachers can help them develop and refine their estimation and measurement skills

Other Resources

7. "Mousemaze Tournament: Connecting Geometry and Measurement" by Shirley Curtis, pp 504-509, *Teaching Children Mathematics*, May 2001, NCTM

   Students are challenged to collaboratively design and construct mouse mazes, based on student-generated criteria, and then participate in a tournament for which they also develop the rules. An open-ended activity which stimulates a high degree of involvement, and appeals to students of varying levels of mathematical experience.

8. "Using a Lifeline to Give Rational Numbers a Personal Touch" by Wanda Weidemann, Alice Mikovch, and Jane Braddock Hunt, pp 210-215, *Teaching Children Mathematics*, December 2001, NCTM

   A creative activity in which students develop a personal 'time-ruler' for their lives. Using one year as the unit of time prompts the need for representing and manipulating non-integer numbers. An assessment rubric and cross-curricular connections are included.

9. "How Big Was the Cat?" by L.E. Sakshaug and K.A. Wohlhuter, pp 350-351, *Teaching Children Mathematics*, February 2001, NCTM

   This 'problem solver' poses the problem faced by a boy who sees the 'biggest' cat while on a walk in the woods, but has difficulty convincing his family that it really was the 'biggest'. Students are invited to help him justify his claim, stimulating an exploration of comparative methods for estimating size. Variations are also suggested.

10. "How Big Is Your foot?", by Suzanne Levin Weinberg, pp 476-481, *Mathematics Teaching in the Middle School*, April 2001, NCTM

    Using the length of his/her own foot, the student measures and compares such lengths as arm span, height, hand span, and length of hand. Good discussion on the inexact nature of measurement.

11. "Power over Trash", by Robert Ronau and Karen Karp, pp 26-31, *Teaching Children Mathematics*, September 2001, NCTM

    The activity described herein is 'environmentally friendly' (the students collect trash from the school yard), and provides a good illustration of the link between science and mathematics through the detailed analysis of the garbage data (Circle, bar, and strip graphs are all put to good use). In addition, there are suggestions for further investigations using graphing calculators and/or the Internet.

12. "The Environmental Math Workbook", by Evannah Sakamoto, Courseware Solutions Inc., Toronto, 1992.

    This book provides activities tied to environmental issues, such as waste collection and analysis. Activities connect mathematics with environmental studies in a "user-friendly" way.

13. "Food for Thought", by G.C. Poke, N. Ewing, and D. Stevens, pp 148-150, *Teaching Children Mathematics*, November 2001, NCTM

    This 'Math by the Month' activity suggests a multitude of ways to use food to prompt mathematical thinking, e.g., (i) use a carrot as a measuring tool for a desk, door or other classroom objects; (ii) collect soft-drink cups and explore their relative sizes; (iii) weigh apples daily in three modes: whole, peel plus apple, and sliced, and chart the changes over one week.

Other Resources

14. "Honoring Traditions: Making Connections with Mathematics through Culture", *Teaching Children Mathematics* by Deborah McLean, pp 184-188, November 2002, NCTM.

    This article presents a brief history of measurement, moving from the informal to the formal. A specific example from the Yup'ik culture of southwest Alaska involves the design of a hooded shirt using the distance from shoulder to wrist and across the back of the shoulders measured in 'handwidths'. Uses of such nonstandard units in other contexts and its importance both historically and in the present day are explored, along with the importance of teaching such culturally-based information.

15. Science Is ..., (2nd ed.), by Susan V. Bosak et al, Scholastic Canada Ltd., 1991

    This practical book has an extensive collection of activities/experiments encompassing environmental studies and mathematics, developed with input from students, teachers and parents. While directed at children ages 6-14, these hands-on activities provide fun and ample learning opportunities for all. Permission to copy for classroom use is granted.

16. "Weather Watch" by Valerie Wyatt, 1990, Kids Can Press, Toronto.

    This easy-to-read book gives information about such things as the nature of lightning, types of clouds, and how to measure rain drops or wind direction. Included are simple experiments for students to create and measure various weather phenomena (e.g., 'Smog In A Jar', 'How Wet Can It Get?', or 'Thunder In a Pop Can').