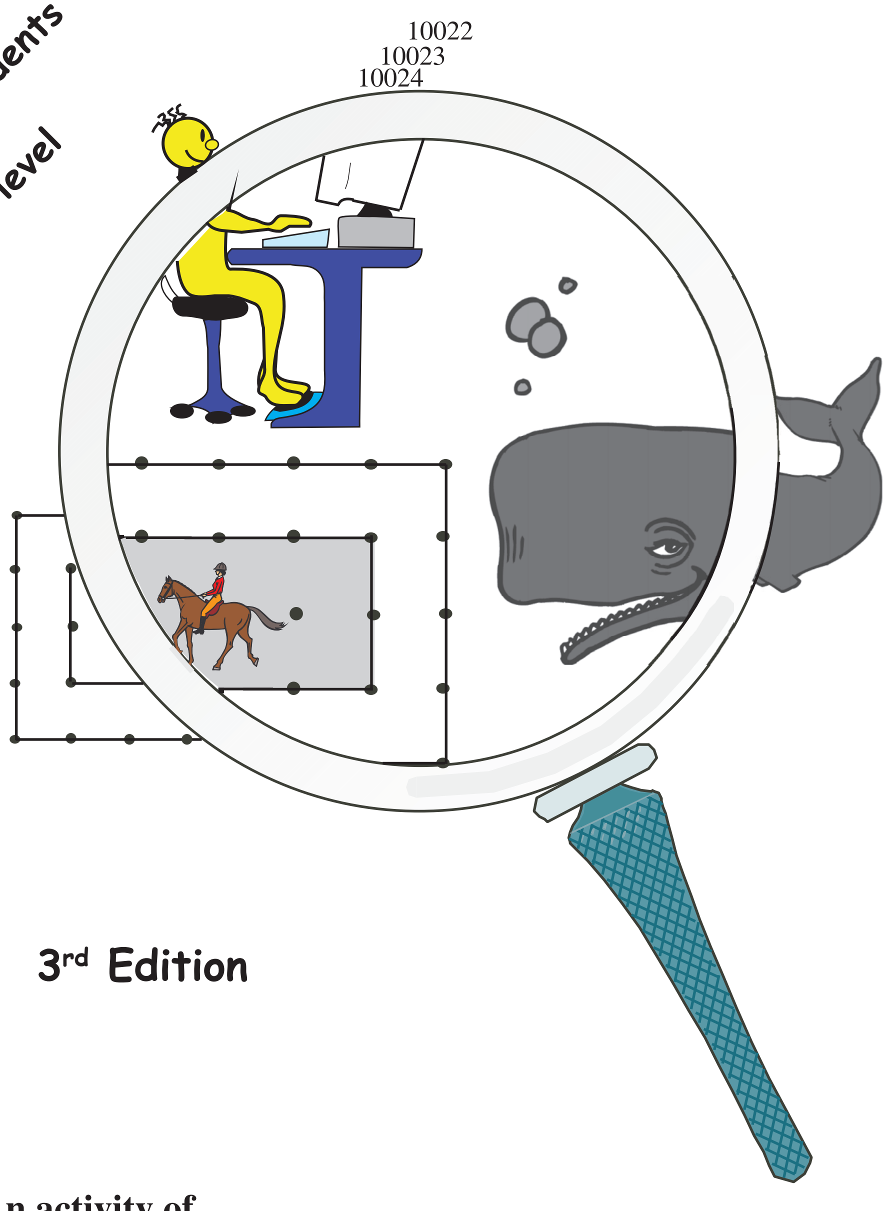
Invitations to Mathematics

Investigations in Measurement

"Around and About"

Suggested for students at the Grade 4 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the extensions and related activities included with individual activities/projects, provide ample scope for all students' interests and ability levels. Related "Family Activities" can be used to involve the students' parents/care givers.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

"Investigations in Measurement" is comprised of activities which explore estimation and measurement, and the selection of appropriate tools and units. Measurement provides the means to describe and analyse the everyday world in concrete terms, from grocery shopping through car assembly to building a space module. The activities involve making and testing hypotheses, and other forms of problem solving, as well as connecting strands of mathematics to each other and to other curriculum areas.

Acknowledgements

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We wish to acknowledge the support of the **Centre for Education in Mathematics and Computing**, and in particular the efforts of Ron Scoins, Gord Nichols, and Carolyn Jackson. A special thank you goes to Bonnie Findlay for prompt, accurate type-setting and creative diagrams.

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Overview

Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and measurement specifically. Some of these beliefs are described below.

Measurement provides the means for describing and quantifying our everyday lives. As such, it is rich with opportunities for exploring both mathematical concepts and their applications in the 'real' world.

Dynamic interaction between students and their environment is essential to developing skill with the processes of measurement and deriving the mathematical possibilities. Concrete, hands-on activities involving estimating, then measuring, in both non-standard and standard units, promote awareness of what units and tools are appropriate for a specific task, why standard units are necessary, and that all measurements are approximations with varying degrees of human and instrument error. Activities with ordinary objects/attributes (e.g., eraser, desk, blackboard, height, heartbeat, gasoline consumption, shadows, packaging) provide limitless opportunities for problem-solving, involving concepts such as length, area, volume, time rates, ratio and proportion, similarity and congruence, as well as connecting mathematics directly to the physical world.

Justifying their own reasoning and discovering the patterns and logical connections which lead to mathematical formulas, or to a deeper understanding of their everyday world, increases the students' ability to reason analytically and to express their thoughts clearly and concisely.

Essential Content

In the activities herein, students will explore the process of measurement and its implications in a variety of contexts, both inside and outside the classroom, with the goal of developing a solid foundation for using instruments and formulas with skill and precision, and analysing the meaning of their measurements. In addition, there are Extensions in Mathematics, Cross-Curricular Activities and Family Activities. These may be used prior to or during the activity as well as following the activity. They are intended to suggest topics for extending the activity, assisting integration with other subjects, and involving the family in the learning process.

During this unit, the student will:

- estimate and measure lengths using both standard and non-standard units;
- use a homemade device to estimate heights of tall objects;
- use shadows to make a sun clock;
- measure areas using grid paper;
- draw figures of given areas or perimeters on dot paper;
- estimate and measure time intervals;
- decide when an estimate is acceptable;
- explore relative sizes by comparing objects;
- participate in a measurement pentathalon, which includes estimating and measuring distance, area, and time intervals;
- use mathematical language to express their results;
- work with others to achieve success.

Notes

"Curriculum Expectations" are based on current Ontario curricula.

Curriculum Expectations

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **How Long?**	• establishing the need for standard units • measuring lengths, using both standard and non-standard units • estimating the heights of tall objects (e.g. tree, flagpole) • measuring the motion of the sun by shadows	• select the most appropriate standard unit • estimate lengths in ... centimetres ... and kilometres • use linear dimensions, perimeter and area measure with precision • solve problems related to their day-to-day environment using measurement and estimation
Activity 2 **Areas in Abundance**	• measuring small areas, using centimetre grid paper • comparing areas and perimeters of simple polygons	• estimate ... area in square centimetres using grid paper • understand that different two-dimensional shapes can have the same perimeter or area • estimate, measure, and record the perimeter and area of two-dimensional shapes, and compare the perimeters and areas
Activity 3 **Time and Time Again**	• estimating and measuring time intervals • establishing when an estimate is appropriate and/or more useful than attempting an accurate measurement	• estimate and measure time intervals to the nearest minute • distinguish between estimated and precise measurements and know when each is required
Activity 4 **How Big?**	• estimating and measuring classroom dimensions and comparing them with various large animals • exploring size by sketching life-size drawings of large animals	• solve problems ... using measurement and estimation • draw items given specific lengths
Activity 5 **Junk Pentathalon**	• estimating and measuring distance, area, and time in a variety of team competitive events	• estimate and measure time intervals to the nearest minute • select the most appropriate standard unit ... to measure linear dimensions • estimate ... area in square

Overview

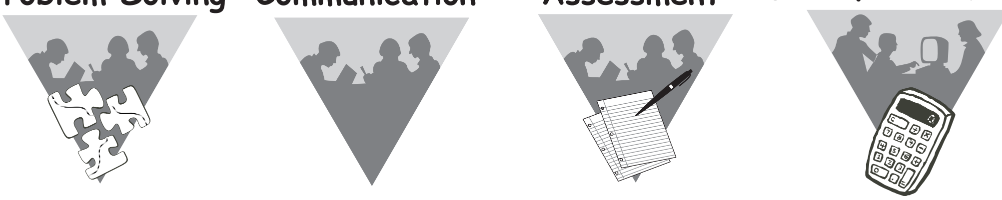
Prerequisites

Students who have completed Grade 3 are expected to have some familiarity with basic units such as cm, m, gm, and L and terms such as perimeter and area. Beyond this basic knowledge there are no prerequisites for the activities in this Grade 4 booklet.

Logos

The following logos, which are located in the margins, identify segments related to, respectively:

Problem Solving | Communication | Assessment | Use of Technology



Snippets

"Snippets" that appear as small notebook pages in the margins are bits of data somehow related to the measurement tasks the students are being given. Sometimes these snippets will include a problem posed for the students. For others, questions will no doubt come to the teacher's mind even as he/she is sharing the snippet with students. Students themselves may identify related questions that they would be interested in pursuing. It is hoped that students will find these bits of information interesting and will realize how frequently measurements are used in everyday life.

Rules of Thumb

R of T

"Rules of Thumb" are ways to help in estimating. For example, the rule-of-thumb "Two pages written by hand will give one page when typewritten" will give an author some idea of the length of his/her erudite article, so he/she knows when the article has reached a permissible length. Rules of Thumb (R of T) have been placed in margins (on file cards) alongside the Activity notes. They can be used as jumping off points for good problems, or just enjoyed for their (possible) values. A worthwhile activity is trying to decide whether each R of T is valid. All R of T's in this book are gleaned from "Rules of Thumb" and "Rules or Thumb -2" by Tom Parker. See "Other Resources" on page 48 for more detail.

Notes

Notes

Overview

Materials

ACTIVITY	MATERIALS
Activity 1 **How Long?**	• Copies of BLM 1 • Toothpicks, paperclips, scissors • Copies of BLMs 2 and 3 (optional) • Cardboard tubes, acetate, fine point markers (optional)
Activity 2 **Areas in Abundance**	• Two paper copies and one acetate copy of BLM 2 for each pair/group • Scissors • Drawings/models of figures for 'New Shapes from Two Shapes' • Copies of BLMs 4, 5, 6, 7 (optional)
Activity 3 **Time and Time Again**	• Copies of BLM 8 • Copies of BLM 9 (optional) • Clock with a second hand
Activity 4 **How Big?**	• Metre sticks or tapes, string • Copies of BLM 10 • Copies of BLM 11 (optional) • Newspapers (optional)
Activity 5 **Junk Pentathalon**	• Cotton balls, rulers, large buttons, marbles (or small balls), rice (coloured with food colouring), newspaper, metre stick/tape • Copies of BLMs 12, 13, 14

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week or so, students in our classroom will be participating in a unit titled “Around and About”. The classroom activities will focus on estimating and measuring length, area, volume, and time with commonly used (metric) units. The emphasis will be on achieving familiarity with these units so that estimates become more accurate.

You can assist your child in understanding the relevant concepts and acquiring good measurement skills by working together to perform simple tasks (e.g., cooking from a recipe, sewing a tablecloth, building a chest, ...), helping to explore everyday ways measurement is used.

Various family activities have been planned for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with measurement in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher’s Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Notes

Activity 1: How Long?

Focus of Activity:

- Establishing the need for standard units in measurement

What to Assess:

- Accuracy of measuring with non-standard and standard units
- Understanding of the need for standard units

Preparation:

- Make copies of BLM 1 for each pair/group.
- Provide toothpicks, paper clips, scissors.
- Make copies of BLMs 2 and 3 (optional).
- Provide cardboard tubes, acetate, and fine-point markers.

Activity:

The purpose of BLM 1 is to provide a simple way to re-introduce students to measuring lengths and to establish the need for standard units. Distribute copies of BLM 1 to each pair/group of students, along with several toothpicks of equal length, and several paper clips of equal length. You may wish to give one group small paperclips and another group larger paperclips. Comparing the results will reinforce the need for standard units.

If students find it difficult to manipulate toothpicks without breaking them, some other item may be substituted. Flat plastic coffee stirrers are substantial enough not to break easily and they are relatively inexpensive. If identical paint brushes or markers are available, they, too, could be substituted for the toothpicks.

Explain that in this activity they will be measuring lengths in different ways, using a variety of tools. As they complete 1 and 2 have them compare their results with another pair/group and try to explain any differences. Ask them to describe how they found the perimeter of the page and note which students used their answers to (a) and (b) to determine the perimeter.

Since photocopies often vary measurements slightly, have students measure their decimetres to be sure they are 10 cm long. Have them add or subtract a millimetre or two to correct the decimetre if necessary.

As they work with the decimetre ruler, ask them how they could mark a half-decimetre on the ruler. Elicit the idea that they can simply fold the ruler in half.

Question 4 asks students to estimate in decimetres, now that they have had a little experience with the unit. Students may need help interpreting "hand span" as the distance from thumb tip to little finger tip when they are spread as far apart as possible.

When students have completed the page, bring out the following important ideas in measurement during discussion of the questions that follow.

(i) Standard units are used so everyone's measurements will be the same or almost the same.

Notes

See page 3 for an explanation for "Snippets", an example of which is given here.

The nepenthe, an insect eating, jungle climbing plant, grows up to 18 m long.

Assessment

Activity 1: How Long?

(ii) A string of several units of the same size is more useful than trying to measure with just one example of the unit. (Imagine trying to measure the width of a room when all you have is a 1 cm ruler.)

(iii) All measurement is really estimation because of slight inaccuracies in the measuring instruments we use and in our own vision, among other things.

Communication

Discuss the following questions with the class:

- Which items were easy/hard to measure with toothpicks/paper clips/decimetre rulers?
- How could you have made it easier to measure the height of the chair with any of these units? Bring out the idea that they could make longer "rulers" if they taped several toothpicks together or linked paper clips, etc.
- Were your measurements the same as other groups'? Why or why not?
- Would you measure things at home using toothpicks or paper clips? Why or why not?
- If you were selling fabric or lumber, why would you not use toothpicks or paper clips to measure the amount sold?

As a review, ask students what units they remember using in the past to measure lengths. List these in order of size. The ones most likely to be mentioned are millimetre, centimetre, metre, kilometre. Ask where in this ordered list the decimetre belongs.

A kangaroo can jump 2.5 m high. A kangaroo rat can jump 48 times ▯ its own height and can run 6 m/s. How fast can you run?

Extensions in Mathematics:

1. BLM 3, The Tubescope, gives students a way to make indirect measurements of the heights of such things as trees or buildings. Students will need small pieces of acetate and fine point pens that will mark permanently on the acetate. Instructions are given on BLM 3. Encourage students to be very careful in drawing the lines on the acetate. The more accurately these lines are placed, the more accurate will be their measurement/estimate of tree height.

Note: A toilet paper roll is very close to 10 cm in length. This will simplify the computation in #9 on BLM 3.

It is better to use short cardboard tubes (e.g., from toilet paper) than long ones (e.g., from paper towels) because with the longer tube it is necessary to stand much further away from the object whose height is being estimated.

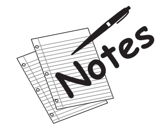
For step 2, drawing the lines on the acetate, have students lay the piece of acetate over cm^2 paper (BLM 2) and trace the lines which are 1 cm apart.

Comments in italics are explanatory, and need not be conveyed to the students.

Once the Tubescopes are made, have students practice viewing things in the classroom or out a window and estimating the height of the picture on the acetate to the nearest half centimetre.

When students are familiar with the instrument, have them list the measurements they will need to take (i.e., distance between student and object, length of the Tubescope, and height of the object's picture), and then go outside to complete #11 on BLM 3.

Activity 1: How Long?

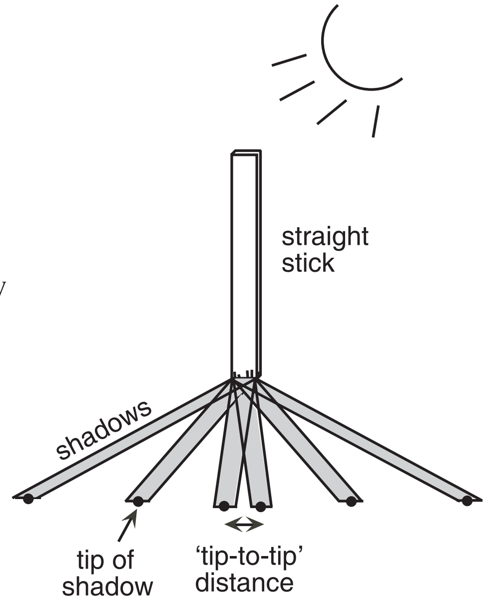


Cross-curricular Activities

1. On a bright sunny day, have students fix a metre stick (or any tall stick) upright at a spot with full exposure to the sun. (If the classroom has a southern exposure, this could be done by fixing a stick with modelling clay on the windowsill.)

   Have the students record the position of the shadow every half-hour during the school day by marking the end of the shadow in some way.

   Ask: What happens to the length of the shadow during the day?
   When is the shadow the shortest? Why?
   Does the tip of the shadow travel the same distance every hour?
   Are any of the distances travelled the same?
   If so, which ones? Why do you think this happened? Did you expect this?
   Why or why not?
   Would you get the same results if you hadn't used a straight stick?
   Why or why not?
   How could you use the shadow marks to make a clock?



Communication

Family Activities:

1. Give students the following exercise:

   Next time you take a trip in a car, have the driver signal the beginning and end of a one-kilometre distance several times. Then estimate where the car will be after travelling another kilometre by picking a spot that you think is about the right distance ahead. Check your estimate by having the driver tell you how far you have gone when you actually reach this spot. Try several times to become better at estimating the length of a kilometre.

   Try to estimate where the car will be after travelling 2 km.

   Can you use this technique to estimate a distance of 10 km? Why or why not?

The siamang, a monkey of Sumatra, has a loud call that can be heard 1km away. How far will your voice carry?

Activity 2: Areas in Abundance

The giant sequois has a trunk so big that a slice of it would stretch from one side of a city street to the other side.

Focus of Activity:

- Exploring the meaning of "area"
- Determining area by counting squares
- Comparing areas and perimeters of different figures

What to Assess:

- Understanding of 'area' as 'surface covered'
- Understanding that figures of the same area need not have the same shape
- Understanding that figures having the same perimeter need not have the same area and vice versa

Preparation:

- Make both paper and acetate copies of BLM 2 for each pair/group.
- Provide scissors.
- Make drawings or models of the 6 figures made with two triangles as described below under 'New Shapes from Two Shapes'.
- Make copies of BLMs 4, 5, 6, 7 (optional).

Activity:

Finding Areas Using a Grid:
Distribute acetate centimetre grids to each pair/group of students. Ask them to use the grid to estimate, in square centimetres (cm^2), the areas of some small items such as a pencil case, or a textbook cover. Show them how the acetate can be laid on top of the item being measured and the squares counted to determine the area.

Once students have determined the area of three or four small items, ask for their estimates. Ask them why the estimates differ for different pairs/groups. One reason for differences may be how they counted partial squares.

Before going further, decide with the students how they will count "partial squares". For example, note the diagrams below where some squares are designated "more than half" and some "less than half".

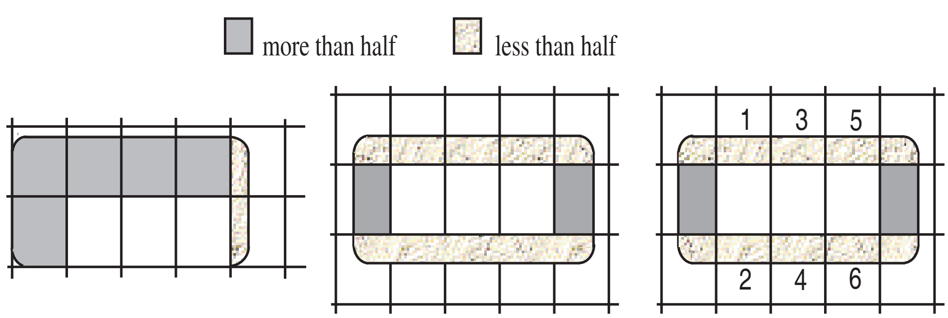


Figure 2.1 Figure 2.2 Figure 2.3

Activity 2: Areas in Abundance

If the more-than-half-squares are counted as full squares and the less-than-half-squares as no squares, then the area of the eraser in Figure 2.1 would be estimated as 8 cm^2. However, if the grid is placed differently as in Figure 2.2, the area would be estimated as 5 cm^2, because there are a number of less-than-half squares. Students may suggest that when "almost-half" squares occur, they can be put together in pairs to make "almost-whole" squares. Thus, in Figure 2.3, squares 1 and 2 could be joined, 3 and 4 could be joined, and 5 and 6 could be joined. This would give an estimated area of 8 cm^2. Students may also suggest that the four corner squares could be combined to make another "almost-whole" square, giving an estimated area of 9 cm^2. Discuss with students which of these techniques they think should be used when determining area by counting squares. You may wish to have all students use one method of counting partial squares, or have some use one method and some the other. Comparison will then show which is likely to be more accurate.

Ask students to estimate the areas of their hands and feet. Record the estimates. Then have students determine the approximate areas of their feet and hands. There are two ways to do this:

(1) Have students trace around feet and hands on plain pieces of paper and cover with an acetate copy of BLM 2 to count squares.
(2) Distribute paper copies of BLM 2 and have students trace hands and feet directly on the square centimetre grid.

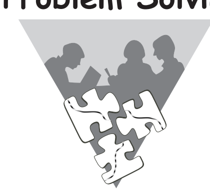
Optional: Classroom data could be used for a graph to compare sizes of hands (or feet). Count the number of students whose hand area is from, say, 121 cm^2 to 130 cm^2 , and the number of students whose hand area is from 131 cm^2 to 140 cm^2, and so on.

For more on graphs, see the series "Investigations in Data Management" for Gr. 4. An order form can be found at the back of this book.

New Shapes from Two Shapes

Distribute paper copies of BLM 2 to each pair/group of students. Have them draw and cut out six rectangles 6 cm by 8 cm. Challenge them to cut all six rectangles from one sheet of BLM 2.

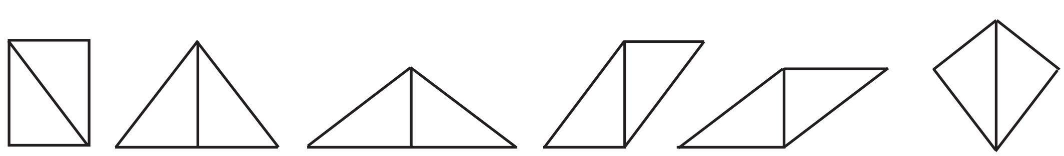
Problem Solving



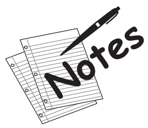
Then have them cut the rectangles apart on one diagonal. Ask if the two halves are the same or different. Students should realize that the two triangles are identical (congruent), and that if one triangle is turned over it will fit exactly on top of the other triangle.

Ask them what the area of the rectangle is. A simple square count should give them the area as 48 cm^2. Ask them to place two of the triangles together to make a figure that is not a rectangle. As students find these figures, they should tape the triangles together or paste the triangles onto another piece of paper or in a notebook.

Challenge students to find as many different figures made up of two triangles as they can. Ask them how they are sure they have them all.



Activity 2: Areas in Abundance



The Himalayan mountains cover nearly 600 000 square km. Is this as big as Ontario? as Yukon? as PEI?

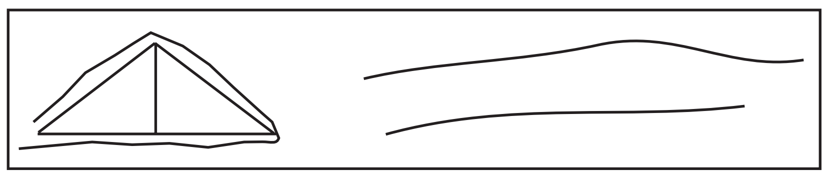
Students should recognize that the triangles can be matched on one of only 3 sides and that there are two ways to orient the triangles for each of these three sides. For example, figures 3 and 5 above have the shortest sides of the triangles together; figures 1 and 6 have the longest side (the diagonal of the original rectangle) together; and figures 2 and 4 have the third sides of the triangles together.

Ask them what the area of each figure is. Students should realize that all figures have the same area of 48 cm^2.

Ask students if they think all the figures will have the same perimeter. Have them give reasons. If they think the perimeters will differ, ask which figure(s) they think has/have the greatest perimeter.

Problem Solving

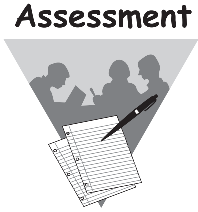
Students can compare perimeters with string.



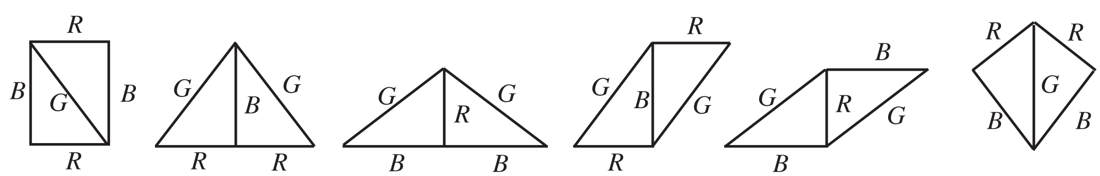
Cut a piece of string to fit around each figure. Stretch the strings as straight as possible and compare the lengths.

If students are reasonably careful they should discover that there are only three distinct perimeters. Figures 3 and 5 have the same perimeter, as do figures 1 and 6, and figures 2 and 4. Ask them why this is so. Ask students how they could have decided which figure(s) have the greatest perimeter without measuring.

Assessment



Have students colour the sides of each triangle, colouring 'R' in red, 'B' in blue, and 'G' in green. Then they can describe the perimeter of Figure 1, for example, as "2 red and 2 blue", and easily see that the perimeter of Figure 6 is also "2 red and 2 blue".



If the sides of the triangles are labelled '*R*', '*B*', and '*G*' we can see that

figures 1 and 6 have perimeters of $R + R + B + B$;
figures 2 and 4 have perimeters of $R + R + G + G$, and
figures 3 and 5 have perimeters of $B + B + G + G$.

Since 'G' is longer than 'B', then $R + R + G + G$ is longer than $R + R + B + B$.
That is, triangles 2 and 4 have perimeters greater than triangles 1 and 6.
Since 'G' is longer than 'R', then $B + B + G + G$ is longer than $B + B + R + R$.
That is, triangles 3 and 5 have perimeters greater than triangles 1 and 6.
Since 'B' is longer than 'R', than $G + G + B + B$ is longer than $G + G + R + R$.
That is, triangles 3 and 5 have perimeters greater than triangles 2 and 4.
Thus, the order from greatest perimeter to least perimeter is triangles 3 and 5, triangles 2 and 4, triangles 1 and 6.

Activity 2: Areas in Abundance

Notes

Extensions in Mathematics:

1. Explore area and perimeter further using BLM 4. Distribute copies and have students do the problems given. Students may need to be reminded that a square is just a rectangle with special properties and is therefore a legitimate answer for either problem. Copies of BLM 6 (Centimetre Dot Paper) should be available for students who need more room than is available on BLM 4.

2. BLM 5 provides another problem ('The Shrinking Rectangle', #1-5) designed to show students that figures having the same perimeter does not necessarily mean they have the same area. Students will need copies of BLM 6 (Centimetre Dot Paper) to record their drawings.

Problem Solving

The 'Border Challenge' (#6 on BLM 5) is simplified if students analyse the problem and approach it by making small changes in the rectangles and looking for a pattern. See "Solutions and Notes" for more on this and the earlier problems.

Use of Technology

3. Use a computer program such as "Geometer's Sketchpad" to
   (i) draw the figures for the problems above; or
   (ii) draw several rectangles and record their measurements (length, width, perimeter, area) and look for patterns. For example, length + width + length + width gives the perimeter.

Family Activities:

1. Have students determine the area of hands and feet of family members. Ask if the sizes of hands and feet are related to general size or related to age, and if this is what students expected.

2. BLM 7 gives rules, playing pieces, and board for an area game that students can take home to play with their families. (The game could also be part of a math centre.)

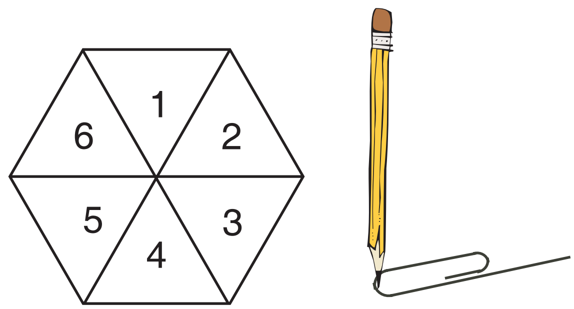
The rafflesia has a flower 1 m across. Draw a circle 1m across. How many of you can stand in it comfortably?

Students should make the pieces and the Game Board to take home. The Game Board can be drawn on BLM 2 or BLM 6. The board could be made a larger size, say 12 cm by 12 cm; piece '1' would then be 2 cm by 2 cm or 4 cm^2. Alternatively, if the board is 6 cm by 6 cm as on BLM 7, then the numbers on the playing pieces will be their areas in square centimetres. Several of each piece should be drawn.

This is a game for 2 to 4 people. The playing pieces can be cut out and pasted onto heavy paper. Students may wish to colour code them for easy play. That is, the '1' pieces might be yellow, the '2' pieces green, and so on. Notice that there are two different pieces numbered '3' and two numbered '4'. Once students are familiar with the game they could design other pieces for 5 and 6.

Activity 2: Areas in Abundance

To play the game, several (4 to 6) of each piece will be needed. These are placed in the middle of the playing area so each person can reach them. Also needed are two regular dice or number cubes. If these are unavailable, students can design spinners, such as the one shown below. For the marker, partially straighten a paper clip and hold in place at the centre of the spinner with the point of a pencil or pen.



The game can also be played like Scrabble. Place all pieces in a bag/box so they cannot be seen. Each player draws 5 pieces to start. In turn, each player then places a piece on his/her board, and draws another piece from the bag. If a player cannot use a piece in his/her hand, that player may put 2 pieces back into the bag, and draw 2 new ones, but will not place a piece on the board in that turn.

Other Resources:

For additional ideas, see annotated Other Resources list on page 48, numbered as below.

3. Addenda Series, Grades K-6: Geometry and Spatial Sense.
4. By the Unit or Square Unit by B.B. Ferrer.

Activity 3: Time and Time Again

Focus of Activity:

- Distinguish between ‘accurate’ numbers and estimations; identify situations for which estimates are appropriate

What to Assess:

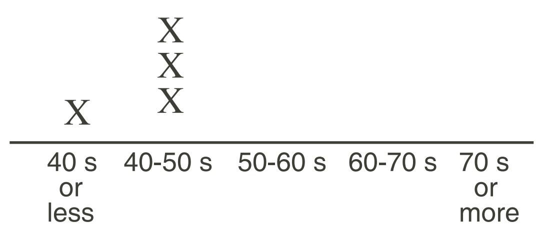
- Identification of situations where estimated answers are appropriate
- Identification of situations where accurate answers are impossible/not required
- Clear and logical reasoning
- Accuracy of timing

Preparation:

- Make copies of BLM 8 for each pair/group.
- Make copies of BLM 9 (optional).
- Provide clock with second hand.

Activity:

Begin by asking students if they think they can estimate one minute accurately. Then have them close their eyes and raise their hands when they think one minute has passed. Record estimates on a line plot adding an ‘X’ as each student raises his/her hand.



When all students have participated, examine the line plot. Repeat the experiment and discuss the results. Were the estimates better or worse for the second trial? Ask if they would find this problem easier or harder if they were doing something during the minute rather than just sitting still. Ask why they think so. Students may say that time seems to go more quickly when they are doing something they enjoy. An example may be the 15 minutes once a parent has said that bedtime will be in 15 minutes, and the student is watching a favourite video.

Tell them they are going to try to estimate how many times they can do a certain task in one minute. Distribute BLM 8. Bring students’ attention to the Part 1 instruction. Discuss what “reasonable estimate” might mean. Students may suggest that “reasonable” could mean “close to the actual value” or “within 5 seconds of the actual value” or “not silly, like estimating ‘25’ for #1”.

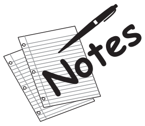
Avoid calling estimates “good” or “bad”. The very nature of an estimate means that it cannot be evaluated in this way. Estimates can be spoken of as “close to” or “not close to” the actual value. Because of flaws in measurements (Are all your clocks, for example, always exactly on time?) and flaws in human vision and reaction time, all measurements are considered estimates or approximations.

Notes

Once an insect lands on a nepenthe, the two edges of the leaf close in less than $\frac{4}{10}$ of a second.

For more on line plots see the series “Investigations in Data Management” for Grade 4. An order form can be found at the back of this book.

A spider can spin a web in 30 minutes. Spiders spin 3-5 new webs each day.



Activity 3: Time and Time Again

Assign Part 1, making sure that students know that they are not to do any timing, but just estimate answers for each problem. Each pair/group of students should agree on one estimate, giving their reasons. For some problems, this may make it necessary for the group to select one person's capabilities. For example, answers to #1 will differ greatly depending on the length of a student's name.

Students should also include in their responses their interpretaion of the problem. For example, for #5 they will need to decide how long a handshake takes; for #3 they will have to decide when a fast walk actually becomes a run, and should be asked to demonstrate this when giving and justifying their answers.

Assessment

Ask a few students for their answers and then assign Part 2 in which students test their estimates from part 1. Observing students will give an indication as to whether or not they are timing things accurately, or interpreting a problem in an unusual way.

Discuss the results of these experiments. Have students explain why they think their estimates and their final answers were very close or were not close.

Communication

Assign Parts 3 and 4. For part 4 you may wish to add conditions. For example, have only one student in each group test problem 7 in order to keep the noise down; have students test #11 at recess, and #12 at home.

Most students should recognize that #12 will take longer than a minute. As an extension have students identify the times for favourite pieces of music, then round to the nearest 30 seconds, and make a bar graph for comparision (and a review of bar graphs).

After discussing responses for Parts 3 and 4 of BLM 8, ask students if one minute seems like a long time or a short time, and to give reasons for their answers. They should be beginning to realize that a period of time can be judged to be "long" or "short" depending on what one is doing during that time. Ask students if one minute would seem long or short if they were watching a favourite TV show, shovelling snow, reading an encyclopedia, eating a chocolate bar, and so on. Have them identify similar situations in which they would judge one minute to seem long or short.

See page 3 for an explanation of "Rules of Thumb."

R of T

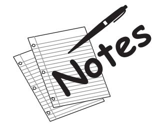
It takes about one minute to read 15 double-spaced typewritten pages aloud, or about 4 s/line. Do you think this is accurate? Why or why not?

Extensions in Mathematics:

1. BLM 9 asks students to identify cases in which the time taken can or cannot be accurately determined. Students should also be able to say whether an estimate is appropriate. For example, 1(a) can be determined very accurately if you know the moment of your birth, but an estimated answer is certainly appropriate. If you have a good memory, 2(a) can be answered accurately, but is this any more useful than an estimate?

   Part 2 asks students to suggest how they could determine an accurate answer or an estimate, but does not ask them to carry this out. Part 3 asks them to select one particular problem and, using their method as outlined in their response to Part 2, determine a "reasonably accurate answer" for this problem. What a "reasonably accurate" answer might be should be discussed with the students.

Activity 3: Time and Time Again



Students should be allowed to use calculators for the necessary computation in Part 3. Otherwise they can get "bogged down" in the computation and lose sight of the problem.

Use of Technology

This BLM can be used in a variety of ways. For example:

(i) Assign #1 to one group, #2 to another group, and #3 to a third group.

(ii) Use # 1, Parts 1, 2, and 3 one day, # 2, Parts 1, 2, and 3 another day, and #3, Parts 1, 2, and 3 a third day.

(iii) Discuss one of the problems with the class, then have them work Parts 2 and 3 in their groups, for that problem.

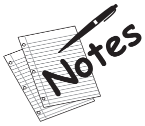
Family Activities:

1. Have students test family members to see who is best at estimating one minute using the technique described at the beginning of this Activity.

Other Resources

For additional ideas, see annotated Other Resources list on page 48, numbered as below.

8. Using a Lifeline to Give Rational Numbers a Personal Touch by W. Weidemann and A. Mikovch.
10. 'Working Cotton': Toward an Understanding of Time by E. Monroe, M. Orme, and L. Erickson.



Activity 4: How Big?

Focus of Activity:

- Estimating length, area, and volume

What to Assess:

- Clarity of descriptions of patterns
- Use of mathematical language

Preparation:

- Provide metre sticks or tapes and string.
- Make copies of BLM 10 for each pair/group.
- Make copies of BLM 11 (optional).
- Provide newspaper (optional).

Activity:

Begin by asking students to look at their classroom. If you are in an open area, establish imaginary walls and have 2 or 3 students stand in positions to provide a visual reference for the walls.

Problem Solving

Ask students to think about the following problem:

Is it farther (i) from the front of the room to the back,
or (ii) from one side of the room to the other,
or (iii) from the floor to the ceiling?

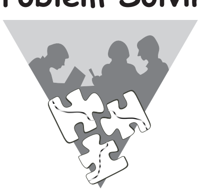
Give students time to come to some decision before asking them to give their answers and their methods for arriving at those answers. Keep a tally of answers, showing how many students choose each of (i), (ii), or (iii).

Communication

Have students think about how they could determine which distance is the greatest, then ask for suggestions. Suggestions may range from "Measure with a ruler." to "Measure one floor tile, (one ceiling tile, one cement wall block), and count them to find which distance is greatest." to "Find the plans of the school, and read the measurements from them."

Have different groups of students measure each distance and record their findings. Round to the nearest 10 or 20 centimetres. Compare answers from different groups. Compare the measurements for height, width, and length of the room and determine which measure is the greatest.

Problem Solving



Students will need these measurements for BLM 10 in which they will compare the size of the classroom to various animals. Distribute copies of BLM 10 to each pair/group. Read over problem 1. Encourage students to think about dinosaur pictures or models they have seen to decide whether or not they think one would fit in the classroom. Explain that they should estimate answers to all the problems on the page, but they should try to make sensible estimations, basing them on their own knowledge.

Activity 4: How Big?

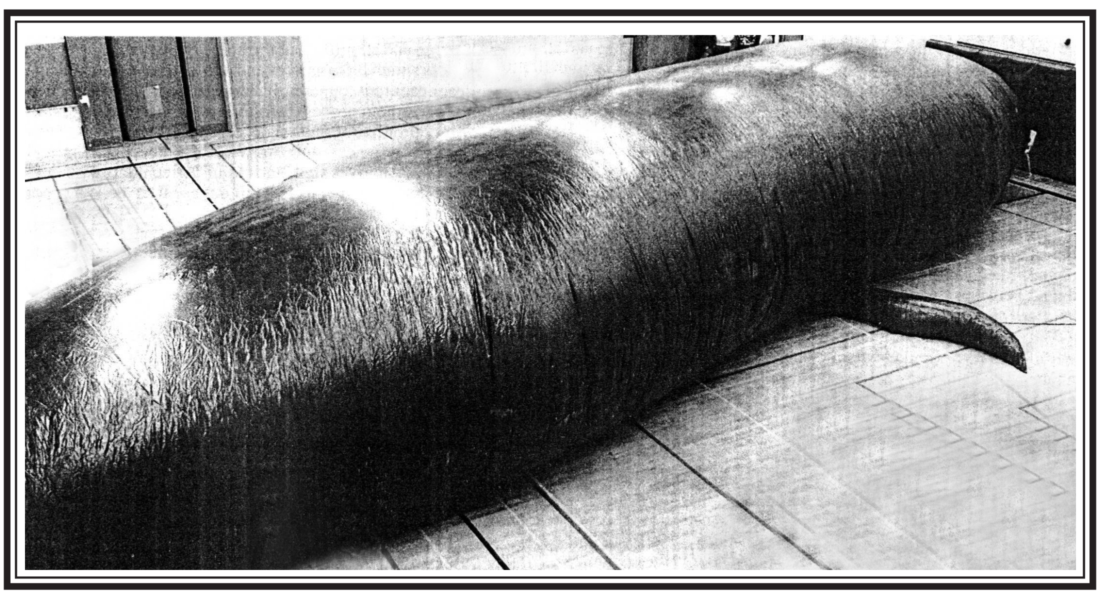
BLM 11 gives data about several large animals. You may wish to distribute this to students to help them with BLM 10 or give measurements as students appear to need them. You may wish to have students do some research in a library or on the internet to find more data about large animals such as a yak, reindeer, or anaconda.

Ask students why measurements are given as "about" or "up to" so many metres (#1 BLM 10). Students should be aware that all these measurements are estimates. Ask students why exact measurements could not be given. This may lead to a discussion of 'average' and its usefulness.

Students who believe that dinosaurs were all huge may be surprised by the measurements given for, say, the ornitholestes. Compare the size of the plant-eaters with the size of the meat-eaters. Have students suggest reasons why the meat-eaters were not the largest dinosaurs. Students may suggest that the meat-eaters had to be able to run fast to catch their food.

Students may also be surprised that the blue whale is larger than any dinosaur that ever lived, and that the African elephant and white rhinoceros, the two largest land mammals of today, are considerably smaller than the blue whale.

The picture below shows a 24-metre long inflatable blue whale in the gym at Sydenham High School, Sydenham, Ontario. The whale is from Canadian Wildlife Federation. The photo is from "International Wildlife", the CWF magazine, May-June, 1991.



There are some problems for which students will need to estimate measurements not given. For example, to estimate the number of baby or adult alligators (#3) that will fit on the floor of the classroom requires an estimate of width as well as the given measure of length.

Notes

Use of Technology

The Goliath Frog grows to be 80 cm long. It weighs up to 15 kg.

R of T

To estimate the weight of a gray whale, figure 30 kg per centimetre of length. Use this to complete the "whale" chart on BLM 11.

Notes

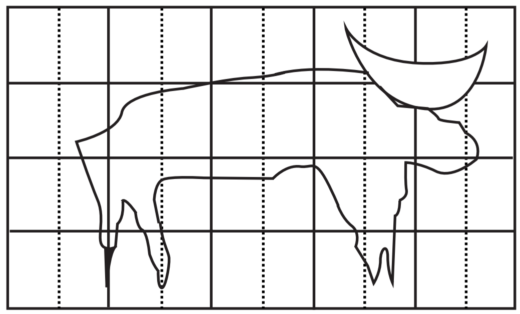
R of T
The distance between an alligators eyes is $\frac{1}{12}$ its total length. Is there a similar R of T for people? for cats? for dogs?

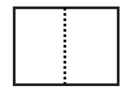
The total distance a worker bee can fly in its lifetime is about 800 km. The flight muscles contain a fixed quantity of glycogen. When this is used up, the bee dies.

Activity 4: How Big?

Extensions in Mathematics:

1. Have students tape together several newspaper pages to make a piece of paper big enough to illustrate one of the animals. Students should draw just a rough outline of the animal chosen. For example, a moose diagram will need 20 sheets of newspaper — that is, 40 pages.





One 2-page spread

Even if the animals are not sketched, figuring out how many pages of newspaper are needed, and taping these together, will give the students a better idea of the sizes of some of these animals.

Cross Curricular Activities:

1. Explore science and other reference books to find sizes of other animals.
   For example, What sizes are various types of elk or bears?
   Find animals that are so small they would fit comfortably on your hand.

   See "Solutions and Notes" for some data on bears.

2. Explore migration distances of birds and other animals. For example, Arctic terns fly from the Arctic to the Antarctic and back every year. A one-way trip is 14,400 km. The Monarch butterfly travels from eastern Canada and the U.S. to Mexico, a trip of 4000 km, taking 2 months to complete the trip. Migrating animals include several, such as caribou, wildebeest, and whales, that follow availability of food.

Other Resources:

For additional ideas, see annotated Other Resources list on page 48, numbered as below.

7. Mousemaze Tournament: Connecting Geometry and Measurement.
9. How Big Was the Cat?, by L.E. Sakshaug and K.A. Wohlhuter.

Activity 5: Junk Pentathlon

Focus of Activity:

- Estimating and measuring distance, area, and time.

What to Assess:

- Facility with measurement
- Reasonableness of estimates

Preparation:

- Provide cotton balls (the type that is often found in the top of a pill bottle will do), rulers, large buttons (at least 2 cm wide), marbles or small balls, rice (coloured with food colouring), newspaper and metre tape/stick.
- Make copies of BLMs 12, 13, and 14 for each team.

Activity:

Ask students if they know what a pentathlon is. Explain that when the pentathlon was first held as an Olympic event in 1912, athletes competed in five different events, gaining points for each. The overall winner was the one who had the most points after all five events. The five events were horse-riding, fencing, pistol-shooting, swimming, and cross-country running. In 1948 a winter pentathlon was introduced. It replaced the swimming with a 3-km downhill race on skis, and the running with a 10-km race on cross-country skis.

You may wish to point out that the prefix "penta-" means "five" as in pentagon (either the mathematical figure or the building in Washington, D.C.).

Tell students that they will be participating in a pentathlon, but the events will be quite different from the Olympics, and they will be competing as teams, not individuals. Distribute copies of BLMs 12, 13 and 14 and read them over with the students. Have students identify an appropriate unit of measurement for each event, and give reasons for their choices. Once the class has come to a decision on this issue, have students record the units at the top of the second, third, and fourth columns in the charts on BLM 14. Tell students that their scores will depend on their estimating skills.

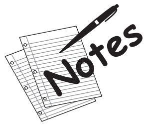
Rather than have all students do all events at the same time, teams could begin at different events. For example, teams 1 and 2 could begin with Event 1, Teams 3 and 4 could begin with Event 2, and so on. Each team should cycle through all five events. This organization is particularly helpful if equipment is limited.

Students should write the names of their team members in the first column of each chart before beginning. If more than 4 spaces are needed, students should add them or record on a separate piece of paper. Alternatively, have only 4 students from a group participate, with a different 4 students participating in each event, and the non-participating student(s) taking turns acting as recorders, timers, etc.

Notes

On snow and ice, otters take 3 jumps and then slide for up to 7 m.

The smallest humming-bird weighs $\frac{1}{2}$ the weight of a new penny.



The largest rain drop ever recorded, 8 mm across, fell in Hawaii.

Assessment

The record for the largest hailstone is 1 kg. If tell April 14, 1986 in Bangladesh.

Problem Solving

R of T

The distance from your elbow to your wrist equals the length of your foot.

Is this true for you?

Activity 5: Junk Pentathlon

A chart such as the one below should be available on blackboard, or chart paper, on which students should record their team totals after each event.

Team	Event 1	Event 2	Event 3	Event 4	Event 5
1					
2					

As students are at work observe them to see if later members of each team give estimates that are closer to the actual measure than those who tried the activity earlier. That is, are students learning from the examples they have seen?

Students in each team will have to agree, for Event 2, just when a button stops spinning. Is it when the botton is no longer vertical or when the button has come to rest? The interpretation can be different for each team without affecting the scores, since it is the difference between estimated and measured times that gives the score.

When all scores have been recorded, ask students to identify the winning team for Event 1. Students should realize that the winner will be the one with the lowest score, not the team with the highest. The winning team for each event should be identified, and then the overall winning team.

Extensions in Mathematics:

1. Have students devise a new event for the pentathlon. The event should meet certain conditions. It must be safe, it must use easily available equipment, it should take only a short time for each person to participate, and so on.

   Some students may simply use one of the given events, with changes in equipment. For example, Event 2 might become Coin Spinning; Event 3 might use a rubber ball instead of a marble. Ask students if they expect different results with different equipment and why.

   For other suggested events, see "Solutions and Notes".

Cross-curricular Activities:

1. Have students explore Olympic records and events. Is the pentathlon as described above still a part of the Olympics? Is there a different pentathlon? Is there a pentathlon in both the Summer and Winter Games? Are there other "-athlons"? (e.g., decathlon [10 events], biathlon [2 events]). What is the record for each of the events?

Family Activities:

1. Suggest that students try some of the "junk pentathlon" events at home with family members.
2. On a Parents' Night, have some students demonstrate the "junk pentathlon" events for/with parents and other care-givers.

Activity 5: Junk Pentathlon

Other Resources:

For additional ideas, see annotated Other Resources list on page 48, numbered as below.

6. What Is A “Good Guess”, Anyway? by F. Kuwahara Lang.

BLM 1: Toothpicks and Decimetres

1. Measure each of the following to the nearest half-toothpick:

   (a) the width of this page ___________________

   (b) the length of this page___________________

   (c) the perimeter (distance around) this page___________________

   (d) the height of your chair ___________________

2. Measure each of the items in #1 to the nearest half-paper-clip.

   (a) _______________ (b) _________________

   (c) ________________ (d) _______________

3. At the bottom of this page is a decimetre (10 cm) ruler. Cut it out and measure the items in #1 again, this time to the nearest half-decimetre.

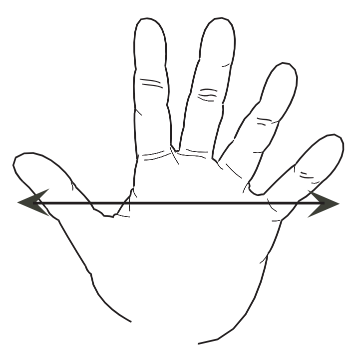
   (a) _______________ (b) _________________

   (c) ________________ (d) _______________

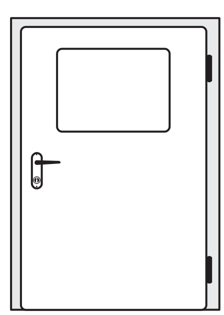
4. Using your decimetre ruler, estimate and then measure the items in the chart below to the nearest half-decimetre.

Item	Estimate	Measurement
(a) the width of your desk		
(b) the width of the classroom door		
(c) your hand span		
(d) the width of a classroom window		
(e) the length of the blackboard		

Hand span



one decimetre

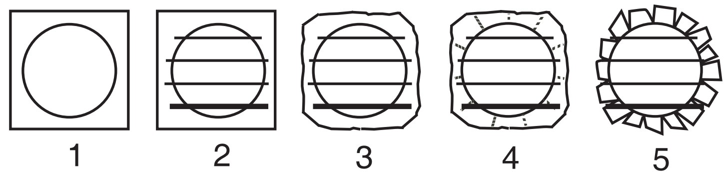


BLM 2: Centimetre Grid

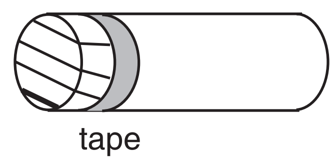
BLM 3: How Tall is That?

A Tubescope is a measuring instrument that will help you estimate the heights of things such as trees or buildings. To make your tubescope you will need a cardboard tube about 10 cm long and a piece of acetate (stiff transparent plastic) big enough to cover the end of the tube.

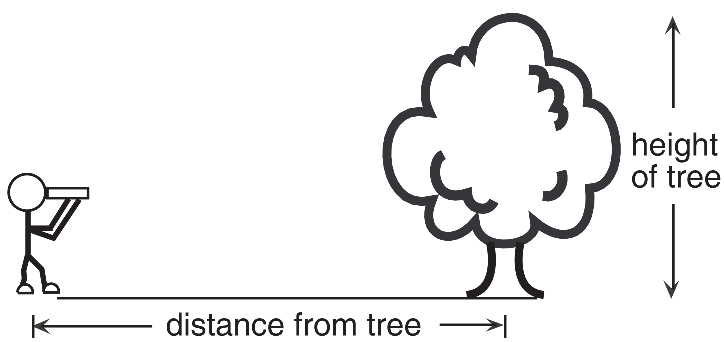
CONSTRUCTION OF A TUBESCOPE:

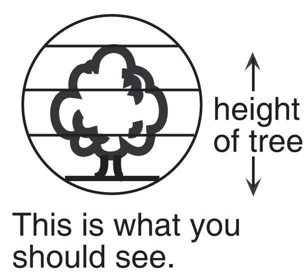


1. Trace around the end of the tube on the piece of acetate (plastic).
2. Draw a thick line across the bottom of the circle. Draw thin lines 1 cm apart over the rest of the circle.
3. Cut the corners off the acetate.
4. Cut from the edge of the acetate in to the circle as shown by the dotted lines. This will make tabs to be taped to the tube.
5. Fold the tabs backwards.
6. Place the acetate circle over the end of the tube and tape the tabs to the outside of the tube as shown below. Masking tape is better than transparent tape for this.



7. To use the "scope" to estimate the height of a tree, look at the tree through the tube, standing far enough back so that the tree is completely visible through the tube.





8. Measure the distance you are from the tree, in metres.
9. Read the height of the tree's 'picture' from the acetate. The tree's "picture" above is about 3.0 cm.
10. To estimate the height of the tree in metres, use the formula:

    height of tree (m) = height of the tree's picture (cm)
    times the distance from you to the tree (m)
    divided by the length of the tube (cm)

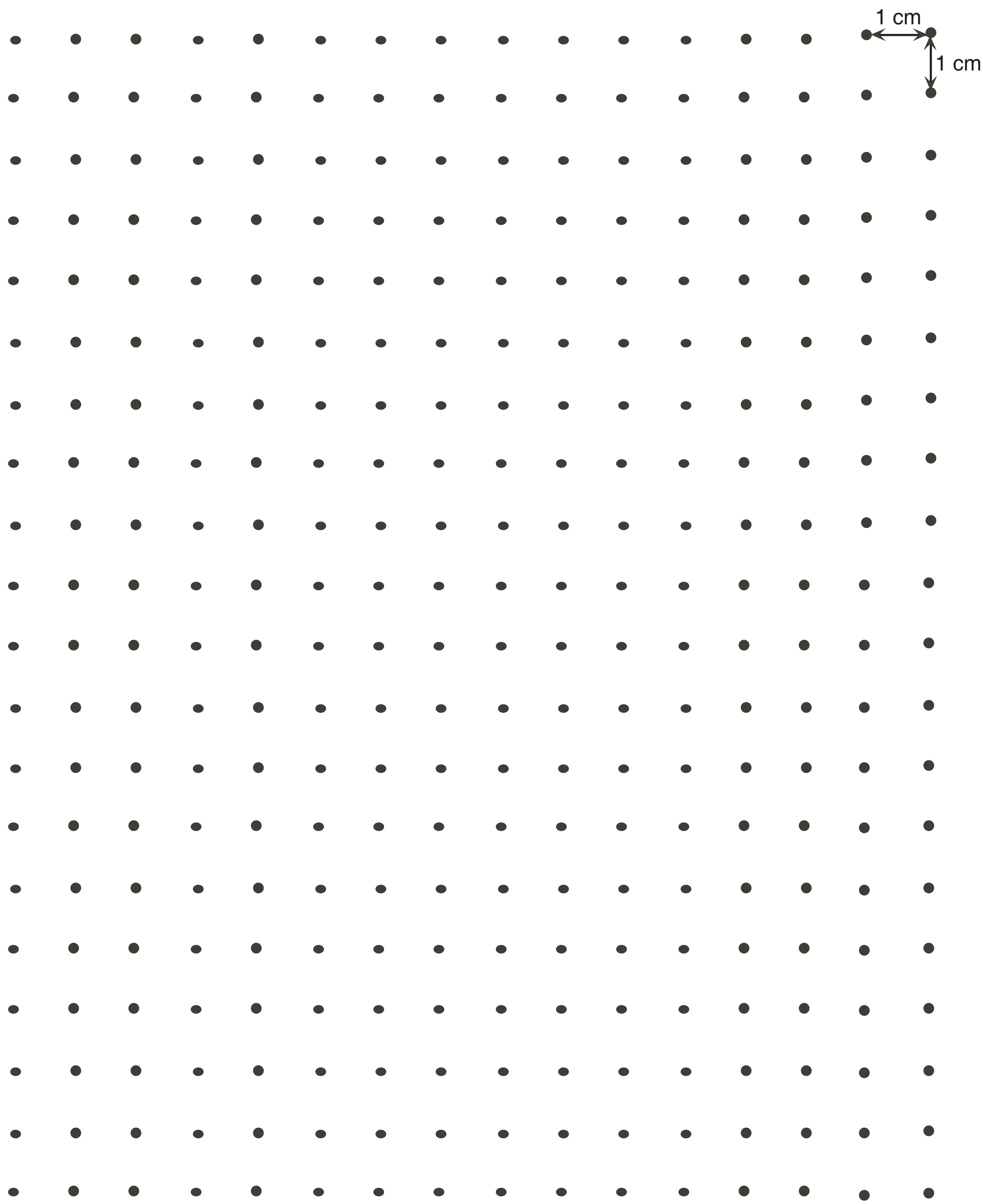
11. Use the Tubescope to estimate the height of
    (a) the school (b) the flagpole (c) a nearby building
12. Are your answers reasonable? Explain.

BLM 4: Dot Paper Areas and Perimeters

Draw your answers on the dot paper below. The dots are 1 cm apart both horizontally and vertically. All rectangles you draw should have their corners on dots.

1. (a) Draw as many rectangles as you can, so that each one has an area of 24 cm^2.
   (b) Determine the perimeter of each.

2. (a) Draw as many rectangles as you can, so that each one has a perimeter of 24 cm.
   (b) Which one has the least area?



BLM 5: The Shrinking Rectangle

For these problems, the perimeter will be in centimetres and the area in square centimetres. Draw your figures on centimetre dot paper.

1. Look at the rectangle below in Figure 1. What is its perimeter? What is its area?

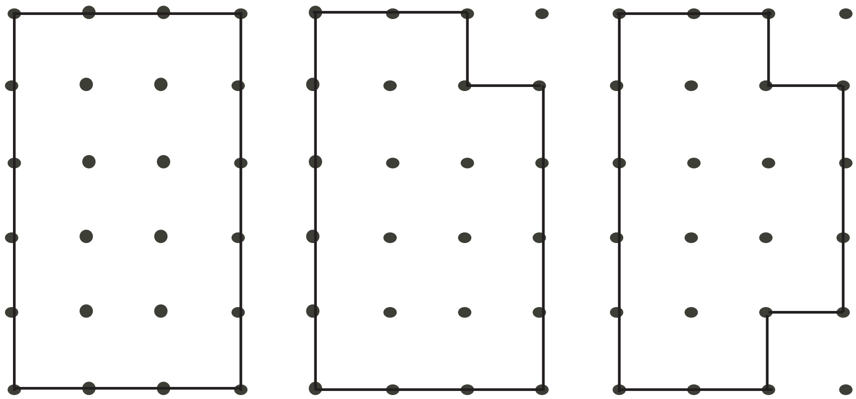


Figure 1 Figure 2 Figure 3

2. Push in one corner of the rectangle as shown in Figure 2. What is the perimeter of the new figure? What is its area?

3. Push in another corner as shown in Figure 3. What is the perimeter of this new figure? What is its area?

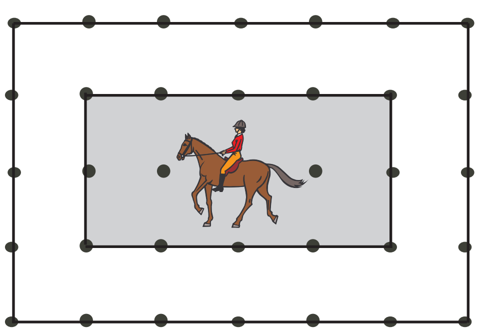
4. Continue the pattern by pushing in another corner, and then another. For each new figure give the perimeter and area.

5. What do you think will happen if you keep pushing in corners as long as you can? Test your theory by drawing the figures. Give the perimeter and area of each one.

A Border Challenge

6. In the diagram below, the frame (white area) of the picture has an area of $16\ \text{cm}^2$. The picture (the shaded part) has an area of $8\ \text{cm}^2$. The shaded (picture) area is half the area of the frame.

Draw a smaller rectangular picture and frame so that the picture area is one-third the area of the frame.



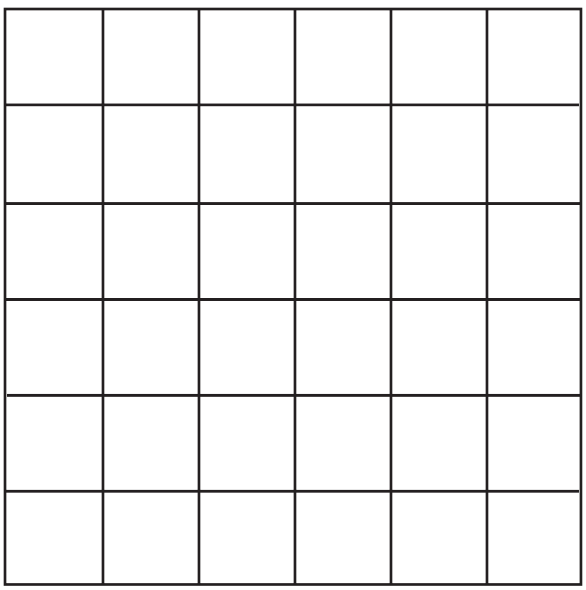
BLM 6: Centimetre Dot Paper

BLM 7: Area Game

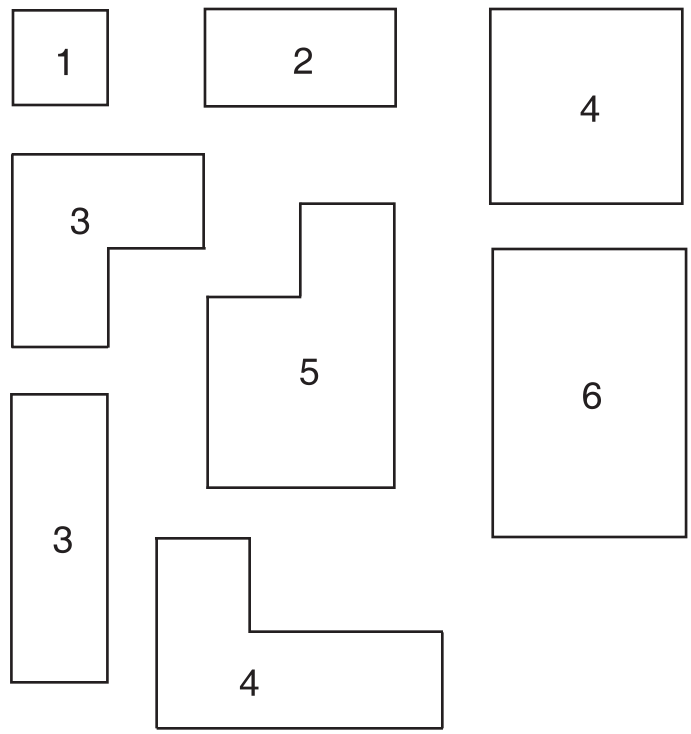
Before you can play the game you need to make the playing pieces and game board. Draw these on either Centimetre Dot Paper or Centimetre Grid Paper, and cut them out. Each player needs a Game Board. Cut five or six of each Playing Piece.

You will also need 2 dice or a spinner with the numbers 1 to 6.

Game Board



Playing Pieces



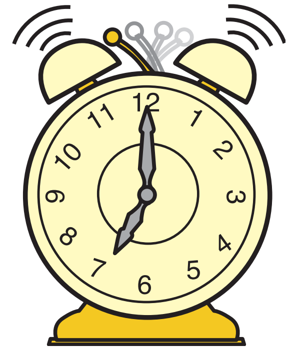
Rules for playing:
1. Roll the two dice. Suppose you roll 2 and 6. You may take either a '2' piece or a '6' piece and place it on your board.
2. Another player rolls the dice and places a piece on his/her board.
3. Once a piece has been played on a board, it cannot be moved.
4. Suppose you roll a double, such as 3, 3. You must take a '3' piece, but you get another turn because you rolled a double.
5. If you roll numbers of pieces you have no room for, you miss a turn.
6. The first player to fill a board completely with no overlaps and no pieces sticking outside the board wins the game.

Variations:
1. On your turn, instead of rolling the dice you may exchange one of the pieces on your board for any available piece.
2. Try playing the game on a board with more than 36 squares. You may need to make more playing pieces.
3. Try playing the game on a board that is not square. What effect does this have on the game? Why?
4. Try playing with a partner on only one board. What effect does this have on the game?

A Challenge:
Find different ways to cover the board so that no two identical pieces are touching each other, except at a corner.

BLM 8: How Long is One Minute?

Part 1: Estimate an answer for each of the following. Tell why you think your estimate is reasonable.



1. How many times can you write your name neatly in one minute?

2. How high can you count in one minute?

3. How far can you walk in one minute?

4. How many times can you take your shoes off and put them on again in one minute?

5. With how many people can you shake hands in one minute?

6. How many times can you put on your coat and take it off in one minute? Don't forget to do up the zippers or buttons.

Part 2: Using a clock with a second hand, calculate answers to Problems 1 to 6 as accurately as you can for one person in your group. Use a different person for each problem.

Part 3: Which of the following things do you think you can do in one minute or less? Explain.

7. Stand up and sit down 10 times;

8. Add together all the numbers from 1 to 20;

9. Write out the 5 times table;

10. Look up the word "estimate" in the dictionary;

11. Run around the bases of a baseball diamond;

12. Listen to your favourite piece of music.

Part 4: Using a clock with a second hand, calculate answers to Problems 7 to 12 as accurately as you can for one person in your group. Use a different person for each problem.

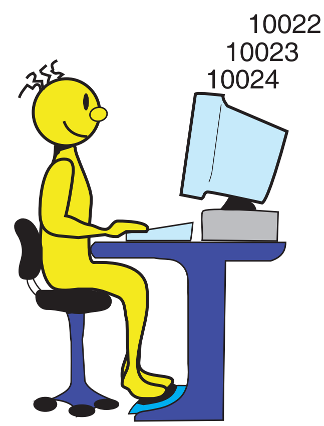
BLM 9: Estimations About You

PART 1: Tell which of the following you can measure accurately, and which you cannot. Give reasons for your answers.

1. The number of hours
   (a) you have been living;
   (b) you are in school each year;
   (c) you have been in school today;
   (d) you have been in school so far in your life.

2. The number of hours
   (a) you watched TV last week;
   (b) you watched TV last year.
   (c) you plan to watch TV during the next 5 days;
   (d) you plan to watch TV in the next year.

3. The length of time in days that it would take you to count to 1 million, if
   (a) you said one number per second without stopping to eat or sleep;
   (b) you said one number per second but stopped to eat and sleep;
   (c) you wrote each number;
   (d) you used a calculator or computer to count.

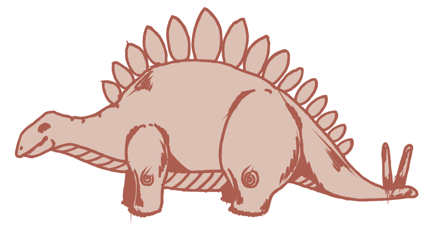


PART 2: Describe a method you could use to give either an accurate answer or an estimate for the problems in #1 or #2 or #3.

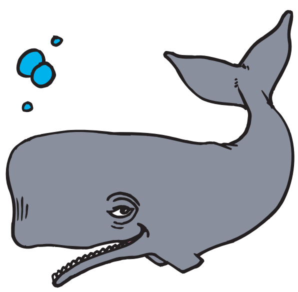
PART 3: Using the method you described in Part 2, give a reasonably accurate answer for one part of problem 1, 2, or 3. Use a calculator to help.

BLM 10: Whales and Elephants

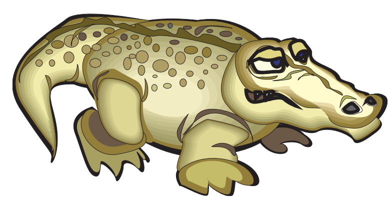
1. (a) Could a dinosaur fit in your classroom? Explain.
   (b) Could a dinosaur fit through the door of your classroom?
   (c) An allosaurus was about 11 m long and 5 m high. Could an allosaurus fit in your classroom?
   (d) An ornitholestes was up to 2 m long and 1 m high. Could an ornitholestes fit in your classroom? How many would fit into your classroom? Explain.



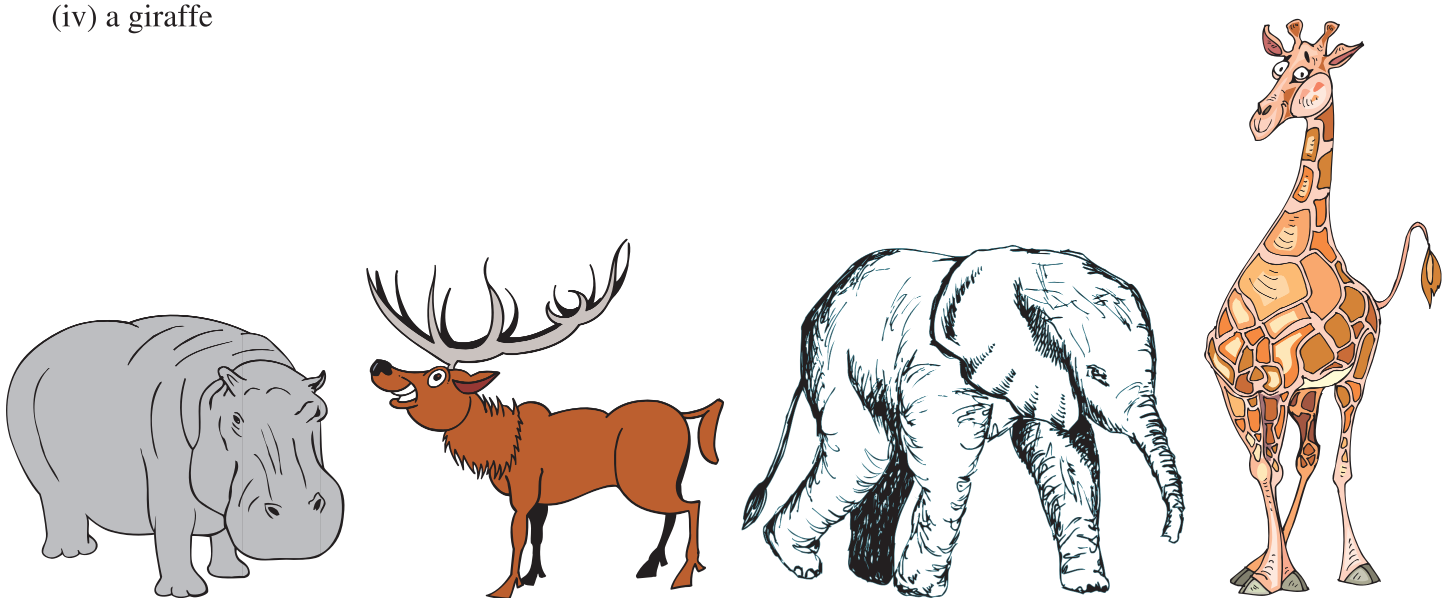
2. The largest whale, the blue whale, is about 33 times as big as an elephant.
   (a) Would an elephant fit in your classroom? Explain.
   (b) Would a blue whale? Explain.
   (c) Would any other kind of whale? Explain.



3. Baby alligators are about 23 cm long when they hatch. They grow to about 3 m in length.
   (a) How many baby alligators would fit on the floor of your classroom? Explain.
   (b) How many adult alligators would fit on the floor of your classroom? Explain.



4. Which of the following large animals do you think would fit in your classroom? Explain why you think so.
   (i) a hippopotamus
   (ii) a rhinoceros
   (iii) a moose
   (iv) a giraffe



BLM 11: Data Sheet

Dinosaurs	Length	Height	Dinosaurs	Length	Height
Allosaurus	11 m	5 m	Brachiosaurus	23 m	12 m
Diplodocus	27 m	7 m	Ornitholestes	2 m	1 m
Segisaurus	1 m	30 cm	Stegosaurus	7 m	4 m
Triceratops	9 m	6 m	Tyrannosaurus	14 m	6 m

Of these dinosaurs listed above, Brachiosaurus, Diplodocus, Stegosaurus, and Triceratops were plant-eaters; the others were meat-eaters.

Rhinoceroses	Height	Weight	Rhinoceroses	Height	Weight
White	1.8 m	2000 kg	Black	1.5 m	1400 kg
Indian	1.65 m	1800 kg	Javan	1.5 m	1600 kg
Sumatran	1.35 m	900 kg			

The White Rhino is the second largest land mammal in the world. The African elephant (3.9 m tall, 6500 kg) is the largest.

Whales	Length	Weight	Whales	Length	Weight
Blue	30 m	90 000 kg	Killer	9 m	5200 kg
Sperm	18 m	55 000 kg	Narwhal	5 m	1600 kg
Gray		30 000 kg	Minke	9 m	7200 kg

The blue whale is the largest creature ever to live on earth.

Other large animals:

The Australian salt-water crocodile weighs up to 900 kg and is up to 6 m long.
The Nile crocodile weighs up to 900 kg and is up to 5.4 m long

A moose is approximately 3 m long, 2.3 m high at the shoulders, and weighs up to 630 kg.

Great white shark: 5.4 m long, 1400 kg
Hippo: 2700 kg, up to 1.4 m high at the shoulders, and up to 3.6 m long
Whale shark: 7.5 m long, 8000 kg
Giraffe: up to 3.6 m at the shoulders, and up to 5.7 m in full height
Nurse shark: 2.4 m long, 160 kg

BLM 12: Junk Pentathlon, Events 1, 2, and 3

EVENT 1: THROWING COTTON

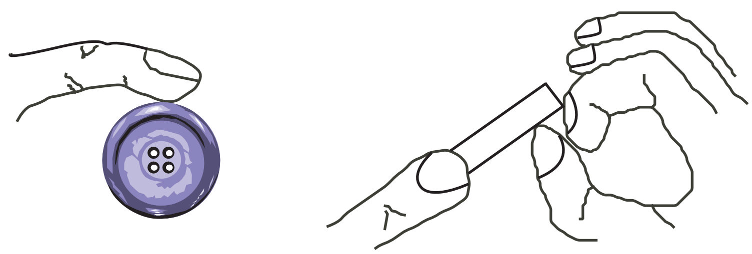
Materials: a cotton ball 3 to 4 cm in diameter, a ruler

Rules:
- Throw the cotton ball. Estimate how far you threw it.
- Write your estimate in the chart.
- Measure the distance you threw it. Write this in the chart.
- Write the difference between your estimate and your throw. This is your score.

EVENT 2: SPIN A BUTTON

Materials: one or two large buttons at least 2 cm wide, clock or watch with a second hand

Rules:
- Estimate how long you can make a button spin.Write this in the chart.
- Spin the button and record your time. Write this in the chart.
- Write the difference between your estimate and your actual time. This is your score.



EVENT 3: MARBLE ROLLING

Materials: a marble or ball, a ruler, a smooth surface near a wall

Rules:
- Estimate how far you think the marble or ball will rebound if you roll it lightly at the wall. Record your estimate in the chart.
- Roll the marble or ball lightly at the wall. Measure the rebound and write it in the chart.
- Write the difference between the estimate and the actual measurement. This is your score.

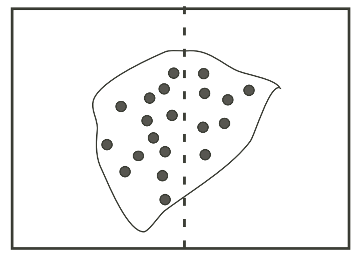
BLM 13: Junk Pentathlon, Events 4 and 5

Event 4 : A Fall of Rice

Materials: 5 ml of rice coloured with food colouring, a sheet of newspaper, a sheet of cm^2 paper

Rules:

- Drop the rice — all at once — from a height of 10 cm above the newspaper. Estimate the area containing rice kernels. Write this in the chart.
- Measure the total area that the rice has landed in as accurately as you can. Use the cm^2 paper to help you. (The diagram shows how to mark the area in which the rice has landed.) Record this in the chart.
- Write the difference between your estimate and your measurement. This is your score.

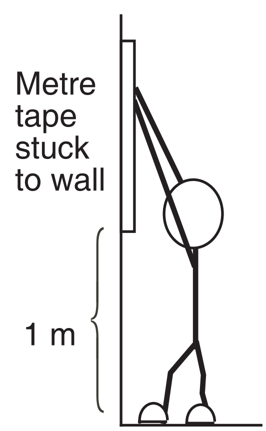


Event 5: The Stretch

Materials: metre stick or tape stuck to the wall 1 m above the floor.

Rules:

- Estimate how tall you can stretch. Write this in the chart.



BLM 14: Score Sheets

Names for Event 1	Estimate in	Actual measurement in	Difference in ....

Names for Event 2	Estimate in	Actual measurement in	Difference in ....

Names for Event 3	Estimate in	Actual measurement in	Difference in ....

Names for Event 4	Estimate in	Actual measurement in	Difference in ....

Names for Event 5	Estimate in	Actual measurement in	Difference in ....

Solutions & Notes

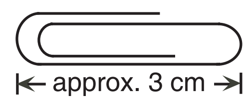
Activity 1: How Long?

BLM 1: Toothpicks and decimeters.

1. Using a (cocktail) toothpick of this length, answers should be:

   (a) 3 toothpicks (b) 4 toothpicks
   (c) 14 toothpicks if they simply add their answers to (a) and (b); or 15 toothpicks if they take into account the "partial toothpicks" of both length and width of page.
   (d) answers will vary.

2. Using a paper clip of this size answers should be:



   (a) $6\frac{1}{2}$ or 7 paper clips (b) $8\frac{1}{2}$ or 9 paper clips
   (c) whether students use their measurements from (a) and (b) or measure all around the page with paper clips, their answers should be 30-32 paper clips
   (d) answers will vary.

Some students may note that the paper clip is approximately half the length of the toothpick and will therefore double their answers to #1 to get answers for #2. This is an opportunity to recognize this as a good estimation technique and to note how such estimations may be less accurate than measuring with the paper clips but that such estimates are still valid.

3. Answers should be:
   (a) 2 decimetres
   (b) 3 decimetres
   (c) 10 decimetres
   (d) answers will vary.

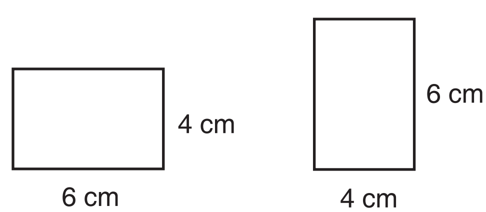
4. Answers will vary.

Activity 2: Areas in Abundance

BLM 4: Dot Paper Areas and Perimeters

1. (a) Possible rectangles with area 24 cm^2 will have the following measurements:
   (i) 6 cm x 4 cm (ii) 3 cm x 8 cm (iii) 2 cm x 12 cm
   (iv) 1 cm x 24 cm (This rectangle will just fit on BLM 6 if students draw 2 extra rows of dots at the bottom of the page.)

   Students may draw identical rectangles in different orientations. Decide with the class whether or not these should be called different rectangles.



   (b) The perimeters of the rectangles listed above are:
   (i) 20 cm (ii) 22 cm (iii) 28 cm (iv) 50 cm

Solutions & Notes

2. (a) Possible rectangles with perimeter 24 cm are:

    (i) 1 cm x 11 cm (ii) 2 cm x 10 cm (iii) 3 cm x 9 cm
    (iv) 4 cm x 8 cm (v) 5 cm x 7 cm (vi) 6 cm x 6 cm

    Students should recognize that the sum of length plus width for each of these rectangles is 12 cm.

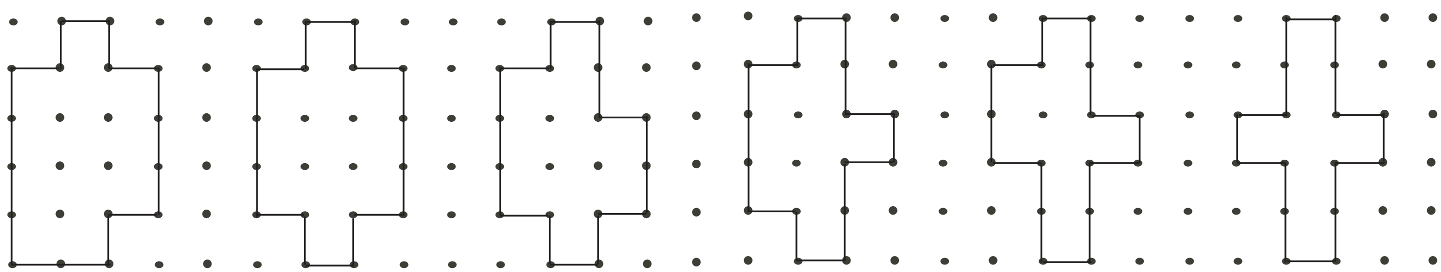
    (b) The areas of the rectangles listed above are:

    (i) 11 cm^2 (ii) 20 cm^2 (iii) 27 cm^2 (iv) 32 cm^2
    (v) 35 cm^2 (vi) 36 cm

BLM 5: The Shrinking Rectangle

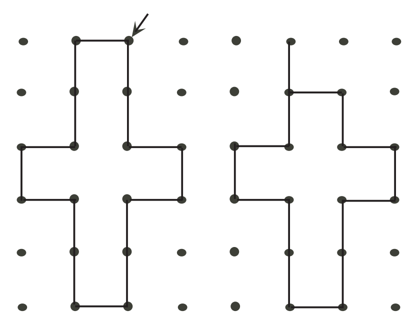
1. Perimeter: 16 cm
   Area: 15 cm^2
2. Perimeter: 16 cm
   Area: 14 cm^2
3. Perimeter: 16 cm
   Area: 13 cm^2

4. & 5. Possible drawings

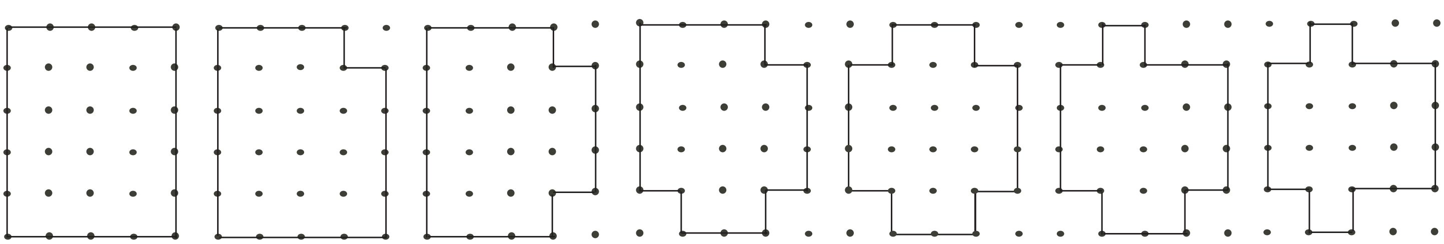


P: 16 cm	P: 16 cm	P: 16 cm	P: 16 cm	P: 16 cm	P: 16 cm
A: 12 cm^2	A: 11 cm^2	A: 10 cm^2	A: 9 cm^2	A: 8 cm^2	A: 7 cm^2

Note that if any more corners are "pushed in" the perimeter will change. For example, if the corner marked by the arrow is pushed in, we get a figure with perimeter of 14 cm, with a 1-cm segment attached.



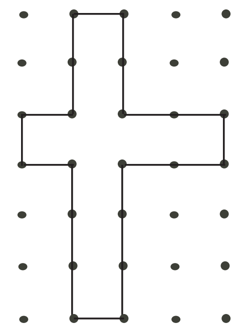
Students might be interested in repeating the problem beginning with a larger rectangle. For example, begin with a 4 cm x 6 cm rectangle:



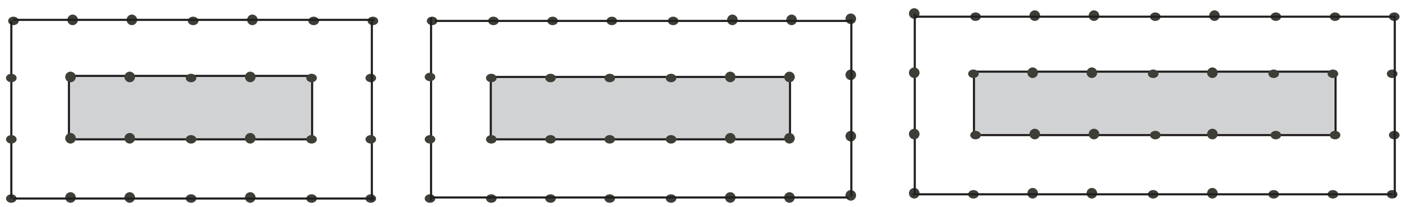
P: 20 cm	P: 20 cm	P: 20 cm	P: 20 cm	P: 20 cm	P: 20 cm	P: 20 cm
A: 24 cm^2	A: 23 cm^2	A: 22 cm^2	A: 21 cm^2	A: 20 cm^2	A: 19 cm^2	A: 18 cm^2

Solutions & Notes

While maintaining the perimeter, this rectangle can be reduced to a figure with area 9 cm^2, although it may not have the configuration shown here.

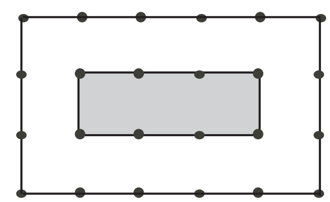


6. If students look for a pattern, they may find the following sequence:



Frame: 14 cm^2	16 cm^2	18 cm^2
Picture: 4 cm^2	5 cm^2	6 cm^2

While trying to solve this problem, students may come up with drawings showing different whole-number ratios. For example, in the drawing on the right, the area of the frame is 4 times the area of the 'picture'.



Activity 3: Time and Time Again

BLM 8: How Long Is One Minute?

Answers will vary.

Students may interpret the questions in different ways. For example, #2 could be interpreted as counting by 2s or 10s or some other way. Estimates based on these assumptions should be accepted if they are reasonable.

BLM 9: Estimations About You

3. (d) Some four-function calculators (i.e., not "scientific" calculators) can be used to 'count' by following the sequence of key strokes: 1 + = = = = = and so on. Other calculators need the following sequence: 0 + 1 = = = =. The automatic memory of the calculator will add '1' for each time '=' is pressed. Students can determine how many such key strokes are possible in one minute and determine how long it would take to count to one million this way. Normally, the speed will not depend on the size of the number. That is, the display late in the counting (999 991, 999 992, 999 993, etc.) will change as rapidly as the display early in the counting (7, 8, 9, 10, etc.).

Solutions & Notes

Activity 4: How Big

BLM 10: Whales and Elephants

Answers will vary.

Cross-Curricular Activities

1. The Himalayan sloth bear weights 115 - 135 kg.
   The brown bear in the taiga region of Russia is almost 3 m tall when standing upright.
   The male polar bear weights 68 kg; it needs 11 kg of food a day on average, though it can eat up to 68 kg in 30 min. of feeding.

Activity 5: Junk Pentathlon

Extension 1:
Other events could include:

- The Paper Plate Discus Throw, in which students throw a paper plate like a discus (or like a frisbee) and estimate, then measure, the distance thrown.
- The Marble Grab in which students reach into a container with one hand and grab as many marbles as they can, then estimate the weight (mass) of the marbles. Materials needed are marbles and a scale.
- The Hot Time event which will need some water kept hot in a thermos. Each team pours a little of the water into a smaller container. Each person then (i) holds a thermometer in his/her hand to determine the temperature of the hand; (ii) feels the hot water and estimates how much hotter it is than his/her hand; and (iii) measures the temperature of the water.
- The Rubber Glove Elastic Stretch: Cut a strip from the cuff of an old pair of rubber gloves. Make this ‘elastic’ about 1 cm wide. Students estimate how much they will be able to stretch the elastic.

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

"For additional ideas, see annotated Other Resources list on page 48, numbered as below."

1. The Ontario Curriculum, Grades 1-8: Mathematics.

2. *Linking Assessment and Instruction in Mathematics: Junior Years*, Ontario Association of Mathematics Educators/OMCA/OAJE, Moore et al., 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, by Jean Kerr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels.
   Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. *How to Evaluate Progress in Problem Solving*, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Suggested Assessment Strategies

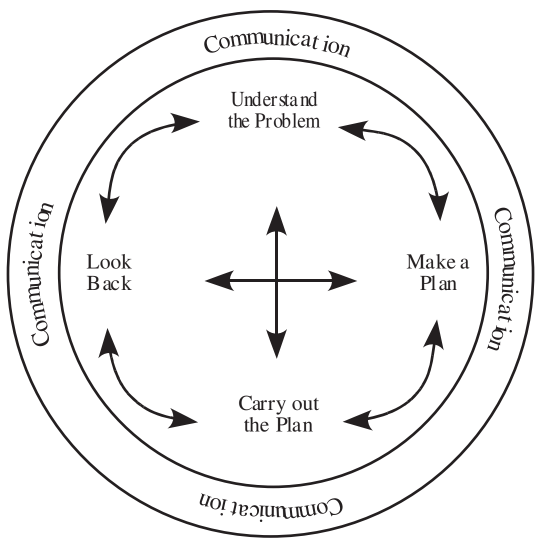
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R's"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;
Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.
Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.
Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
STRATEGIES	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

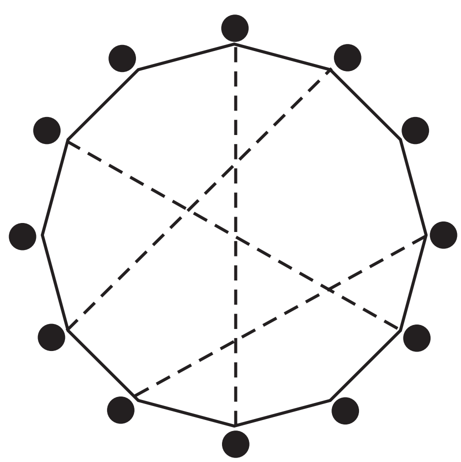
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a “what if...?” problem, such as “What if the question were reversed?”, “What if we had other data?”, “What if we were to show the data on a different type of graph?”.

Suggested Assessment Strategies

Adapting the Rubric

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described' by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

Activity 2, New Shapes From Two Shapes

Strategies and Reasoning: How do students determine all the figures that they can make with the two identical triangles? How well do they justify their answers?

For example,

- The "Limited" student proceeds using trial-and-error, may not find all the answers, and is not sure whether or not he/she has found all possibilities.
- The "Acceptable" student will find the six figures but may use trial-and-error. His/her reasons for the answer may be, "We tried different ways of putting the two triangles together but we kept finding the same figures so we think we have them all."
- The "Proficient" student will have a logical strategy such as, "We put each of the pairs of equal sides together in two different ways, so we got six new shapes."

Activity 4, BLM 10

Strategies and Reasoning: How well do students use estimation along with the given data to answer the questions? How well can they justify their methods and results?

For example,

- The "Limited" student is able to deal with questions involving known measurements (e.g., #1(c)) but is unable to decide on a strategy for more open questions (e.g., #1(d) How many would fit into your classroom?). The student needs to help identifying and determining needed bits of information (e.g., for #3 they need to know/estimate the area taken by a baby alligator).
- The "Acceptable" student may be able to identify the types of data needed for each problem, but may not be sure how to go about finding these.
- The "Proficient" student will not only recognize that estimates are needed but will use good estimating techniques (e.g., "I think a baby alligator is about as long as this sheet of paper. It's probably half as wide. So let's figure out how many pieces of paper this size we would fit on the floor of the classroom.")

Other Resources

1. *Principles and Standards for School Mathematics*, Jean Carpenter and Sheila Gorg (Ed.), 2000, NCTM
Includes a general discussion of measurement expectations for each grade level, and how to help students achieve them. The Grade 6-8 section has an intriguing suggestion for an activity dealing with relative sizes, based on Shel Silverstein's poem 'One Inch Tall'.

2. *Measurement, Addenda Series, Grades 5-8*, Grances R. Curcio (Ed.), 1994, NCTM
This booklet contains problems for grades 5-8 dealing with measurement and estimation. Topics include making, and assessing the validity of, estimates of mass, length, area, volume, capacity, selecting appropriate units of measurement, using a variety of measurement tools, graphing data, and developing formulas based on observations. There is a wide variety of excellent applications to real-world problems.

3. "Addenda Series, Grades K-6: Geometry and Spatial Sense" by Lorna Morrow and John Del Grande, NCTM, 1993.
This book includes detailed lessons for each grade from K to 6, emphasizing manipulatives. BLMs are provided.

4. "By the Unit or Square Unit?" *Mathematics Teaching in the Middle School*, B.B. Ferrer, Nov. 2001, pp 132-137
The article describes teachers' ways of helping students understand perimeter and area and any relationships between them. For example, students were presented with the problem: "For the school carnival, sponsors will pay for advertising their products on signs. How can the school make the most money: by charging the sponsors by the square unit of area, or by measuring the perimeter of the signs and charging by the linear unit?" Both regular and irregular shapes (e.g., pentominoes, students' footprints) are measured, using grid paper and string, bringing out some of the less intuitive aspects of how perimeter relates to area.

5. "Rules of Thumb" and "Rules of Thumb 2", Tom Parker, 1983 and 1987, Houghton Mifflin Company, Boston, ISBN 0-395-34642-8 (pbk.) and 0-395-42955-2 (pbk.)
These two books contain, between them, 1826 "rules-of-thumb" on almost any topic you can think of (e.g., body measurements, children, distance, travel, and weather). The index helps one locate any one of these topics. The rules-of-thumb are presented as they were submitted and sometimes contradict each other. Parker makes no claims for their validity. One good problem for students would be to select a rule-of-thumb and have students devise an experiment to test its validity.

6. "What Is A "Good Guess", Anyway?", by Frances Kuwahara Lang, pp 462-466, *Teaching Children Mathematics*, April 2001, NCTM
This theoretical discussion analyzes the strategies children use in estimating and gives examples of how teachers can help them develop and refine their estimation and measurement skills

Other Resources

7. "Mousemaze Tournament: Connecting Geometry and Measurement" by Shirley Curtis, pp 504-509, *Teaching Children Mathematics*, May 2001, NCTM
   Students are challenged to collaboratively design and construct mouse mazes, based on student-generated criteria, and then participate in a tournament for which they also develop the rules. An open-ended activity which stimulates a high degree of involvement, and appeals to students of varying levels of mathematical experience.

8. "Using a Lifeline to Give Rational Numbers a Personal Touch" by Wanda Weidemann, Alice Mikovch, and Jane Braddock Hunt, pp 210-215, *Teaching Children Mathematics*, December 2001, NCTM
   A creative activity in which students develop a personal 'time-ruler' for their lives. Using one year as the unit of time prompts the need for representing and manipulating non-integer numbers. An assessment rubric and cross-curricular connections are included.

9. "How Big Was the Cat?" by L.E. Sakshaug and K.A. Wohlhuter, pp 350-351, *Teaching Children Mathematics*, February 2001, NCTM
   This 'problem solver' poses the problem faced by a boy who sees the 'biggest' cat while on a walk in the woods, but has difficulty convincing his family that it really was the 'biggest'. Students are invited to help him justify his claim, stimulating an exploration of comparative methods for estimating size. Variations are also suggested.

10. "'Working Cotton': Toward an Understanding of Time", by Eula Monroe, Michelle Orme, and Lynnette Erickson, pp 475-479, *Teaching Children Mathematics*, April 2002, NCTM
    The article discusses using a picture book and some simple activities to assist students in estimating elapsed time, and developing a feel for the difference between seconds, minutes, and hours, and for how to order sequences of events.

11. Science Is ..., (2nd ed.), by Susan V. Bosak et al, Scholastic Canada Ltd., 1991
    This practical book has an extensive collection of activities/experiments encompassing environmental studies and mathematics, developed with input from students, teachers and parents. While directed at children ages 6-14, these hands-on activities provide fun and ample learning opportunities for all. Permission to copy for classroom use is granted.