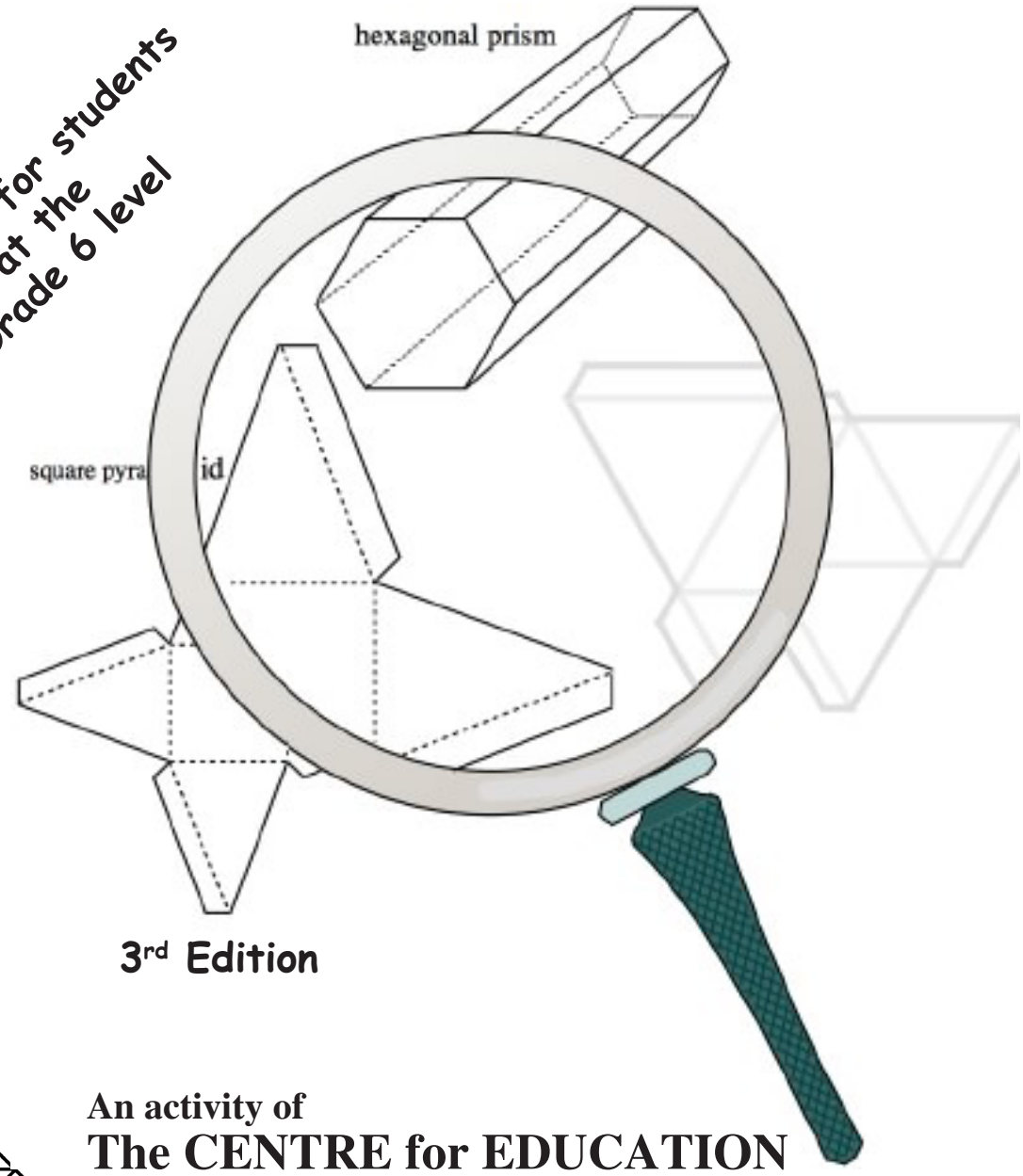
Invitations to Mathematics

Investigations in Geometry:

"Net Quest"

Suggested for students at the Grade 6 level



3rd Edition

An activity of
The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
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Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics, among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will enable them to construct their own mathematical meaning and understanding. This emphasis, plus the extensions and related activities included, provide ample scope for all students' interests and ability levels. Related "Family Math" activities which may be used to involve the students' parents are also suggested.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Outcomes are related to Ministry Curricula for the province of Ontario, but are adaptable to other locales.

Investigations in Geometry is comprised of activities to enhance the students' geometry and spatial sense abilities, as well as creativity and problem-solving skills. Geometry concepts are easily integrated with other subject areas, providing ways to demonstrate that mathematics pervades everyday life.

Due to their nature, geometry activities depend heavily on manipulative materials. Every effort has been made to use materials readily available in most classrooms/schools, and a number of Black-Line Masters (BLMs) are provided to add further manipulatives.

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Table of Contents

Overview

Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and geometry specifically. Some of these beliefs, taken from a variety of sources, and described below.

Geometry is best learned through a combination of active exploration and reasoning, through a cycle of concrete investigations which leads to conjecture, which can then be tested by further concrete investigations.

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Students need a rich learning environment that contains a variety of geometric objects they can manipulate in order to discover the geometric properties of objects and the relationships among them. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures.

The ability to visualize is particularly important in the study of geometry. Students need to be able to draw images in their mind of how figures look, and to be able to manipulate these images mentally.

Students need to develop and use a variety of communication skills in geometry. Geometry uses a formal language to describe objects and their interrelationships and movements in space. In grades 4, 5, 6, however, teaching students the formal language of geometry is not as important as helping them to identify geometric properties and principles and to use pictures or everyday language to explain their observations.

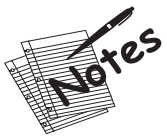
Geometry provides ample opportunity for the development of divergent thinking and creative problem solving, as well as logical thinking ability. Further, exploring problems embedded in 'real-world' settings cultivates the perception that geometry plays a critically significant role in everyday life, rather than being just a set of memorized properties and vocabulary.

Essential Content

The activities in this unit focus on concepts in three-dimensional geometry.

During this unit, the student will:

- review properties of two-dimensional and three-dimensional figures;
- identify characteristics of three-dimensional figures;
- use concrete materials to create two-dimensional and three-dimensional figures;
- apply his/her understanding of two-dimensional figures and three-dimensional figures to a real life situation;
- identify examples of symmetry;
- solve problems using logic and the manipulation of three-dimensional figures;
- utilize spatial abilities.



Curriculum Connections

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Nets of Cubes**	• constructing cubes from given nets • designing different nets for cubes	• draw and construct three-dimensional geometric figures from nets • design nets of cubes and pyramids using grid and isometric paper
Activity 2 **Constructing Three-Dimensional Figures from Nets**	• constructing prisms, pyramids, and other 3D figures from nets • use mathematical terms to describe the nets and the 3D figures	• draw and construct three-dimensional geometric figures from nets • use mathematical language to describe geometric ideas
Activity 3 **Recognizing Nets**	• identifying nets for a given 3D figure or drawing of a 3D figure • using mathematical terms in justifying their choices	• identify nets for a variety of polyhedra from drawings by visualizing the two-dimensional faces of the three-dimensional figures
Activity 4 **The Jolly Postman (2 periods)**	• designing packages that meet post office measurement requirements • creating nets for, and constructing, the packages	• draw and construct three-dimensional geometric figures from nets • recognize and describe in mathematical language the occurrence and application of geometric properties and principles in the everyday world

Overview

ASSESSMENT

Assessment is a process of gathering evidence about a student's knowledge, skills, and values, and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned, but mathematical competence involves such characteristics as the ability to communicate, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. Because of the nature of the activities it is suggested that a variety of types of assessment be used. Suggestions include:

(i) observing students as they work to see if they are applying various concepts; to see if they are working cooperatively; to observe their committment to the tasks;

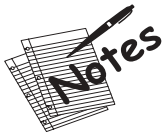
(ii) assessing the completed project to see if instructions have been followed; to see if concepts have been applied correctly; to see if the language of mathematics has been used correctly;

(iii) assessing the students' descriptions of their completed work to see if mathematical language is used correctly; to see if students understand the concepts used;

(iv) providing opportunities for student self-assessment: have students write explanations of their understanding, opinion, or feelings about an activity. One technique is to have them write under the headings What I Did, What I Learned, and How I Felt About It. Students could be asked to write a review of one day's activities or of the whole unit's work.

(v) selecting an exemplary piece of work to be included in a portfolio for assessment purposes or for sharing with parents.

See Suggested Assessment Strategies, page 32, for further discussion and sample rubrics.



Overview

Prerequisites

Some basic vocabulary is assumed, but is not critical. Properties of figures are more easily discussed if students are familiar with such terms as edge, face, vertex. However, these can also be introduced as needed during the activities.

Logos

The following logos, which are located in the margins, identify opportunities for, respectively:

Problem Solving

Communication

Assessment

Use of Technology

Materials

ACTIVITY	MATERIALS
Activity #1 **Nets of Cubes**	• Class set of scissors • Masking Tape • BLMs 1, 2, 3, 4, and 5
Activity #2 **Constructing 3-D Figures**	• One set of three-dimensional figures • Collected three-dimensional objects • Class set of scissors • Masking tape • BLMs 6, 7, 8, and 9
Activity #3 **Recognizing Nets**	• Class set of scissors • Graph paper or geopaper • Masking tape • BLMs 10, 11, 12, 13, and 14
Activity #4 **The Jolly Postman**	• Cardboard • Canada Post brochures • Collected three-dimensional objects • Class set of scissors • Tag manilla paper • Masking tape • Rulers • Measuring tape • Samples of mail • Story book with a mailing theme (e.g., The Jolly Postman...)

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week, students in our classroom will be participating in a Geometry unit titled "Net Quest". The classroom activities will focus on examining, comparing, and constructing three-dimensional figures.

Your child will bring home one or more activities for family participation. These communicate mathematically by encouraging him/her to explain his/her reasoning orally (or in writing).

Thank you for being part of our learning. If geometry is a part of your everyday work or hobbies, please encourage your child to learn about this so that he/she can describe the tasks to his/her classmates.

Sincerely,

Teacher's Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is best to schedule class time for sharing and discussion of results.

Activity 1: Nets Of Cubes

Focus of Activity:

- Visualizing and constructing cubes from given nets
- Designing nets for cubes

What to Assess:

- Correct use of mathematical language
- Use of reasoning in attempting to design a net of a cube
- Willingness to remain at the task and search for alternate solutions
- Ability to adjust an incorrect solution rather than starting over
- The level of cooperative effort

Preparation:

- See the table on page 4 for materials
- Copy BLMs 1, 2 and 3 for each pair of students.
- Provide copies of geopaper - BLM 5.
- Copy BLM 4 for each pair of students (optional).
- Make acetate copies of BLMs 1, 2, and 3 for use with an overhead projector.

Activity:

Distribute BLMs 1 and 2 to pairs of students.

Have the students cut out and fold and tape each net to form a cube. Ask the students to list their observations about cube nets:

- How are the nets alike?

- How are they different?

Problem Solving

Ask each pair of students to use dot paper, BLM 5, to design as many cube nets as they can that are different from those already used. They should design a net, then cut it out and fold it together to check that it will be a cube.

There are 6 other possible nets of cubes. These are given on BLMs 3 and 4. Ask students how they decided that one of their nets would be a cube.

Distribute copies of BLM3 to each pair of students, and pose the following problem:

(i) Before you construct the cube for each net, predict which numbers will be opposite one another.

(ii) Check your predictions by cutting out the nets and constructing the cubes.

Communication

Ask students to explain how they arrived at their predictions. Have them use what they've learned to mark the nets on BLMs 1 and 2 to show opposite faces.

Activity 1: Nets Of Cubes

Ask students for which nets they found this easier.

Usually, nets that involve the simplest folding are easiest for students to visualize being folded into a cube. Thus nets like 3 or 5 will probably be more difficult for students than nets 6, 7 or 8.

Extensions in Mathematics:

1. Distribute copies of BLM4 to each pair of students, and pose the following problem:

   (i) When NET-9 is folded into a cube, vertex (corner) A will meet vertex C. Edge AB will meet edge CB. Write as many matches like this (of vertices, and of edges) as you can for each net.
   (ii) Check your solutions by cutting out the nets and constructing the cubes.

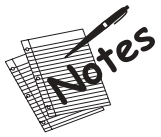
Family Activities:

1. At home, find a cubical box and make as few cuts as possible along the edges in order to open it as a cube net. On dot paper sketch the resulting net.

Other Resources:

For additional ideas, see annotated Other Resources list on page 38, numbered as below.

9. Promoting Visual Imagery in Young Pupils by Erna Yackel and Grayson H. Wheatley in *Arithmetic Teacher*



Comments in italics are explanatory, and need not be conveyed to the students.

Problem Solving

Activity 2: Constructing Three-Dimensional Figures From Nets

Focus of Activity:

- Constructing 3D figures from nets
- Identifying faces, edges, and vertices of three-dimensional figures
- Naming common three-dimensional figures

What to Assess:

- Correct use of geometric terms
- Care in constructing three-dimensional figures
- Level of cooperative effort

Preparation:

- See the table on page 4 for materials
- Prior to these activities, teacher and students should collect a variety of three-dimensional objects such as those on BLM 6.
- Make copies of BLMs 6 and 9 for students.
- Make copies of BLM 7 and BLM 8 on heavy paper for students.
- Make acetate copies of BLMs 6, 7, 8 and 9 for use with the overhead projector.

Activity:

Distribute copies of BLMs 6, 7, and 8 to each group of 3 or 4 students. Compare the nets with the drawings on BLM6. Have each group construct one of each figure.

Problem Solving

Distribute copies of BLM 9 and play a game of "Who Am I?".

Answers are given here for your convenience:

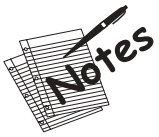
1. cube
2. cone
3. triangular prism
4. triangular pyramid
5. sphere
6. triangular prism
7. rectangular prism
8. square pyramid
9. cylinder
10. any prism

Assessment

Have students compose further "Who Am I?" challenges. Note whether students give sufficient information for accurate identification, whether the clues give too much information, and whether or not students are using mathematical language correctly.

Students should keep BLM 6 and the figures made from BLMs 7 and 8 for use in Activity 4.

Activity 2: Constructing Three-Dimensional Figures From Nets



Extensions in Mathematics:

1. Sort the set of three-dimensional figures using different criteria suggested by the sutdents. You may wish to challenge them by suggesting they try to find a criterion that will result in 2 sets of figures, in 3 sets, in 4 sets.

2. Count edges, faces, and vertices of the three-dimensional figures (excluding sphere, cone, and cylinder). Try to determine the relationship of number of vertices (V) plus number of faces (F) to number of edges (E).

Note: Euler's Theorem (pronounced Oiler) is given as $V + F = E + 2$

Use of Technology

Extension in Mathematics:

1. Have students explore the web for information or software about making "three-dimensional" drawings of the models they have built.

Cross-curricular Activities:

1. Select an empty container (e.g., cereal box) and design a net for that box. Carefully take the box apart to determine the net that was used in its construction. Compare the two. Give reasons the manufacturer might have had for choosing the net used.

Family Activities:

1. Make a list of three-dimensional figures in the home, giving the mathematical name for its shape and the name of the object.
2. Think about which shapes occur most often, and why. Write a possible explanation.
3. Repeat #1 but in a different location — e.g., a supermarket.

Other Resources:

For additional ideas, see annotated Other Resources list on page 38, numbered as below.

2. Addenda Series, Grades K-6: Geometry and spatial Sense

Activity 3: Recognizing Nets

Focus of Activity:

- Identifying nets for a given three-dimensional figure

What to Assess:

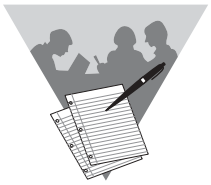
- Ability to identify nets without folding
- Accuracy of construction when necessary
- Use of properties of a figure to identify a net
- Use of mathematical terms, especially when describing techniques used

Preparation:

- See the table on page 4 for materials
- Make copies of BLMs 5, 10, 11 and 12 for each student or pair of students .
- Make copies of BLMs 13 and 14 (optional)

Activity:

Assessment



MATCHING A NET TO A SKETCH OF A FIGURE

Distribute copies of BLMs 10 and 11 to each student or group of students. Explain the task. Students are to examine the sketch at the top of each column. They are then to examine all the nets in the column and decide which are nets of the figure. There will be more than one. Students should attempt to do this mentally. Once their decisions are made, they are free to cut out the nets and try to construct the figure. In this way they are checking their own work. (If the nets prove too small for some students, provide graph paper or geopaper (BLM5) for the students to enlarge the nets before cutting them out.) Observation will indicate which students are comfortable with their visualizing ability and which need more practice.

As students complete the work, you may wish to pause for some discussion of technique. (See next paragraph.) Alternatively, you may wish to give copies of BLM 12 to students who complete BLMs 10 and 11 early.

Communication

DISCUSSION

At some point in the activity, or at the end, stop for discussion. Have students try to explain how they visualized a net "folding", or how they analyzed the component parts to determine which were nets of the given figure. This discussion is important for those with poor spatial abilities since it gives them suggestions for approaching such problems.

Problem Solving

MATCHING NETS AND SKETCHES

Once you have completed the discussion of BLMs 10 and 11, provide all students or pairs of students with copies of BLM 12. This activity is a little more complex, since students must match each incomplete net to one of several figures. There will be more than one incomplete net for each given sketch. Students should be able to match each incomplete net with one of the figures sketched. Once a net is matched with a figure, students should identify the missing piece of the net, and sketch it in. Nets can be cut out and checked. (Only one face of a figure will need to be added to each net.)

Activity 3: Recognizing Nets

DISCUSSION

Further discussion of techniques used by students will be beneficial.

Extensions in Mathematics:

1. Provide copies of BLMs 13 and 14 for each student or pair of students. (Or see alternate suggestion below under "Family Activities".) Students try to complete each net so that the given cube can be constructed. As before, cutting and folding the nets will be the test of accurate solutions.
2. Have students explore the web, their own computer graphics programs, or other resources for examples of different three-dimensional figures used in architecture and make a collection of as many different such figures as possible. They could design and draw buildings using different 3-D figures, or use actual models to duplicate an example they have found.

Problem Solving

Use of Technology

Cross-Curricular Activities:

1. Explore the types of buildings in which certain 3-D figures are used. Compare the architecture of office buildings, castles, cathedrals, temples, mosques, bridges, towers, private homes, etc.

Family Activities:

1. Students could take copies of BLMs 13 and 14 home to work on with their families.

Other Resources:

For additional ideas, see annotated Other Resources list on page 38, numbered as below.

2. Addenda Series, Grades K-6: Geometry and Spatial Sense

Activity 4: The Jolly Postman (Two-periods)

Focus of Activity:

- Design of nets for packages which satisfy Canada Post regulations and are suitable for mailing different sizes and shapes of objects

What to Assess:

- Design plan of the net:
  - produces a package appropriate to the shape of the object
  - follows Canada Post regulations
  - is accurately measured
  - uses knowledge of the properties of three-dimensional figures
- Cardboard construction of the mailing package:
  - uses the net accurately
  - fits the object comfortably
  - is neatly constructed
- The level of cooperative effort

Preparation:

- See the table on page 4 for materials
- Make copies of BLM 15
- Collect Canada Post brochures, and six to ten objects to be mailed, of varying sizes and proportions, including such items as rolled up posters, and a few odd shapes. Create a display area in the classroom for this collection.
- Retrieve overheads of BLMs 6, 7, 8 from Activity 2.

Activity:

Note: *The activity in Extension 1 can replace "The Jolly Postman"; it may however, involve the use of a compass and/or protractor, depending on the accuracy demanded.*

Read the poem "The Jolly Postman or Other People's Mailing Packages" with the students.

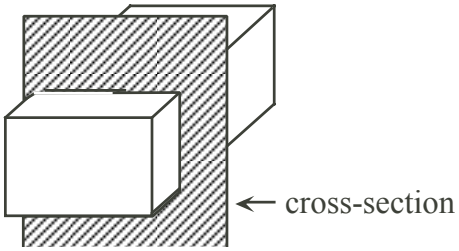
Elicit from the students what features they think might be important when preparing packages for mailing (e.g., weight, length, width, height, fragility, whether its value warrants insurance). Make a list and post it near the display area mentioned above.

Discuss the Canada Post regulations briefly.

Canada Post Priority Courier accepts packages up to a maximum size as given by the rules below.

- Length, width, and height must each be no more than 110 cm.
- Length added to the perimeter of the cross section must not be more than 2 m. The cross section is shown in the diagram.

For more details, see the Canada Post mail brochure.



Activity 4: The Jolly Postman (Two-periods)

Working with the students, create a list of the three-dimensional figures that mailing packages typically resemble. (Rectangular prisms and cylinders are most common).

Working in groups of 2 to 4, students choose an item from the display area and measure the minimal dimensions (length, width, and height) around the object. Referring to BLM 6 and the models made in Activity 2, they decide which three-dimensional figure would best encase the object chosen.

Then they decide on an appropriate net, and use the measurements for their chosen object to assign dimensions to the net, keeping in mind the Canada Post regulations on dimensions. Each student should sketch the design of the net in their math journal, labeling the dimensions of each side carefully, and then compare their sketch with the others in their group.

Note: This may be a good place to end the first period.

Finally, using their design, each group constructs a cardboard mailing package for the object chosen, by drawing and cutting out their net (scaled to fit the package) and then folding and taping it together.

If time permits, the class lists evaluation criteria for the design plan and construction, and students evaluate one another's efforts along with the teacher.

Problem Solving

Assessment

Extensions in Mathematics

1. Pizza Galore:
   Suppose you own a highly specialized pizza franchise which produces not only circular pizzas, but also regular polygons: triangles, quadrilaterasl, pentagons, hexagons and octagons.

   - Working in groups of 2 to 4, students decide how to cut such oddly-shaped pizzas into slices of equal size (possibly using a compass as well as a ruler), and explain their reasoning.

   - Design a net for a box for each type of pizza (pizza plus toppings is about 2 cm thick), using the *least* amount of packaging. Construct the net, and the box (from bristol board or cardboard) and design a logo/advertisement for the cover of the box.

   - Evaluate each other's efforts using agreed upon criteria.

2. Students use CAD (computer-assisted drawing) programs, such as AutoSketch™ or AutoCad™ to design the mailing package. The object can be drawn using CAD software tools which can display the object as it would appear in three dimensions.

Problem Solving

Use of Technology

Activity 4: The Jolly Postman (Two-periods)

Cross-curricular Activities

1. Students examine the samples of mail from the display area and identify what else needs to be done to mail each package. Some students might refer to their own previous experiences, while other students might study the Canada Post brochures. Possible topics to investigate are:
   - decide what type of mailing service is needed (regular, air mail, over-night, same-day, rush);
   - weigh it to determine the cost;
   - write out a return and sender's address using a conventional format;
   - find out the cost of insurance, if the package has significant value;
   - ensure the inside contents do not shake or move around in the mailing package.
   - complete a customs slip for mailing outside Canada

   An example of a mailing address

   Ms. Cinderella Princess
   1 Palace Court
   FAIRY TALE KINGDOM, Imagination

   The students prepare their packages for mailing and return them to the display area when they are finished.

2. Read "The Jolly Postman" by Janet and Allen Ahlberg. Have students create a similar story about mailing packages rather than letters, perhaps a continuation of the poem on page 13. Their story might include a description of the difficulties the characters experience in designing and constructing mailing packages for such unusual items (e.g., Cinderella's glass slipper, Baby Bear's broken chair, or a miniature of the Witch's gingerbread house).
3. Invite someone from the post office to come to the class and discuss the students' parcels with them along with answers to other questions.
4. Bring in stamps from other countries. Discuss stamp design. Have students choose an issue/topic (ecology, science) or a famous Canadian (Wayne Gretzky) and design a stamp portraying that person or idea.

Family Activities

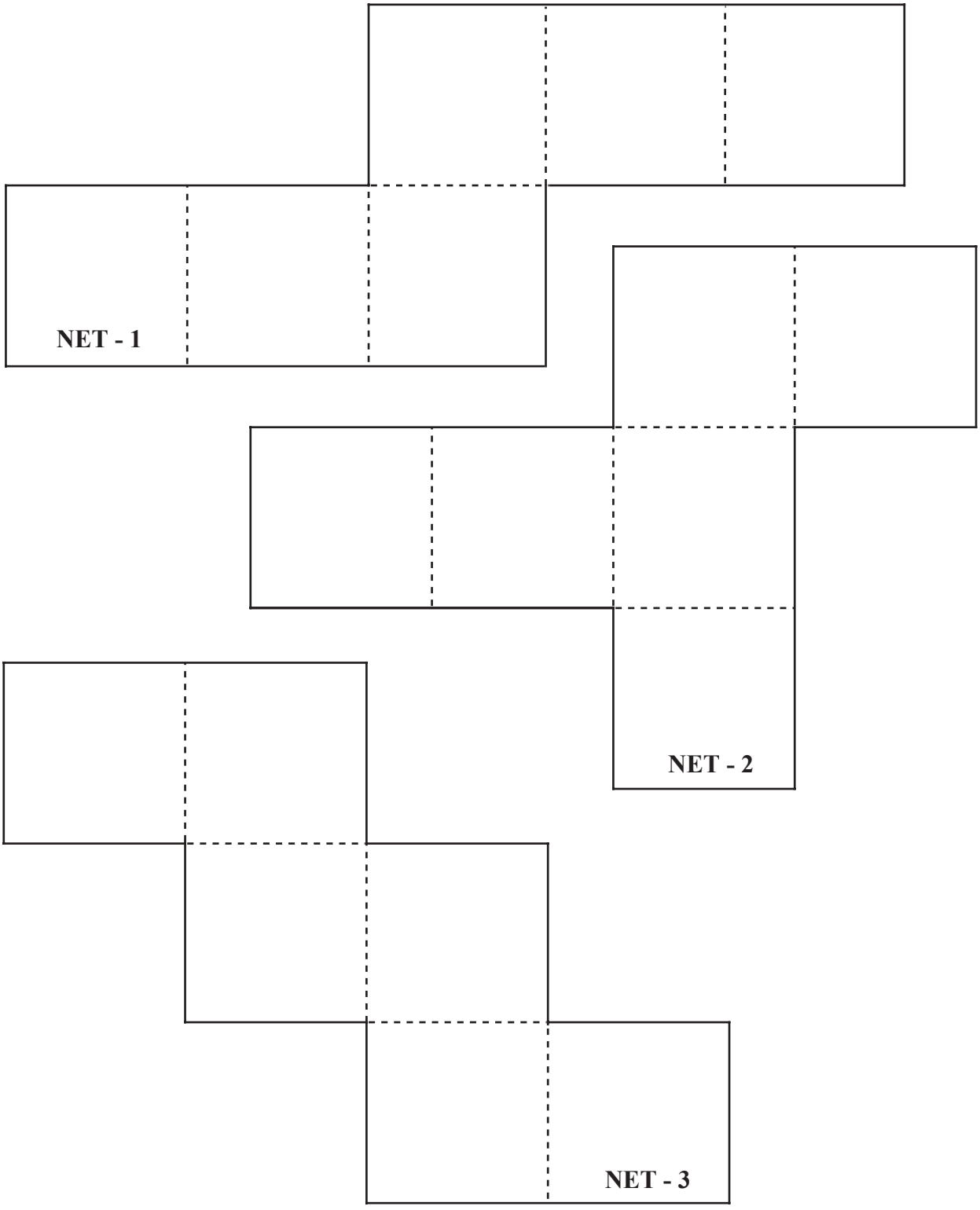
1. Visit your local post office; talk to the people who work there about postal regulations and potential problems with mailing packages.
2. Research the history of Canada Post, and decide how the type of parcels allowed for mail has changed over time. Compare the cost of mailing a letter today with the cost in the students' grandparents time. compare the different services offered. Compare the types of transportation used to deliver mail.

Other Resources:

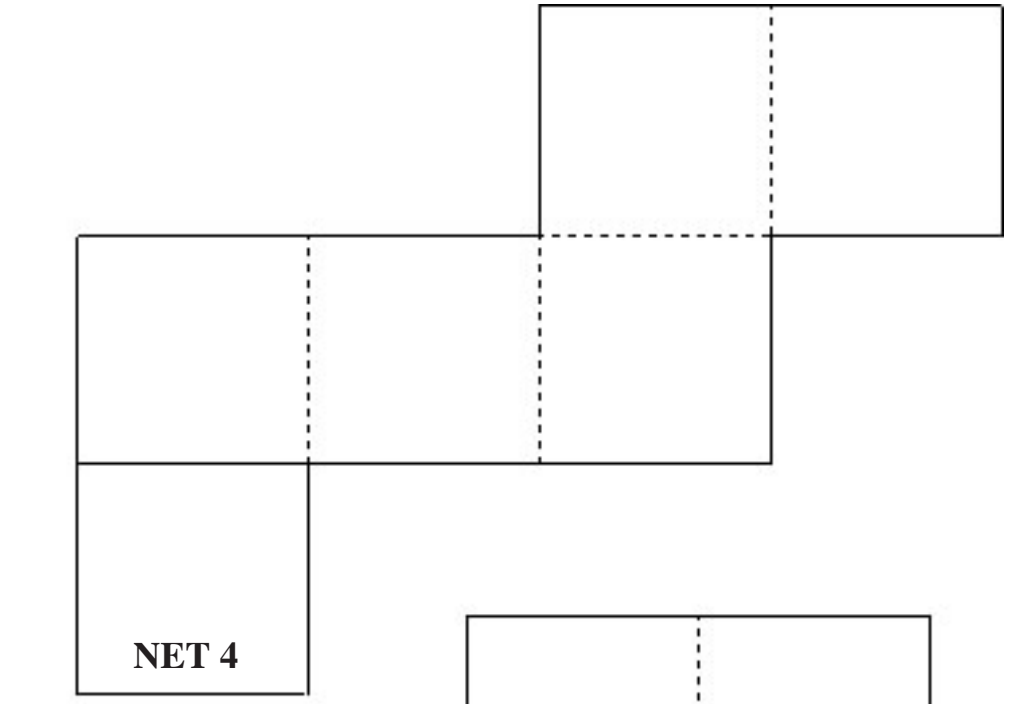
For further details, see annotated Other Resources on page 38, numbered as below.

4. The Jolly Postman or Other People's Letters by Janet and Allan Ahlberg.

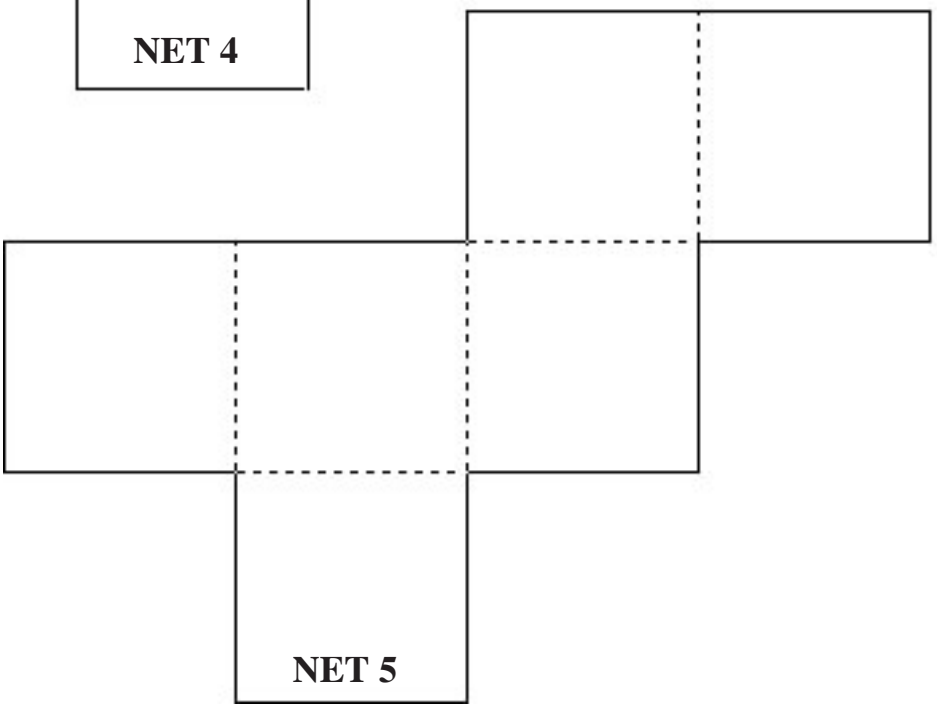
BLM 1: Cube Nets 1



BLM 2: Cube Nets 2

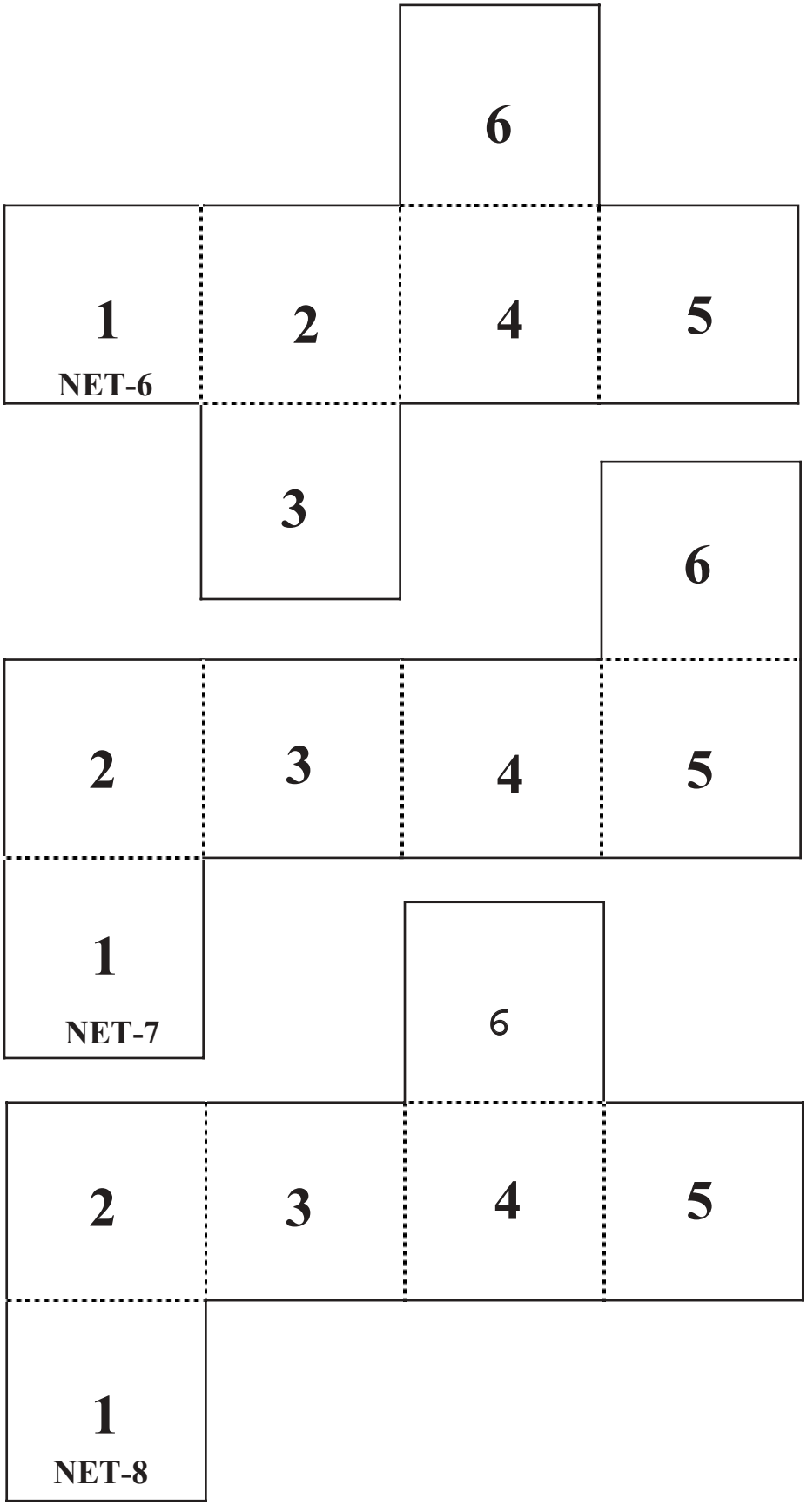


NET 4

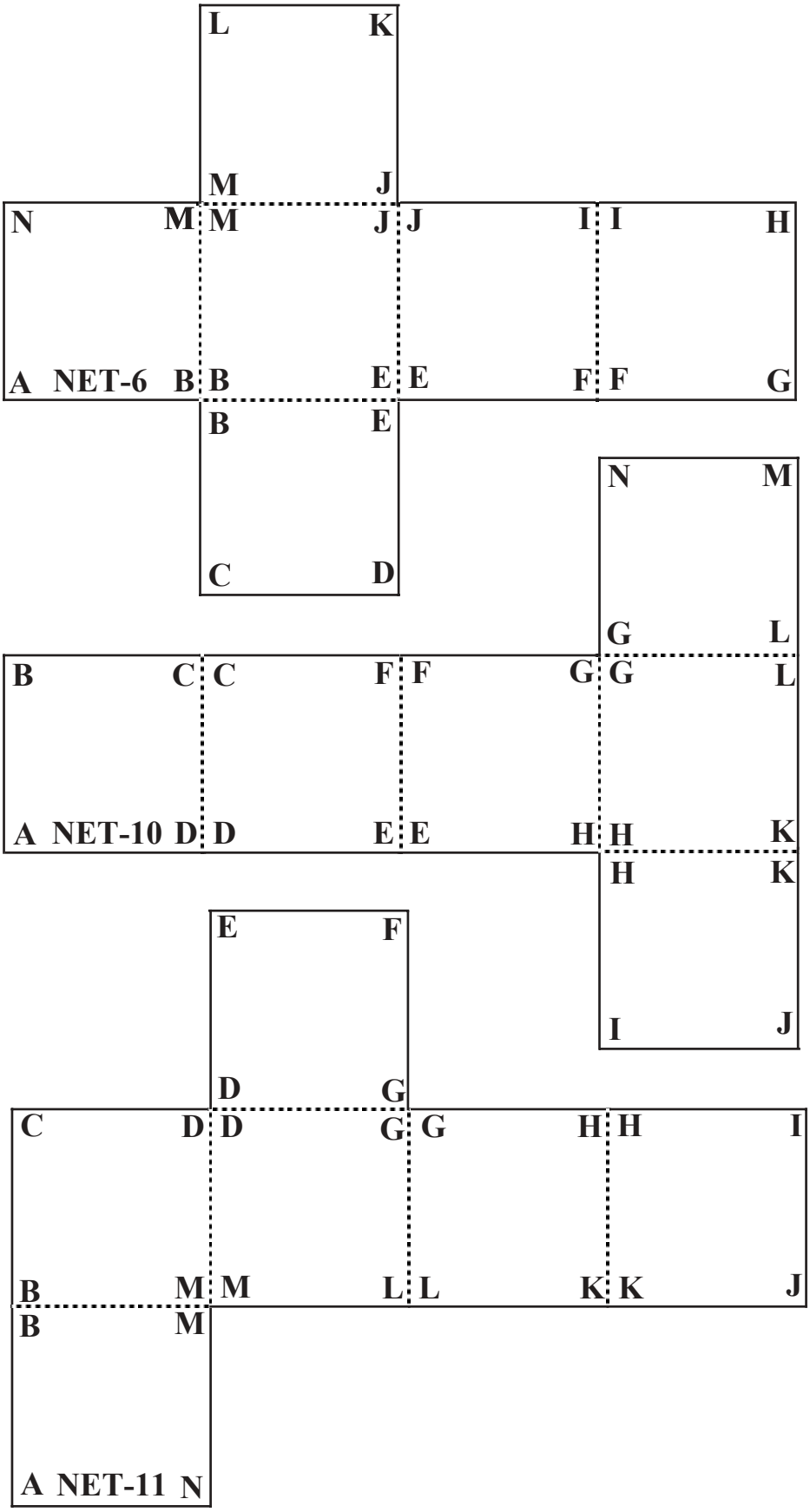


NET 5

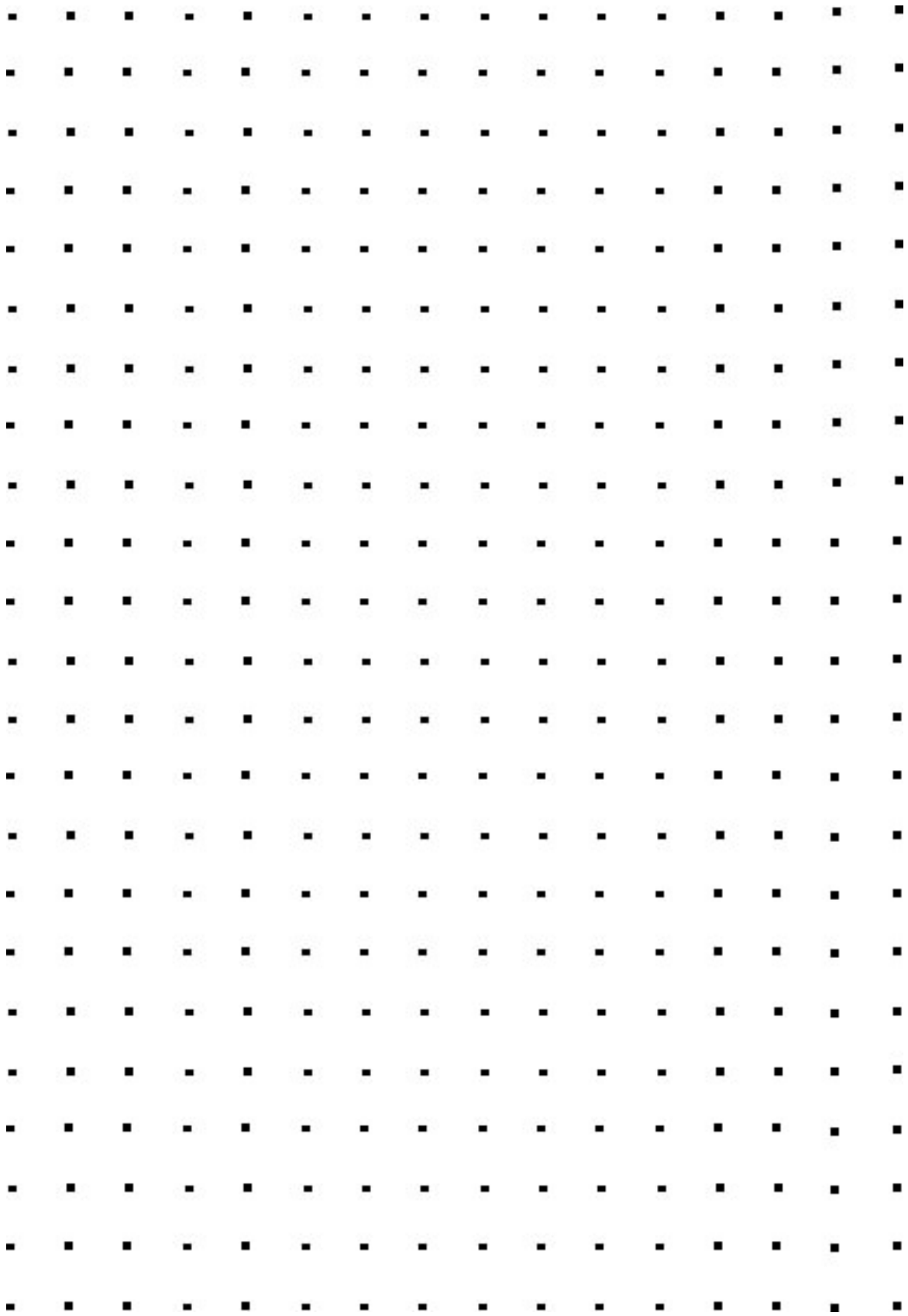
BLM 3: Cube Nets 3



BLM 4: Cube Nets 4

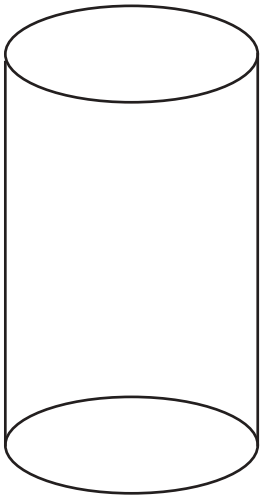


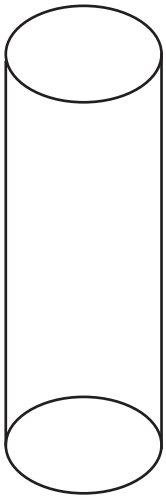
BLM 5: Geopaper



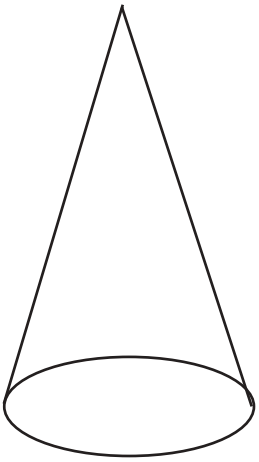
BLM 6: 3D Figures

cylinders

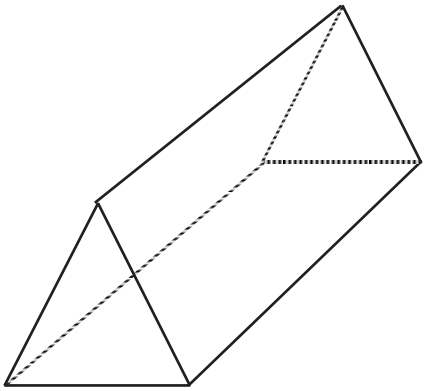




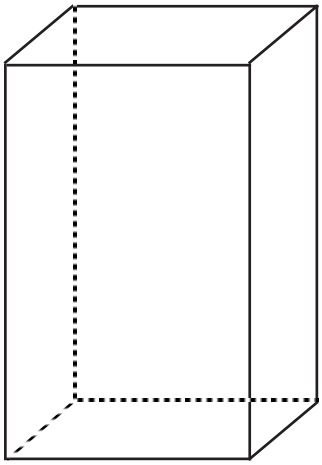
cone



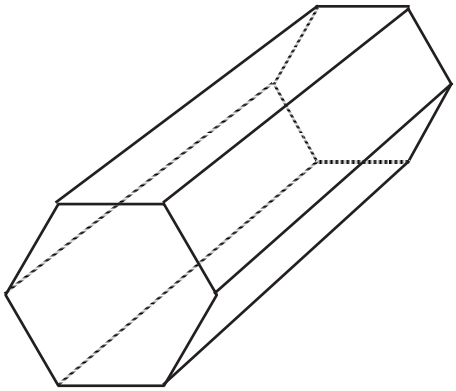
triangular prism



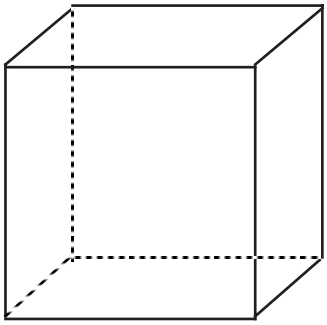
rectangular prism



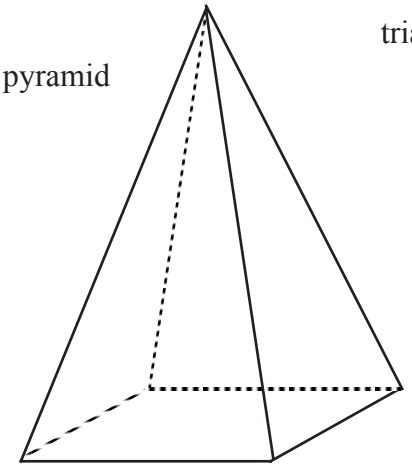
hexagonal prism



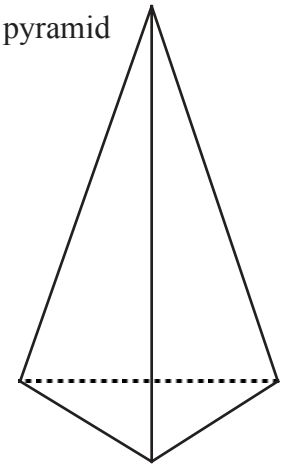
cube



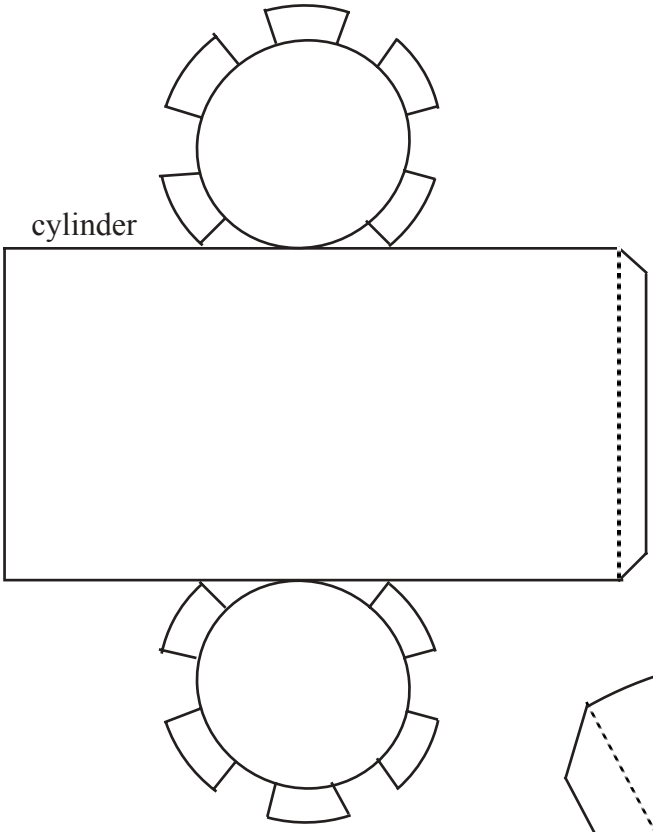
square pyramid

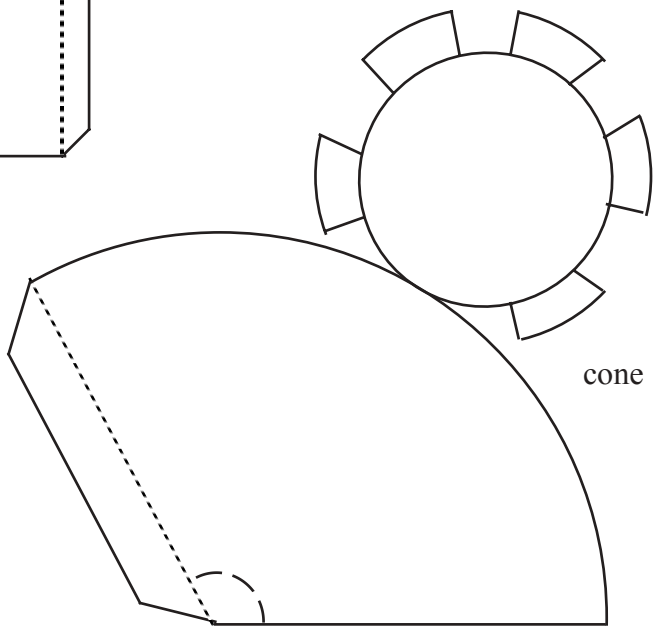


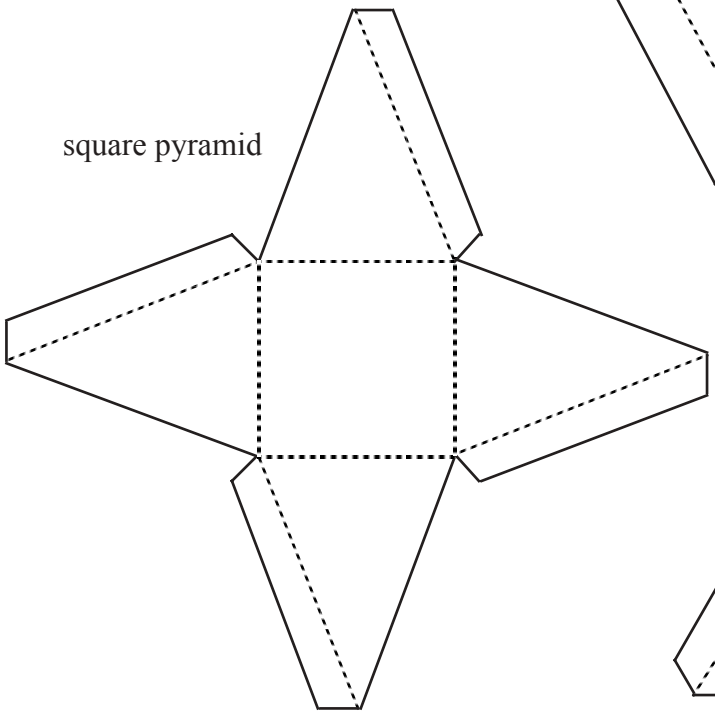
triangular pyramid

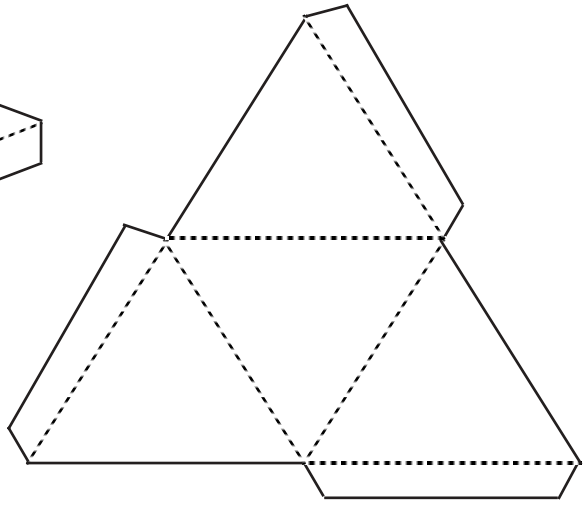


BLM 7: 3D Figures



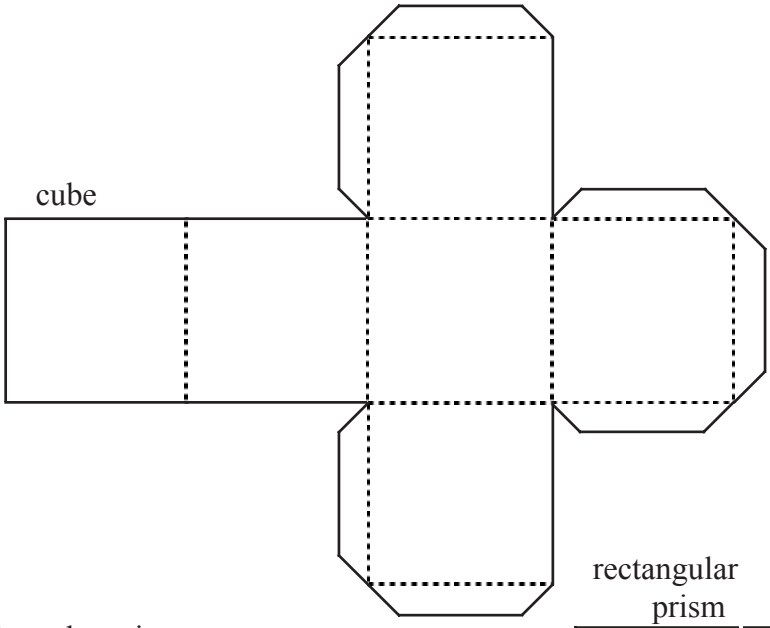




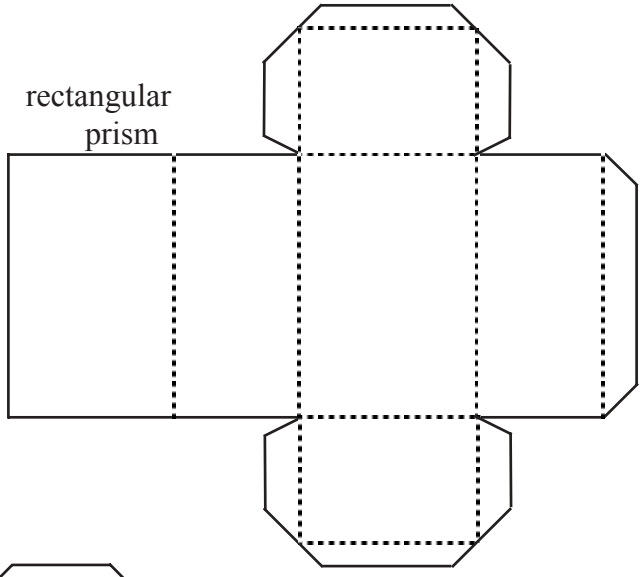


BLM 8: 3D Figures

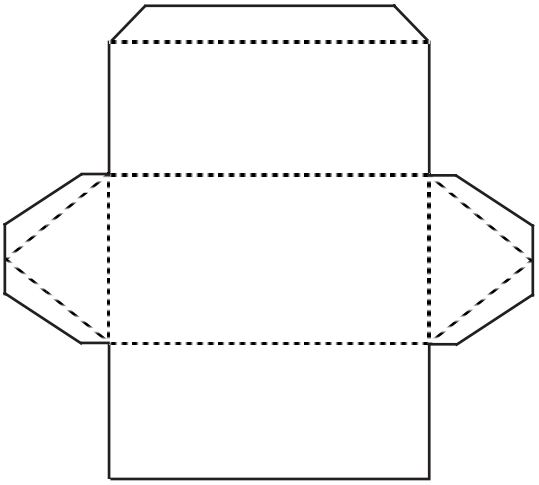
cube



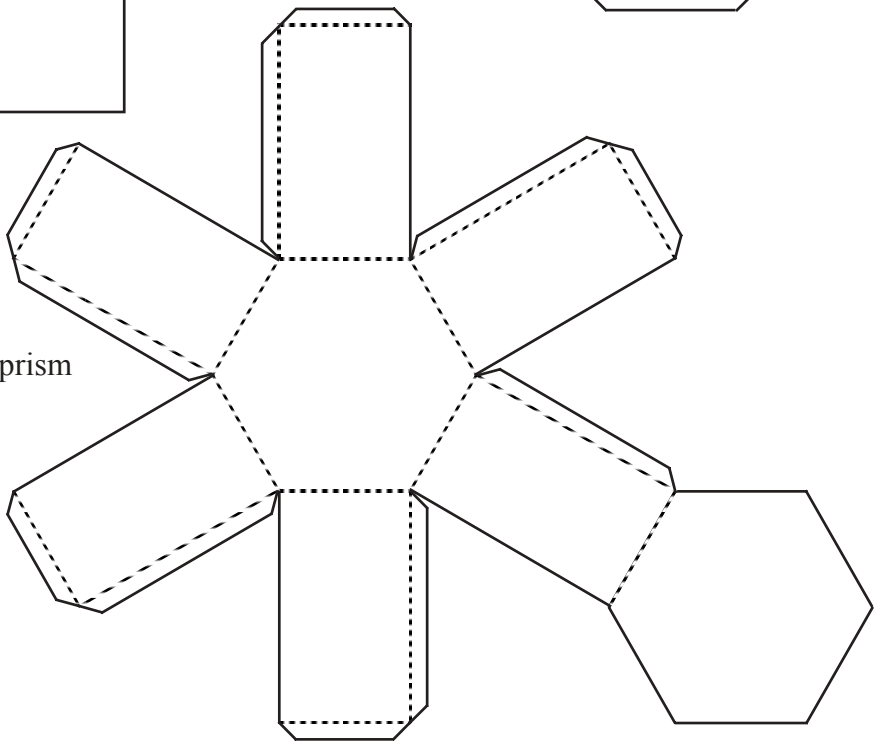
rectangular prism



triangular prism



hexagonal prism



BLM 9: Mystery Figures

For each one, tell what figure is being described. Be prepared to tell why you think your answer is correct. Answers, except for 'sphere', are from figures pictured on BLM6. There may be alternate answers if additional figures are used.)

1. I have 6 faces and 8 vertices
   All my edges are the same length.
   Who am I?

2. I have just 1 vertex.
   Who am I?

3. I have just 2 triangular faces.
   Who am I?
   How many faces have I?

4. I have just 4 vertices.
   Who am I?

5. I have no faces, no vertices, and no edges.
   I am a solid object.
   Who am I?

6. I have just 2 parallel faces.
   I have 6 vertices.
   Who am I?

7. I have 8 vertices.
   I have 3 pairs of parallel faces.
   I am not a cube.
   Who am I?

8. I have 4 faces meeting at 1 vertex.
   Who am I?
   How many faces have I?

9. I have two parallel faces.
   I have no vertices.
   Who am I?

10. I have 3 edges meeting at each vertex.
    What could I be?

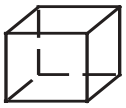
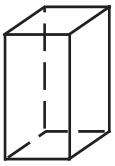
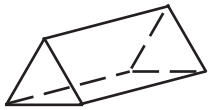
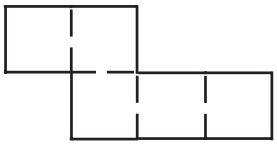
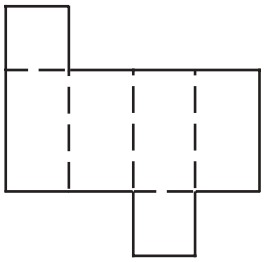
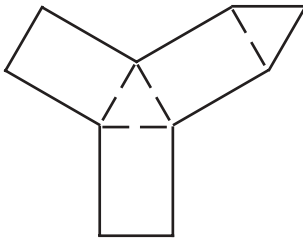
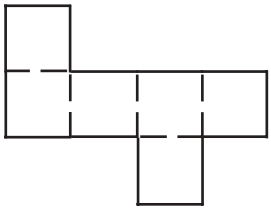
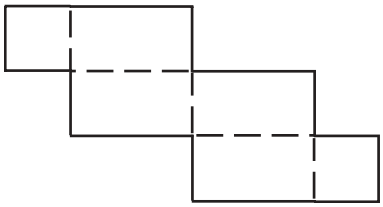
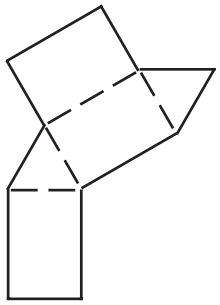
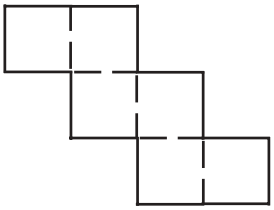
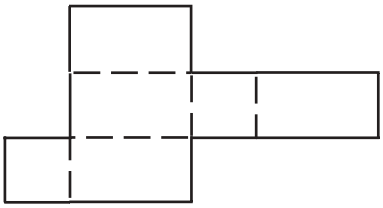
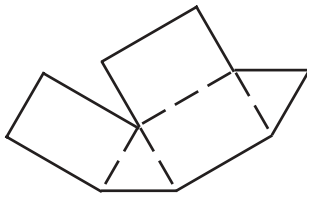
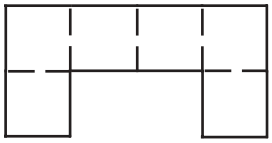
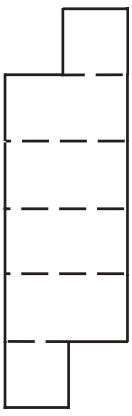
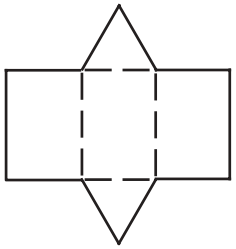
Make up your own.

11.

12.

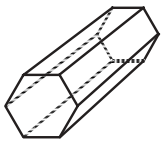
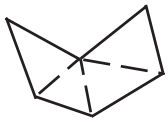
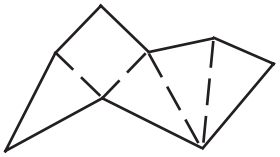
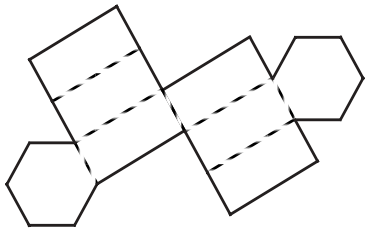
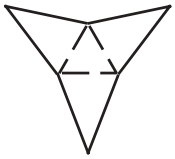
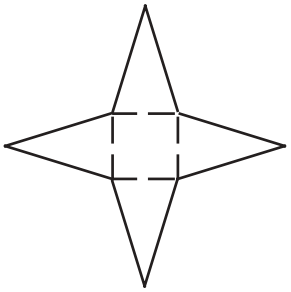
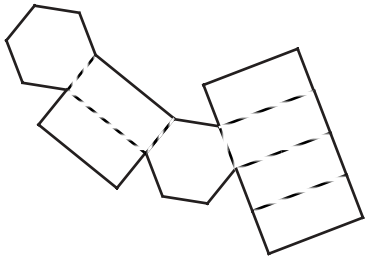
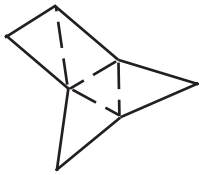
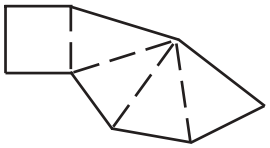
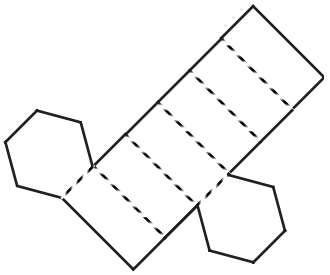
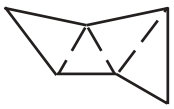
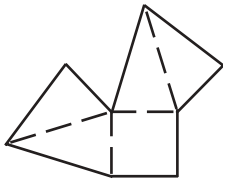
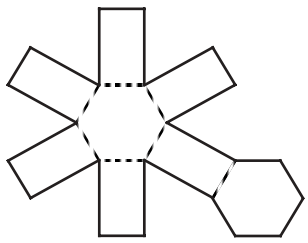
BLM 10: Net Matching

In each column, mark the net or nets that could be used to construct the figure at the top of the column.

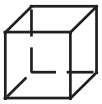
1.	2.	3.
 cube	 rectangular prism	 triangular prism
a. 	**a.** 	**a.** 
b. 	**b.** 	**b.** 
c. 	**c.** 	**c.** 
d. 	**d.** 	**d.** 

BLM 11: Net Matching 2

In each column, mark the net or nets that could be used to construct the figure at the top of the column.

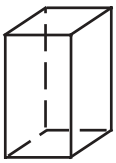
1.  triangular pyramid	**2.**  square pyramid	**3.**  hexagonal prism
a. 	**a.** 	**a.** 
b. 	**b.** 	**b.** 
c. 	**c.** 	**c.** 
d. 	**d.** 	**d.** 

BLM 12: Net Matching



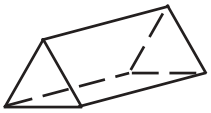
cube

A



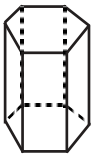
rectangular prism

B



triangular prism

C



hexagonal prism

D

square-based pyramid

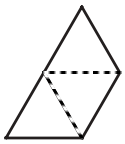
E

triangular pyramid

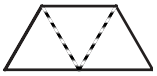
F

Each of the nets below is incomplete. One more piece must be added so the net can be used to construct one of the figures above. Add the necessary piece and tell which figure (A, B, C, D, E, or F) the net can be used to make.

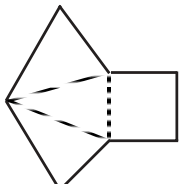
1.



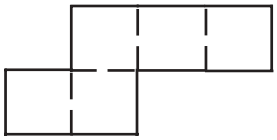
2.



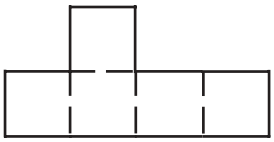
3.



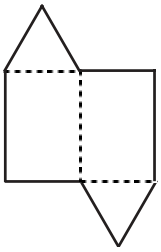
4.



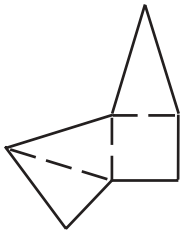
5.



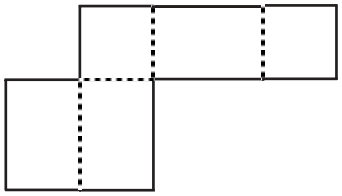
6.



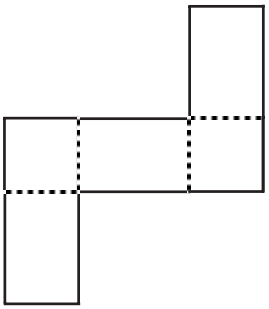
7.



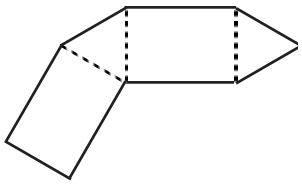
8.



9.



10.



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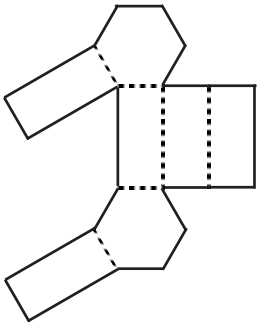

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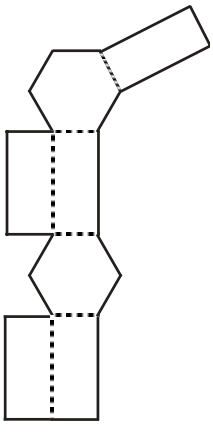
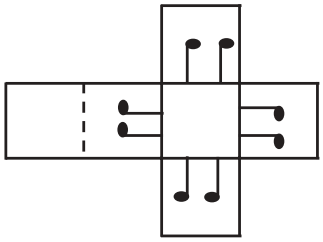


BLM 13: Cube Problems

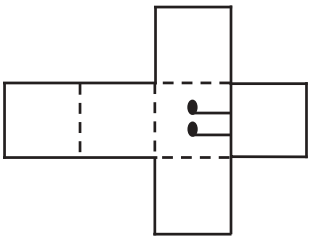
Problem 1:

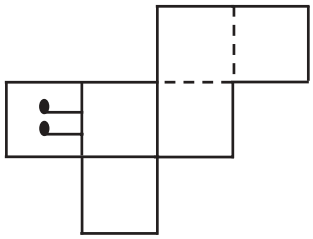
Imagine a cube with one shaded face (the bottom) and flags on the four faces that touch the bottom.

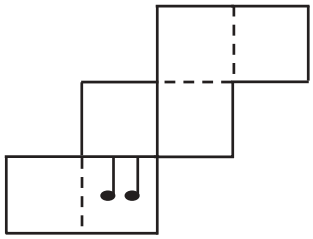
This could be unfolded to give a net like the one shown.



Where would you put the flags on each of these nets so they would give the same cube?





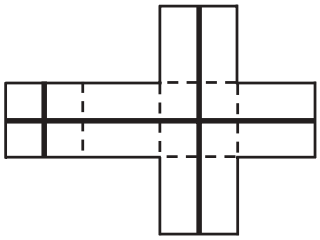


How can you check your answers?

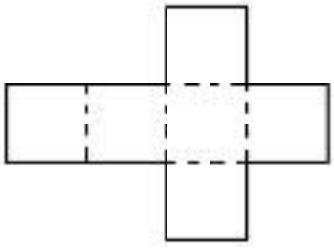
Problem 2:

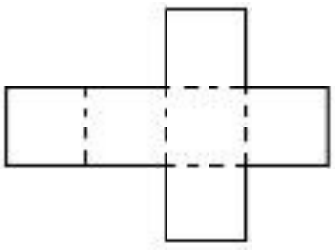
Imagine a cube box with ribbon around it as shown.

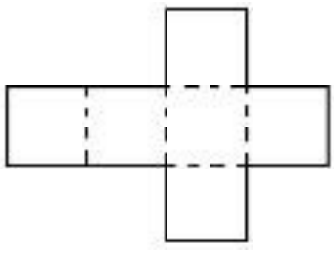
If this cube is unfolded, the net will have "ribbons" on it. Here is one possible net.



Imagine that each of these cube boxes with ribbons were unfolded. Draw the "ribbons" on the nets. You will have to imagine what the back and bottom of the box will look like. (The ribbons go directly around the boxes.)



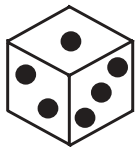




BLM 14: Cube Problems

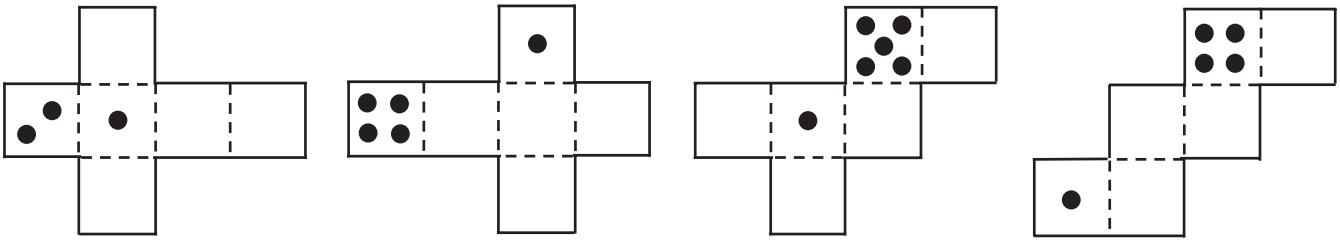
Problem 3:

A regular die has 1, 2, and 3 dots in the positions shown.

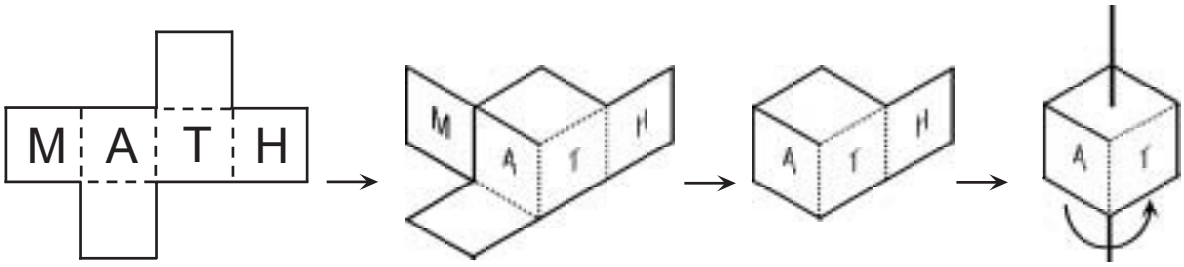


The numbers on opposite faces of a die have a sum of 7. (What number is opposite the '2'? opposite the '1'? opposite the '4'?)

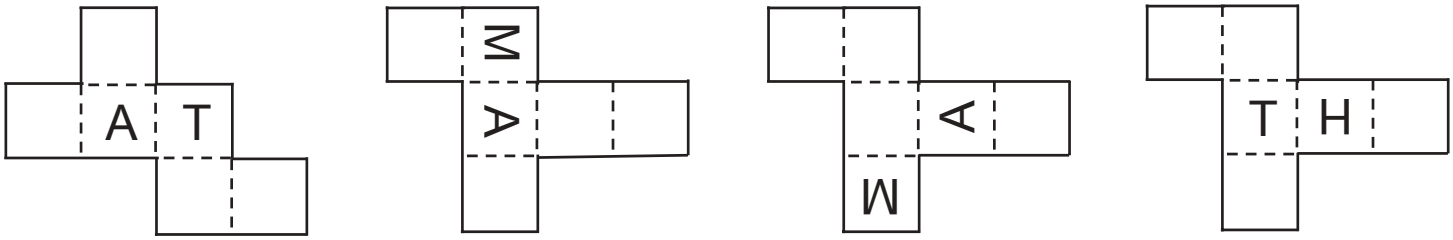
Where would you place the remaining dots on each of the following to make regular dice?



Problem 4:
The cube net below has the letters M, A, T, and H printed on four of its faces. When the net is folded into a cube, the cube can be turned to spell the word "MATH".



The nets below have two of the letters, M, A, T, H, on two of the faces. Print the other two letters where they should go so that the finished cube can spell the word "MATH" when it is turned as shown above.

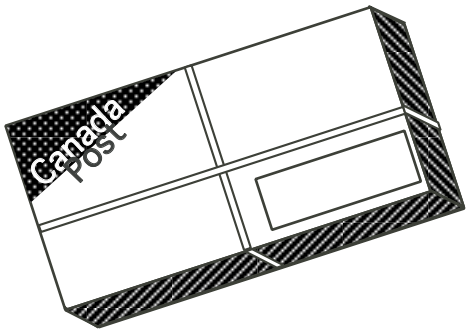


Would it be easier or harder if you did not have to make sure the letters were all "the same way up". Why? Explain.

BLM 15: The Jolly Postman

The Jolly Postman or
Other People's Mailing Packages

(Based loosely on "The Jolly Postman", by Janet and Allan Ahlberg)



Once upon a bicycle, so they say,
A jolly postman came one day
From over the hills
And far way...

To pick up parcels from the Three Bear's cottage.

The postman took the parcels.
They must be mailed today.
The packages are for Goldilocks,
And must go without delay.

"Oh dear," says the postman,
"I'm not sure that they can go.
They must meet the requirements,
Of the post office," and so...

He took out the booklet,
With rules for package size.
"These regulations," he exclaimed,
"Are all organized..."

"Oh my," said Papa Bear,
"Let's check the length, width, and height.
We must ensure the packages,
Will make it there tonight...

Solutions & Notes

Activity 1: Nets of Cubes

BLM 3

For net 6, pairs of opposite sides are 1 and 4, 2 and 5, 3 and 6
For net 7, pairs of opposite sides are 1 and 6, 2 and 4, 3 and 5
For net 8, pairs of opposite sides are 1 and 6, 2 and 4, 3 and 5

BLM 4

For net 9, pairs of matching edges are

LM and NM	AN and GH
AB and CB	KJ and IJ
DE and FE	LK and HI
CD and GF	

For net 10, pairs of matching edges are

AB and KL	IJ and ED	BC and LM
AD and KJ	FG and NG	
EH and IH	CF and MN	

For net 11 pairs of matching edges are

AB and KJ	EF and IH	AN and KL
BC and JI	FG and HG	
CD and ED	MN and ML	

Activity 3: Recognizing Nets

BLM 10

Nets of the cube are b and c
Nets of the rectangular prism are a and c
Nets of the triangular prism are a, b and d

BLM 11

Nets of a triangular pyramid are b and d
Nets of the square pyramid are a, b, and c
Nets of the hexagonal prism are b, c, and d

Solutions & Notes

BLM 12

When the missing pieces are added correctly,

a and b will be nets of triangular pyramids
c and g will be nets of square pyramids
d and e will be nets of cubes
f and j will be nets of triangular prisms
h and i will be nets of rectangular prisms
k and l will be nets of hexagonal prisms

When the students test the "corrected" nets by cutting out and folding, it will be obvious whether or not the nets are correct.

BLMs 13 and 14

Students should check their own work by cutting and folding their nets to see if their cubes will match the given illustrations/descriptions.

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions, by Jean Karr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels. Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. "Assessment", Arithmetic Teacher Focus Issue, February 1992, NCTM.
   This copy of NCTM's journal for elementary school addresses several issues dealing with assessment. It also includes suggested techniques and student activities.

5. How to Evaluate Progress in Problem Solving, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Suggested Assessment Strategies

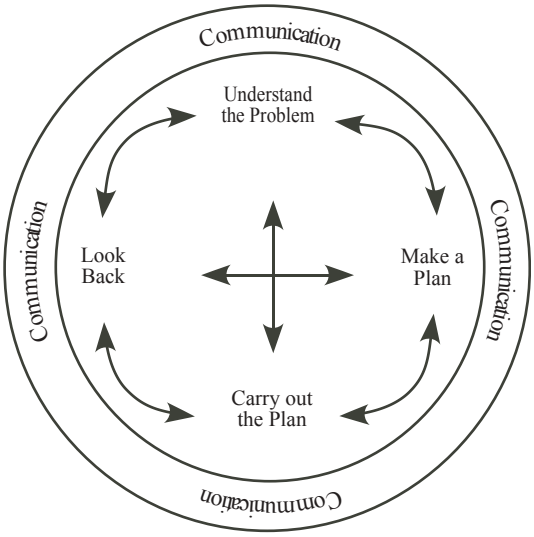
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R's"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;

Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.

Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
STRATEGIES	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
RELEVANCE	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

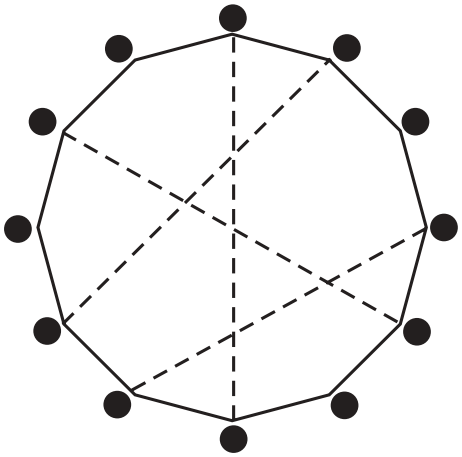
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a “what if...?” problem, such as “What if the question were reversed?”, “What if we had other data?”, “What if we were to show the data on a different type of graph?”.

Suggested Assessment Strategies

Suggested Adaptation of Rubric for Activity 1

Once students have completed the five cubes made from Nets 1 to 5, they are given the task of finding other, different cube nets. The other six possible nets are given in Nets 6 to 11. The following rubric is an adaptation of the one just above, suggesting ways the specific attributes of the "cube net" problem can be incorporated into the assessment.

Some parts of the rubric may not be valid, depending upon how the problem was posed. For example, if students are not asked to explain/justify their findings, either orally or in writing, then the final criterion will not be applicable.

Neatness in cutting out squares and constructing cubes is not given as a criterion for this task, since the focus of the task is to identify cube nets and justify these choices. If the nets/cubes are to be part of a classroom display, you may choose to have students redraw their final responses. It would then be appropriate to include, in the rubric, a criterion dealing with appearance.

Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
The student • identifies only 1 correct net of a cube, and is satisfied with that as a solution • may include incorrect or duplicate nets with insufficient or too many squares	The student • identifies at least 2 correct nets of a cube • may include incorrect or duplicate nets, but such nets will be built with 6 squares	The student • identifies at least 3 correct nets of a cube • may include incorrect or duplicate nets that have been tried and discarded	The student • identifies at least 5 additional correct nets of a cube • may include partial incorrect nets that are discarded even before complete
• needs concrete material; may be more successful manipulating cut-out squares than just drawing the nets on geopaper	• is able to identify one or two nets by drawing, but prefers cut-out squares or other manipulatives	• is comfortable using geopaper to draw nets, but may have some trouble identifying duplicates	• is comfortable using geopaper to draw nets, and can identify duplicates easily
• indicates little or no testing of all nets	• may indicate only partial testing of nets	• gives some indication of testing of most or all nets	• gives evidence of testing for all nets, or can explain how (s)he folded the net mentally
• is unaware that several nets are missing or that nets must contain 6 squares; poor use of language	• may be unaware that not all nets have been found; may not be able to give valid or well-expressed reasons for conclusions	• is aware that there are still nets missing, but may not be able to explain fully how this is known	• shows understanding that problem may not be completely solved; can give carefully analysed proof/reasons

Other Resources

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Addenda Series, Grades K - 6: Geometry and Spatial Sense" by Lorna Morrow and John Del Grande, NCTM, 1993.
This book includes detailed lessons for each grade from K to 6, emphasizing manipulatives. BLMs are provided.

3. "Addenda Series, Grades K-6: Patterns" by John Firkins, NCTM, 1993.
This book contains detailed lessons on various aspects of pattern and challenges students to interpret, develop, and extend patterns.

4. "The Jolly Postman or Other People's Letters" by Janet and Allan Ahlberg, William Heinneman Ltd., 1986.
This book is a rhyming poetry story about a postman who delivers letters, brochures, and postcards to storybook characters, such as Goldilocks, The Big Bad Wolf, and Cinderella. Each letter refers to and extends the details of the different stories.

5. "Promoting Visual Imagery in Young Pupils" by Erna Yackel and Grayson H. Wheatley, in *Arithmetic Teacher* 37(6) Feb. 1990, pp 52-8, NCTM.
Suggestions for activities to help children develop their ability to 'see' images mentally. Use is made of tangram pieces.