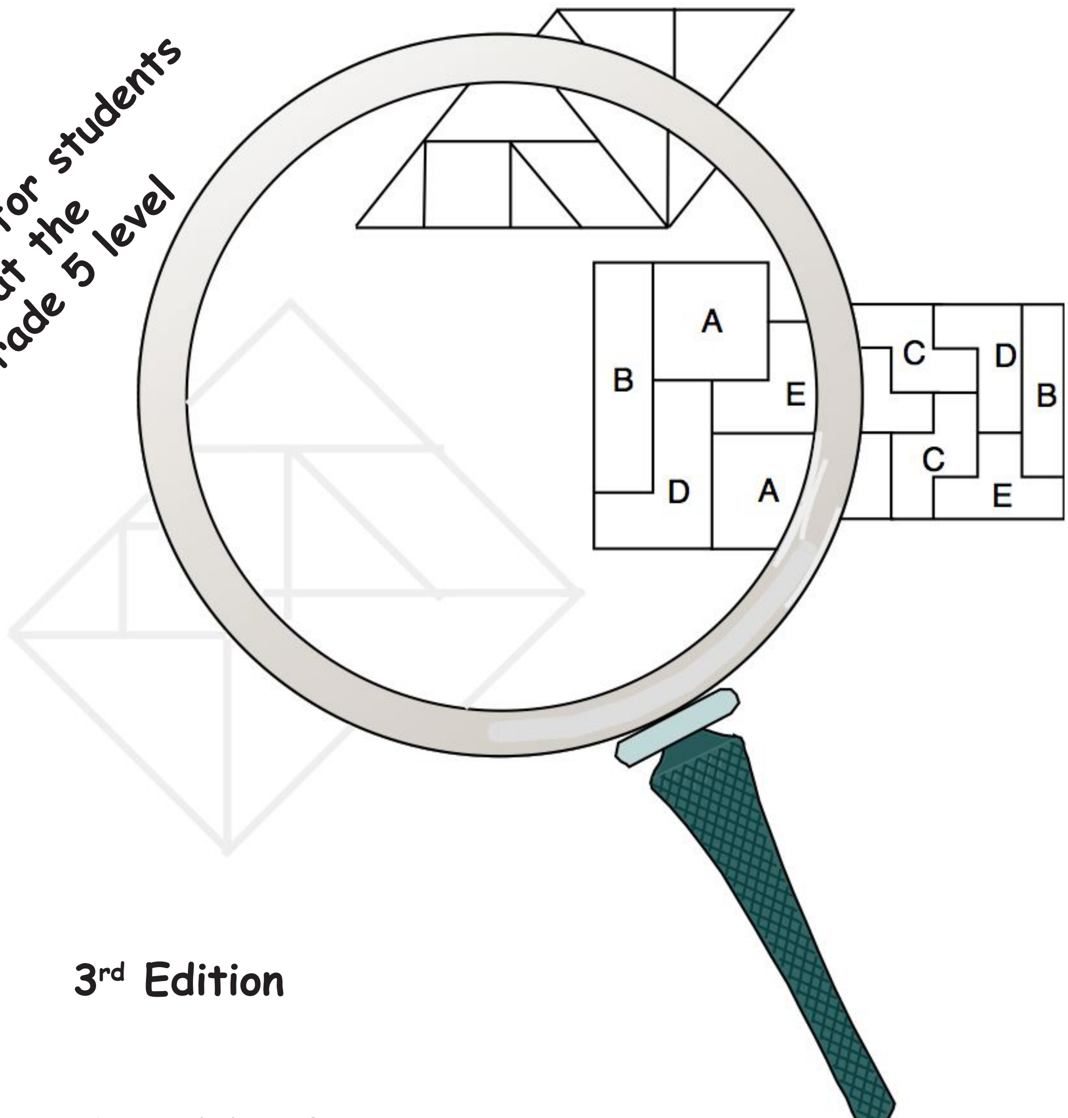
Invitations to Mathematics

Investigations in Geometry:

"We're in Great Shape"

Suggested for students at the Grade 5 level



3rd Edition

An activity of
The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
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Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics, among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will enable them to construct their own mathematical meaning and understanding. This emphasis, plus the extensions and related activities included, provide ample scope for all students' interests and ability levels. Related "Family Math" activities which may be used to involve the students' parents are also suggested.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Outcomes are related to Ministry Curricula for the province of Ontario, but are adaptable to other locales.

Investigations in Geometry is comprised of activities to enhance the students' geometry and spatial sense abilities, as well as creativity and problem-solving skills. Geometry concepts are easily integrated with other subject areas, providing ways to demonstrate that mathematics pervades everyday life.

Due to their nature, geometry activities depend heavily on manipulative materials. Every effort has been made to use materials readily available in most classrooms/schools, and a number of Black-Line Masters (BLMs) are provided to add further manipulatives.

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Table of Contents

Overview

Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and geometry specifically. Some of these beliefs, taken from a variety of sources, are described below.

Geometry is best learned through a combination of active exploration and reasoning, through a cycle of concrete investigations which leads to conjecture, which can then be tested by further concrete investigations.

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Students need a rich learning environment that contains a variety of geometric objects they can manipulate in order to discover the geometric properties of objects and the relationships among them. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures.

The ability to visualize is particularly important in the study of geometry. Students need to be able to draw images in their mind of how figures look, and to be able to manipulate these images mentally.

Students need to develop and use a variety of communication skills in geometry. Geometry uses a formal language to describe objects and their interrelationships and movements in space. In grades 4, 5, 6, however, teaching students the formal language of geometry is not as important as helping them to identify geometric properties and principles and to use pictures or everyday language to explain their observations.

Geometry provides ample opportunity for the development of divergent thinking and creative problem solving, as well as logical thinking ability. Further, exploring problems embedded in 'real-world' settings cultivates the perception that geometry plays a critically significant role in everyday life, rather than being just a set of memorized properties and vocabulary.

Essential Content

The activities in this unit focus on exploring two-dimensional figures. During this unit, the student will:

- identify various polygons by name, as well as subsets such as types of triangles;
- sort and classify polygons based on properties such as parallel sides, equal sides or angles, lines of symmetry;
- combine polygons in a variety of ways;
- identify slides, flips, and turns;
- use a variety of problem-solving strategies.

Curriculum Connections

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Sorting Polygons**	• sorting polygons (with 3 to 8 sides) • using mathematical terms to describe sorting rules	• sort polygons according to the number of sides, angles, and vertices • use mathematical language to describe geometric ideas
Activity 2 **Sorting Triangles**	• sorting triangles according to sides and angles • using mathematical terms to describe classes of triangles	• sort polygons according to the number of sides, angles, and vertices • identify, describe, compare, and classify geometric figures • use mathematical language to describe ideas
Activity 3 **Tetrominoes**	• identifying non-congruent structures made of four identical squares • tiling a given rectangle using tetrominoes	• identify congruent and similar figures using transformations • demonstrate an understanding of congruent figures
Activity 4 **Tangram Quadrilaterals (2 periods)**	• constructing quadrilaterals using Tangram pieces • identifying common quadrilaterals (square, rectangle, parallelogram, trapezoid) and their properties	• discuss geometric concepts with peers and use mathematical language to explain their understanding of the concepts • discuss ideas, make conjectures, and articulate hypotheses about geometric properties and relationshapes

Overview

Assessment

Assessment is a process of gathering evidence about a student's knowledge, skills, and values, and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned, but mathematical competence involves such characteristics as the ability to communicate, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. Because of the nature of the activities it is suggested that a variety of types of assessment be used. Suggestions include:

(i) observing students as they work to see if they are applying various concepts; to see if they are working cooperatively; to observe their committment to the tasks;

(ii) assessing the completed project to see if instructions have been followed; to see if concepts have been applied correctly; to see if the language of mathematics has been used correctly;

(iii) assessing the students' descriptions of their completed work to see if mathematical language is used correctly; to see if students understand the concepts used;

(iv) providing opportunities for student self-assessment: have students write explanations of their understanding, opinion, or feelings about an activity. One technique is to have them write under the headings What I Did, What I Learned, and How I Felt About It. Students could be asked to write a review of one day's activities or of the whole unit's work.

(v) selecting an exemplary piece of work to be included in a portfolio for assessment purposes or for sharing with parents.

See Suggested Assessment Strategies, page 38, for further discussion and sample rubrics.

Notes

Notes

Overview

Prerequisites

If students have some familiarity with polygons and their names, Activity 1 will be largely review. Most students will have encountered these figures in previous grades which should be sufficient introduction to Activity 1.

Logos

The following logos, which are located in the margins, identify opportunities for:

Problem Solving | Communication | Assessment | Use of Technology

Materials

ACTIVITY	MATERIALS
Activity #1 **Sorting Polygons**	• BLMs 1, 2, 3, and 4 • BLM 5 (optional)
Activity #2 **Sorting Triangles**	• Triangles from Activity #1 • BLM 6, heavy coloured paper or bristol board, fasteners, scissors, paper punch (optional) • BLM 7 is needed for Family Activity 2
Activity #3 **Tetrominoes**	• For each pair of students, provide 2 complete sets of the five different tetrominoes from BLMs 8, 9, 10, 11, and 12. If possible, each figure should be of a different colour. • Provide a copy of BLM 13 for each pair of students. • Overhead transparency of BLM 13 for teacher to demonstrate solution • Overhead markers in colours similar to tetrominoes for teacher recording of solution • BLM 15 is needed for Family Activity 1 • Copies of BLM 14 (optional)
Activity #4 **Tangram Quadrilaterals**	• For each pair of students one copy of Tangram Templates (BLM 16) or commercial tangrams • One copy of Tangram Recording Sheet (BLM 17) or chart paper • One set of overhead tangram pieces or run one copy of each of BLM 16 and 17 on acetate

Overview

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week, students in our classroom will be participating in a Geometry unit titled 'We're in Great Shape!'. The classroom activities will focus on examining, comparing, drawing, and constructing two-dimensional figures.

Your child will bring home one or more activities for family participation. These home pursuits will use geometric figures, and are designed to encourage discussion and problem solving. There are many ways of arriving at a solution so it is important that everyone involved discusses his or her ideas and strategies.

You can help your child learn to communicate mathematically by encouraging him/her to explain their reasoning orally (or in writing).

Thank you for being part of our learning. I hope that you enjoy the activities. If geometry is a part of your everyday work or hobbies, please encourage your child to learn about this so that he/she can describe this to his/her classmates.

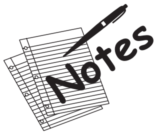
Sincerely,

Teacher's Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is best to schedule class time for sharing and discussion of results.

Notes



Activity 1: Sorting Polygons

Focus of Activity:

- Sorting two-dimensional figures
- Classifying triangles
- Defining characteristics of polygons

What to Assess:

- Facility with terminology
- The level of cooperative effort
- Use of properties of figures to define sorting rules

Preparation:

- See the table on page 4 for materials
- Provide copies of BLMs 1, 2, 3, and 4 reproduced on heavy coverstock if possible.
- Prepare a wall-chart displaying the vocabulary needed, along with sketches.

Activity:

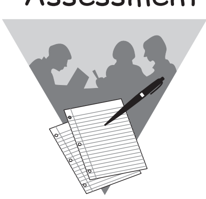
Introduction

Students cut apart the figures along the dividing lines, to make a set of 'cards', and sort them according to the number of sides.

- 3 sides (triangles)
- 4 sides (quadrilaterals)
- 5 sides (pentagons)
- 6 sides (hexagons)
- 8 sides (octagons)
- others

Introduce the terms *polygon* (e.g., a closed figure with sides that are line segments) and *regular polygon* (e.g., a polygon with all sides congruent and all angles congruent). Have students identify the non-polygons and the regular polygons in their sets of figures. Have them explain why the non-polygons *are* non-polygons.

Assessment



Have students sort all of the polygons in at least three different ways and write their sorting rules. Observe and listen to students to determine the names of polygons with which they are already familiar, and what properties of the figures they are using (e.g., parallel sides, right angles, lines of symmetry, and so on).

Determine two or three other sorting rules through discussion.

As a class, sort according to two or more of these properties. For example, you might ask students to find, in the set of polygons, all figures that have both congruent sides and congruent angles, or those that have both congruent and parallel sides. For example, A & D have all sides congruent and all angles congruent; J & L have at least 1 pair of congruent sides and one pair of congruent angles; T has a pair of congruent sides and a pair of parallel sides, but the two pairs are not the same; Q (or any other rectangle) has pairs of sides that are both congruent and parallel.

Activity 1: Sorting Polygons

Students should record descriptions of one or more of the sorting rules they used, the terms *polygon*, *triangle, quadrilateral*, etc., and a definition or a description of the figures represented by each term.

Students should keep the set of polygon and non-polygon cards for Activity 2.

Extensions in Mathematics:

1. Have students construct various figures on a geoboard or geopaper (BLM 5). For example, ask them to construct a triangle with two sides equal, a quadrilateral with one pair of parallel sides, or a hexagon with at least two right angles.

2. Students can make up challenges like these for each other.

3. Any computer draw program (such as Geometer's Sketch Pad™) could be used by students to draw sets of polygons and non-polygons to add to the sets provided.

Family Activities:

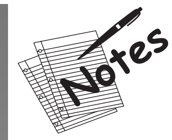
1. Pick a room in your house. Examine things in the room to find out which two-dimensional figures are represented. Make a chart to record your data. What figures occur most often? Which figures occur least often? Why? Write down your reasons.

2. Search for examples of polygons in newspapers, magazines, telephone directory, T.V. Guide, advertising flyers. Sketch three or four examples, and share with the class.

Other Resources:

For additional ideas, see annotated Other Resources list on page 43, numbered as below.

2. Addenda Series, Grades K-6: Geometry and Spatial Sense



Communication

Use of Technology

Activity 2: Sorting Triangles

Focus of Activity:

- Sorting triangles in various ways by features such as congruent sides and/or angles.

What to Assess:

- Facility with terminology
- The level of cooperative effort
- Accuracy of sorting strategies

Preparation:

- See the table on page 4 for materials
- Use the cut-out polygons from Activity 1.
- Make copies of BLM 6 (optional)
- Make copies of BLM 7 (optional)

Activity:

INTRODUCTION

Select all the triangles from the set of polygons (BLM 1, 2, 3, and 4). Students should then sort the triangles. Discuss various sorting rules.

One way to sort the triangles is according to the number of **congruent sides.** Have students sort the triangle cards according to the lengths of their sides. Discuss with students their reasons for sorting triangles as they did (e.g., "In these triangles, one side is longer than the other two".) Introduce the following terms used to describe triangles:

- *equilateral:* all sides are the same length
- *isosceles:* two sides are the same length
- *scalene:* no sides are the same length

Have students sort the triangles this way and record the result of the sorting by writing the letter labels of the triangles in the correct column of a chart similar to the one below.

Equilateral	Isosceles	Scalene
	C	H

Repeat using **angles** as the sorting criterion. Record the letter labels in a chart. First have students develop their own criteria for this type of sorting. Then introduce the following labels:

- *acute-angled*: all angles are acute (between 0° and 90°)
- *right-angled*: at least one angle is a right angle (90°)
- *obtuse-angled*: at least one angle is obtuse (between 90° and 180°)

Acute-angled	Right-angled	Obtuse-angled
C	B	

Activity 2: Sorting Triangles

Using these two charts, probe students with questions such as

"Are there any right-angled scalene triangles?"

"Are there any obtuse-angled scalene triangles?"

Students should record the names of types of triangles in their notebooks and write a definition or description, or sketch the appropriate figure.

Extensions in Mathematics:

1. Have students draw a chart similar to the one below.

	Equilateral	Isosceles	Scalene
Acute-angled			
Right-angled			
Obtuse-angled			

Students should write the letter labels of all the triangles in the appropriate spaces in the chart. If any boxes remain empty, students should attempt to draw an appropriate triangle, or write reasons why such a triangle is impossible.

2. Reproduce the geostrips from BLM 6 on heavy coverstock, if possible, or have students paste BLM 6 on coverstock before cutting out. Have students try to construct triangles with the following geostrips:
   a) 10, 15, 20
   b) 5, 10, 20
   c) 5, 5, 5
   d) 15, 15, 15
   e) 5, 10, 15
   f) 10, 10, 10

   As an alternative to the prepared geostrips, have students measure strips of paper in 5 cm, 10 cm, 15 cm, and 20 cm lengths, each length a different colour. This applies students' measurement skills, and provides visual clues for identification of different kinds of triangles based on colour. For example, if a triangle has 3 sides of the same colour, then it must be an equilateral triangle. Similarly , if two sides are one colour, and the third side is a different colour, then it must be an isosceles triangle.

   Have students identify any that are impossible to construct, explain why, and justify their conclusions.

Cross-curricular Activities:

1. 1. Use triangles in various building projects with simple materials (e.g., toothpick bridges). Consider why triangles are so often used in buildings.
2. Use different colored triangles of different sizes and types to make designs, borders, or pictures.
3. Use Logo software to draw triangles and triangular designs.

Notes

Communication

Problem Solving

Assessment

Use of Technology

Activity 2: Sorting Triangles

Family Activities:

1. Investigate types of triangles used in your neighbourhood, in buildings, packaging, landscaping, etc. Record your observations in some way (e.g., sketches, photos).
2. Work together to solve the toothpick triangle puzzles on BLM 7. Discuss various problem-solving strategies you might use. Record your solutions and share later with classmates.

Other Resources:

For additional ideas, see annotated Other Resources list on page 43, numbered as below.

4. Triangles: Shapes in Math, Science and Nature by Catherine Sheldrick Ross

Activity 3: Tetrominoes

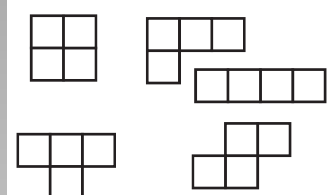
Focus of Activity:
- Covering a rectangular region with tetrominoes.

What to Assess:
- Persistence in solving the problem
- The level of cooperative effort
- Correct use of mathematical language (e.g., slide, flip, turn)

Preparation:
- See the table on page 4 for materials
- For every group of four students provide 24 squares. Have students cut out squares from scrap paper or provide sheets of dot paper or graph paper).
- Prepare a class set of small puzzle pieces (*tetrominoes*) as shown on BLMs 8 through 12, with each different tetromino a different colour. These puzzle pieces are the set of all possible combinations of four squares with any two connecting squares having a side in common. Each pair of students requires **two** complete sets.
- Also provide sheets with a 5 unit by 8 unit grid (10 cm x 16 cm) and a blank rectangle of the same size for recording the solutions to the puzzle. (see BLM 13).
- Prepare an overhead transparency of a complete solution to the problem, using coloured pens to match the colours of the tetrominoes.
- Make copies of BLM 14 (optional)
- Make copies of BLM 15 (optional)

Notes

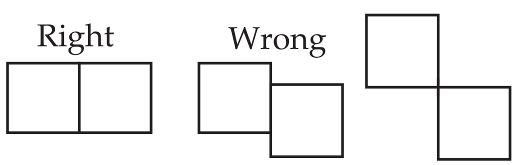
The Set of Tetrominoes



Activity:

INTRODUCTION

Challenge students to find all the possible ways to arrange four squares so that two connecting squares have a side in common. Note the examples below;



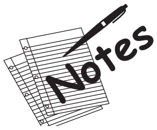
Students will discover that there are only five possible ways to arrange four squares. These five arrangements make up the set of *tetrominoes*.

Provide pairs of students with complete sets of tetrominoes and a grid plus a rectangle of the same size (BLM 13) for recording the solution(s).

Challenge each pair of students to fit the puzzle pieces together to form a rectangle which covers the grid or the blank rectangle of the same size.

Many solutions are possible — pieces may be slid, flipped, or turned in any orientation.

Problem Solving



Assessment

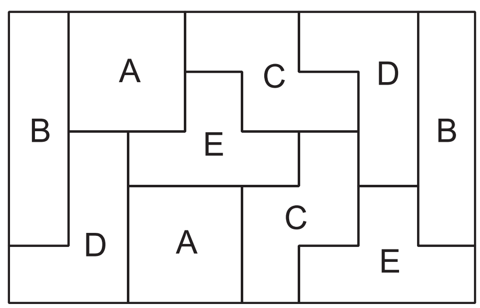
Activity 3: Tetrominoes

If students have had previous experience with slides, flips, and turns, you may wish to question them as they work to determine if they recongnize that the motions might be helpful in solving the problem.

Once a solution has been found, students should outline or paste the pieces in place on either the grid or the blank rectangle of the same size.

Extensions in Mathematics:

1. Students may find additional solutions to the puzzle.
2. Using an overhead transparency of a solution to the puzzle, illustrate and review slides, flips, and turns.
    - In describing a slide, direction and distance may be used to identify the slide. For example (see diagram below), the slide from one square tetromino (A) to another could be described as "1 unit right, and 3 units down". Or the reverse could be used "1 unit left, and 3 units up".
    - In describing a flip, a mirror line may be used to identify the flip. For example, in the diagram below, the two rectangular tetrominoes (B) are related by a flip in a vertical mirror line 3 units from each rectangle.
    - In describing a turn, direction, angle of rotation, and centre of rotation may be used to identify the turn. For example, in the diagram below, one "zigzag" tetromino (C) is related to the other by a 1/4 turn counter clockwise about the lower right vertex of the upper tetromino.



3. Students may be challenged to find all polygons made up of five squares connecting in such a way that any two connecting squares share a common side. Students use the pentominoes (BLM 14) to fill a grid or blank rectangle 6 squares by 10 squares (BLM 13).

Use of Technology

4. Some students may be familiar with a computer game called Tetris™. These students could introduce others to the game or invent a similar game they could play with the cut-out tetrominoes.

Use an actual tetromino piece on top of the diagram to illustrate each motion. Have students identify examples of slides, flips, and turns in their own solutions. They can demonstrate these by sliding, flipping or turning one tetromino to match its twin.

Activity 3: Tetrominoes

Challenge them with further questions such as "Which two pieces (in the solution shown on the overhead) could be related by either a slide, a flip, or a turn? (the rectangle). Which could be related by either a turn or a slide? (the square). Which are related by a slide and then a flip? (the L-shaped piece).

Different solutions will lead to different motions to pair up the tetrominoes. Students could make up some questions about their own solutions and exchange them with their classmates.

Cross-Curricular Activities:

1. Encourage students to think about the different tetromino shapes in terms of modular building units. Which arrangement would be most suitable for a house? Why? Which might be most suitable for a warehouse? Why?
2. Challenge students to think of real-life applications of tetrominoes such as flower beds made up of four square beds all connected by at least one side. Which arrangements would provide the most viewing for people walking alongside the beds?
3. Complement these activities with the computer game Tetris.

Family Activities:

1. Work together to solve the Dissection Squares puzzles on BLM 15. Record your solutions on the given grids to share with your classmates.

Other Resources:

For further details, see annotated Other Resources on page 43, numbered as below.

9. Pentominoes Revisited by Barry Onslow

Notes

Communication

Problem Solving

Use of Technology

Activity 4: Tangram Quadrilaterals (Two periods)

Focus of Activity:

- Forming a variety of quadrilaterals using seven tangram pieces.

What to Assess:

- Persistence in solving the problem
- The level of cooperative effort
- Willingness to search out alternate solutions
- Use of descriptive language related to geometry

Preparation:

- See the table on page 4 for materials
- For each pair of students provide one copy of the Tangram Templates (BLM16) made with heavy paper or coverstock for durability.
- Also provide a copy of the Tangram Recording Sheet (BLM17) for students to record solutions, or provide chart paper so that students may prepare a large wall chart for eventual display.
- Prepare an overhead set of the seven tangram pieces by copying BLM 16 on acetate.

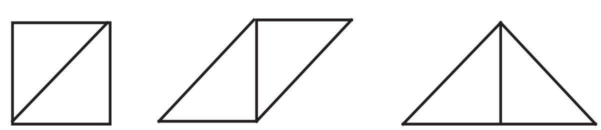
Activity:

Introduction

Discuss with the students some of the background history of the tangram. For example, the tangram is a puzzle created by Chinese mathematicians about 4000 years ago. It is a square subdivided into seven pieces — 5 triangles, a square, and a parallelogram. These pieces can be combined in various ways to make different geometric figures.

Distribute copies of BLM 16 and have students cut out both the white set and the shaded set. (One set can be kept at school, and the other taken home.)

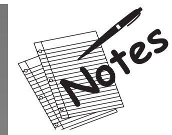
Use the overhead set to familiarize students with the seven tangram pieces. Discuss their names and characteristics. For example, all the triangles are isosceles right-angled. The two small triangles can be put together in one way to form the square, in another way to form the parallelogram and another way to form the middle-sized triangle.



This illustrates that the areas of the square, parallelogram, and middle-sized triangle are equal. Instead of telling students all this ask a series of questions to help students discover some of these properties for themselves. For example

- Are any pieces congruent?
- How could you describe the pieces of the tangram?
- Which pieces have the same area? (Encourage students to justify their responses.)
- What different shapes can you make using just the two small triangles?

Activity 4: Tangram Quadrilaterals (Two periods)



Distribute copies of BLM 17. The purpose of this activity is to form as many of the quadrilaterals (squares, rectangles, parallelograms, and trapezoids) as possible, using an increasing number of tangram pieces (from one piece to seven pieces). Caution students that some figures are impossible to create. Stress the importance of keeping an accurate record of the solution(s).

Challenge pairs of students to form a square (if possible) using just one tangram piece, then two pieces, three pieces, and so on up to seven pieces. Have them record their solution(s) on the Tangram Recording Sheet, or on a large sheet of chart paper. Invite students to demonstrate their solution(s) on the overhead.

Problem Solving

Note: *This may be a suitable place to end the first period of this activity.*

Instruct pairs of students to create the other quadrilaterals. The solutions could be recorded by each pair of students on their own sheet. Alternatively, the solutions could be recorded on a large cooperative class chart by the student pair who first discovers the solution(s). You may wish students to draw their solution on the large chart with pencil or crayon, or you might want them to make paper cut out solutions from the templates, and glue them on the chart. Encourage students to find more than one solution if possible. This could be an ongoing activity.

Extensions in Mathematics:

1. Students can be challenged to create additional tangram puzzles for themselves and peers/parents. For example, animal shapes could be designed by the students.

Family Activities:

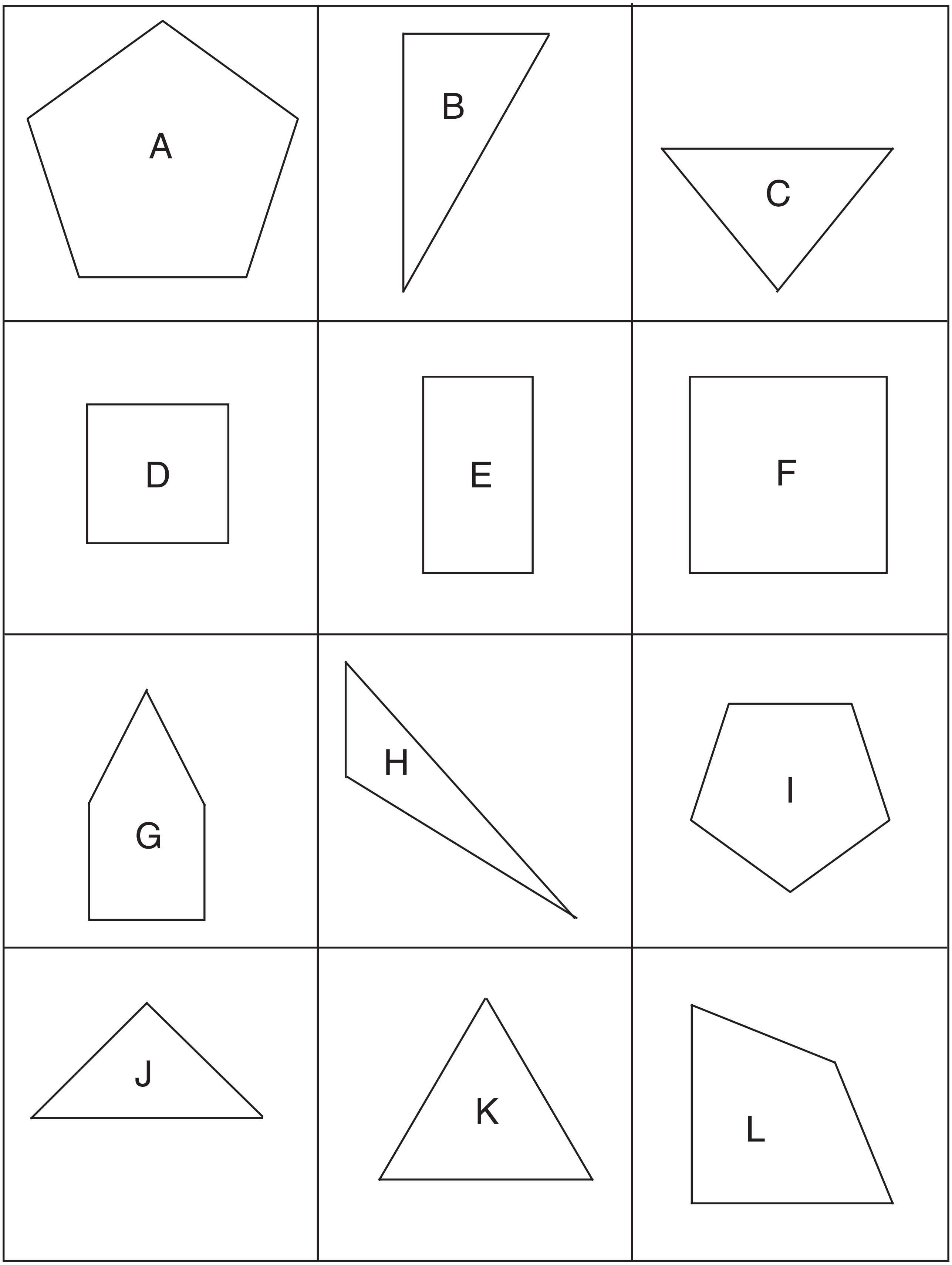
1. Have students take their tangram pieces home and challenge their families to re-assemble the pieces to form a square or any of the figures recorded on BLM 17.

Other Resources:

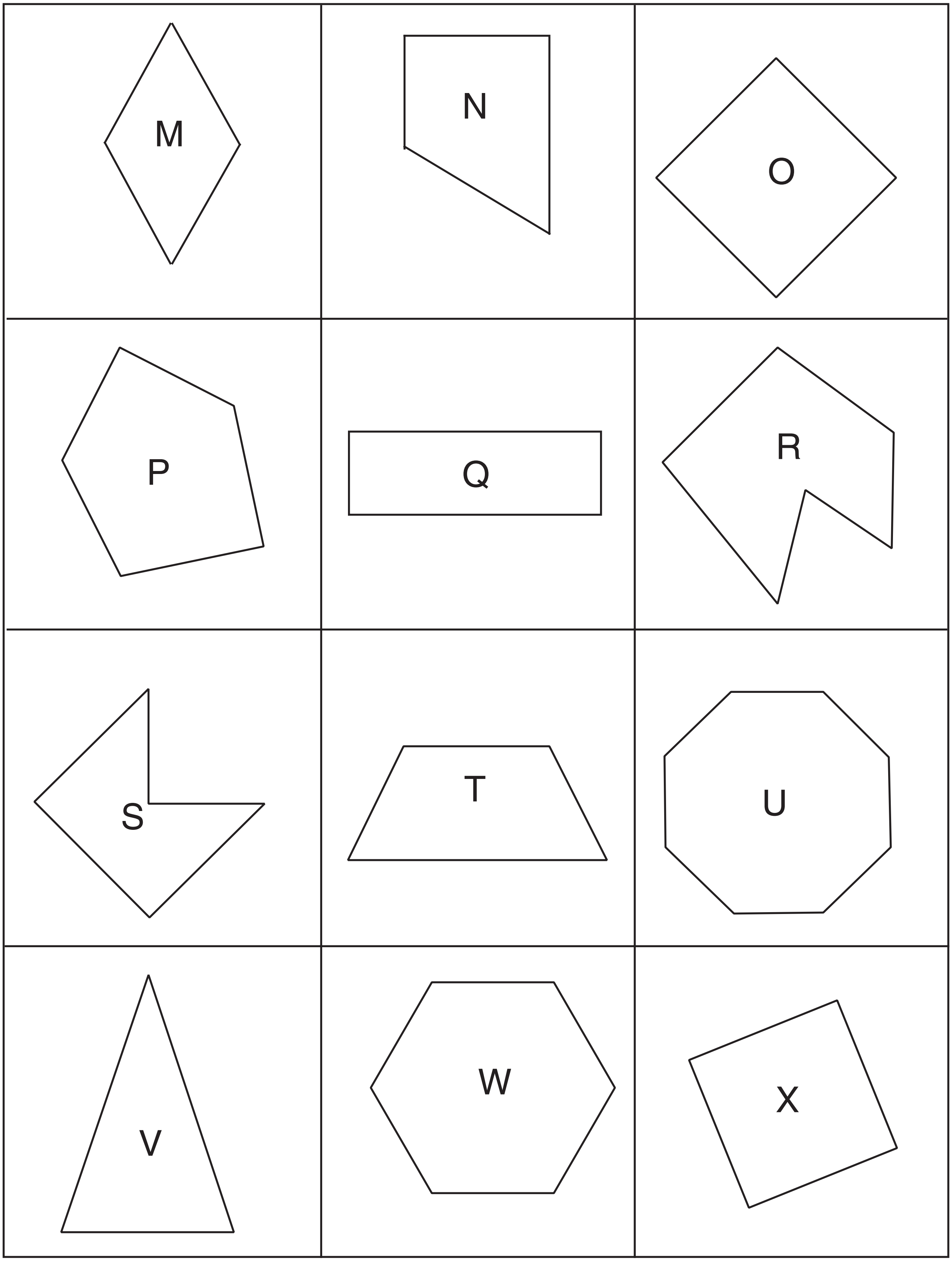
For further details, see annotated Other Resources on page 43, numbered as below.

2. Addenda K-6: Geometry and Spatial Sense
5. Grandfather Tang's Story by Ann Tompert
6. Moving on With Tangrams by Judy Goodnow
7. Cooperative Problem Solving with Tangrams by Ann Roper
8. Making and Exploring Tangrams Andrejs Dunkels

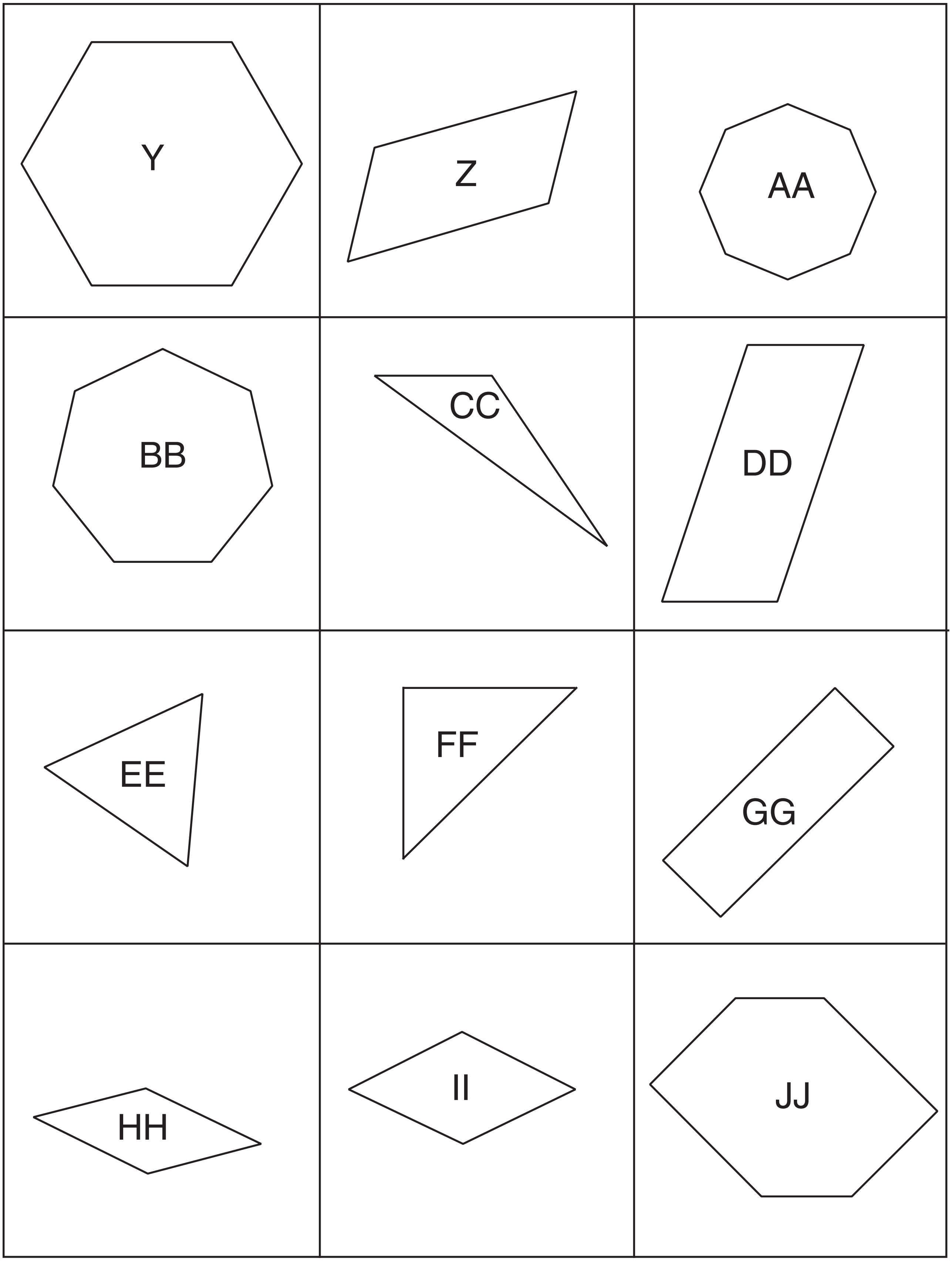
BLM 1: Polygons 1



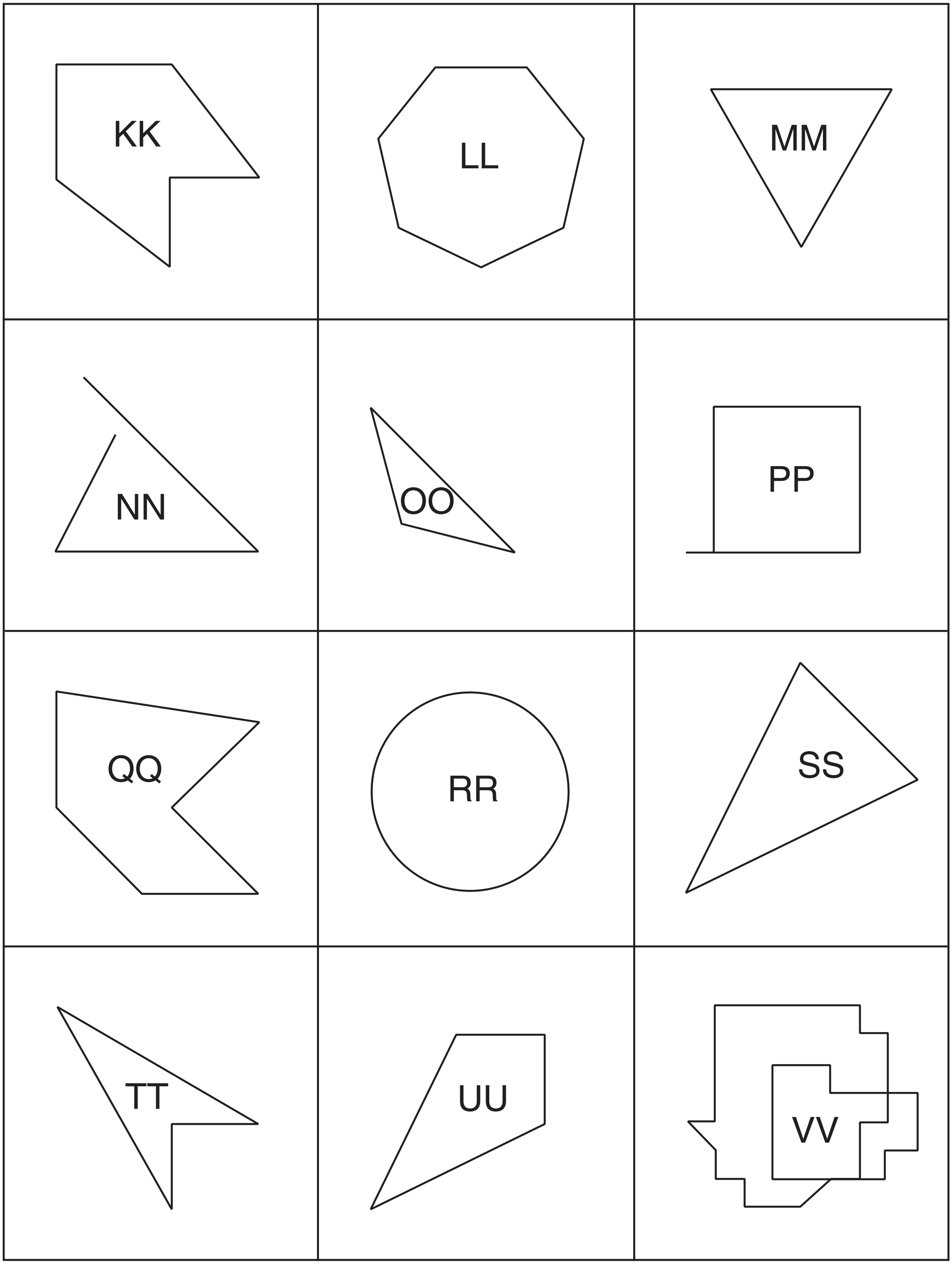
BLM 2: Polygons 2



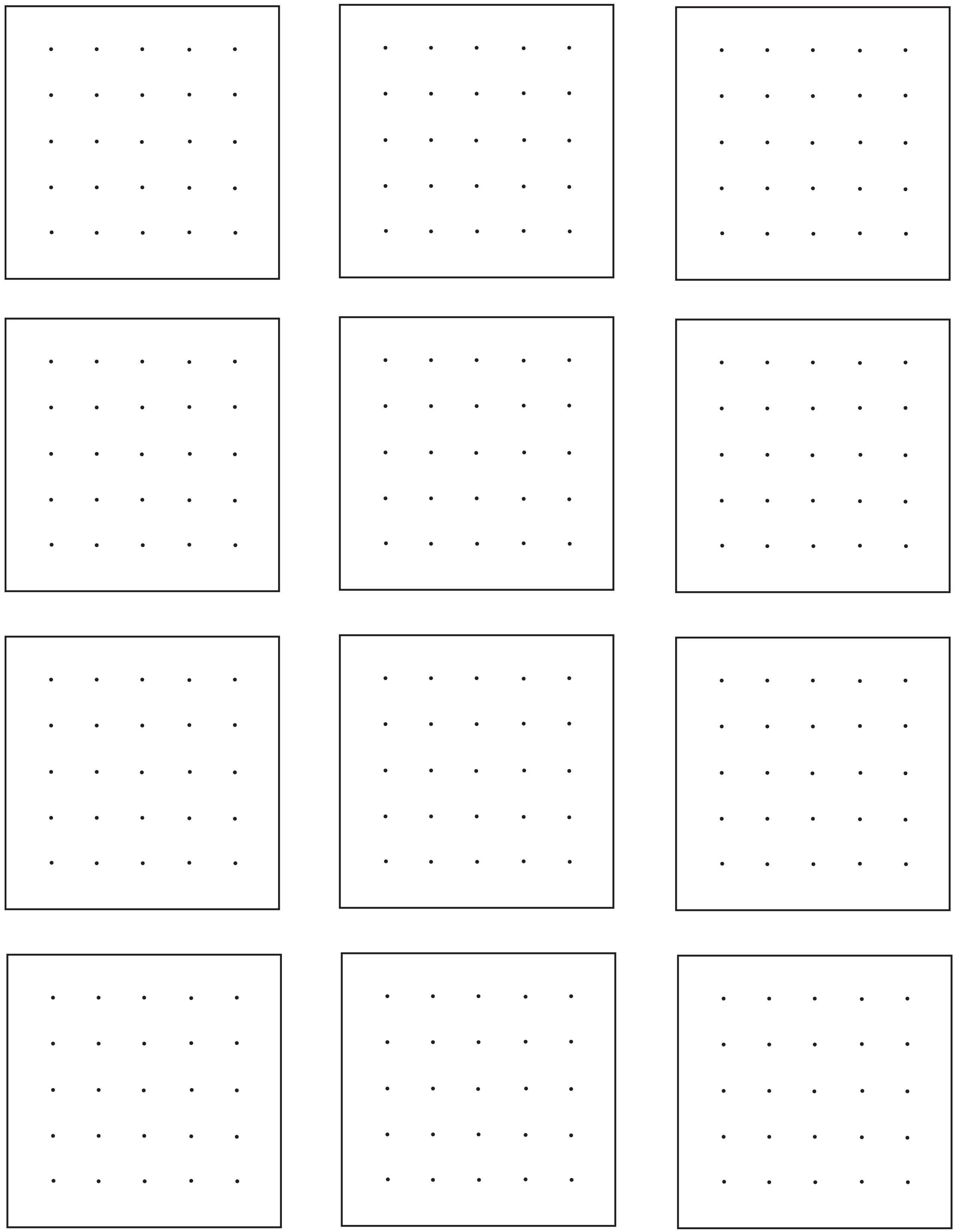
BLM 3: Polygons 3



BLM 4: Polygons 4

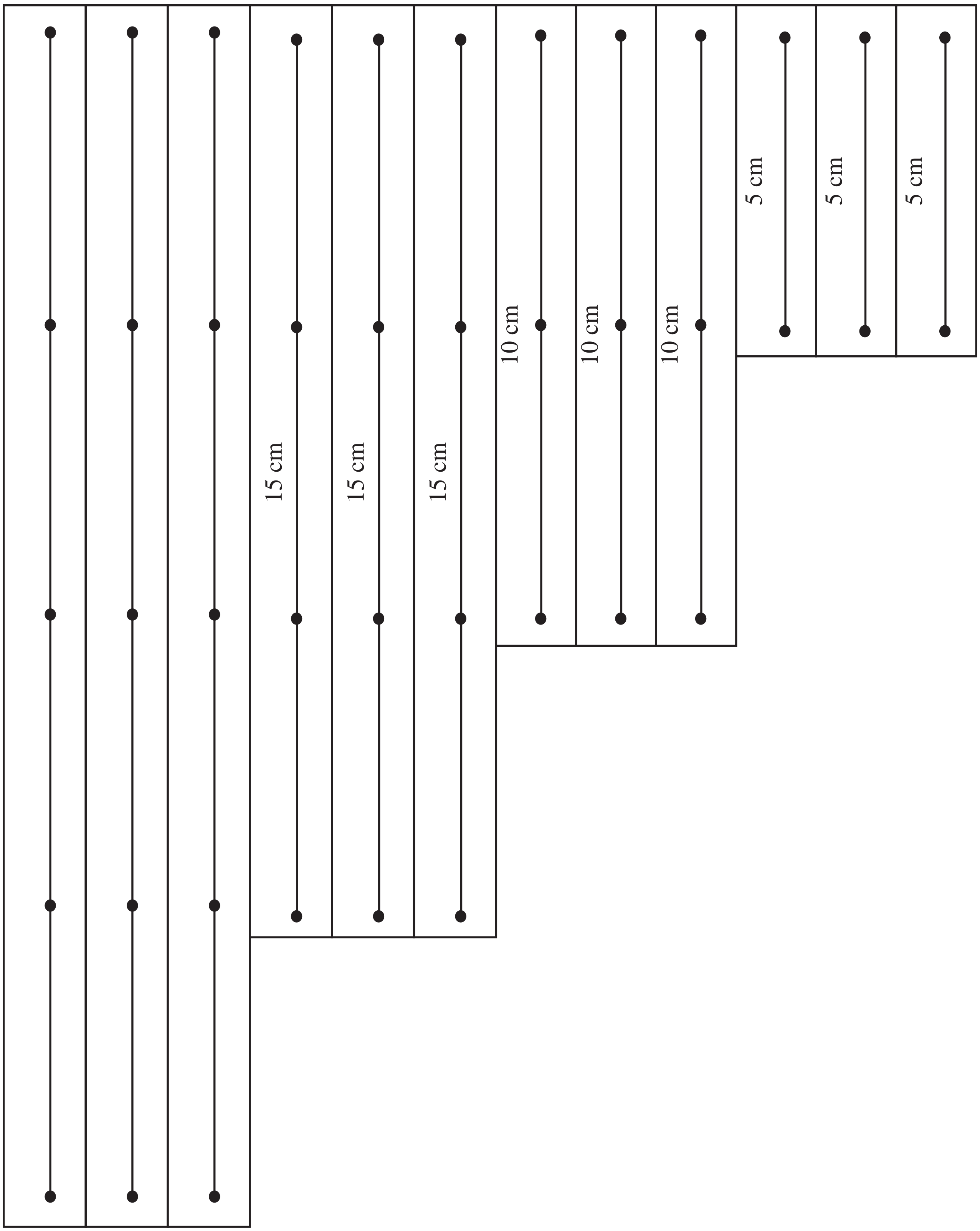


BLM 5: Geoboards

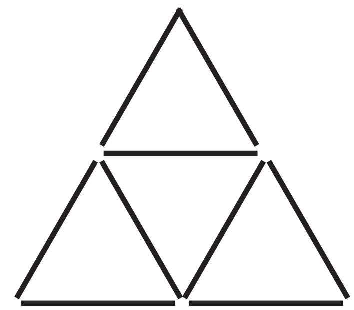


BLM 6: Geostrips

Join with paper fasteners at dots.



BLM 7: Toothpick Triangles



Construct this figure with 9 toothpicks.

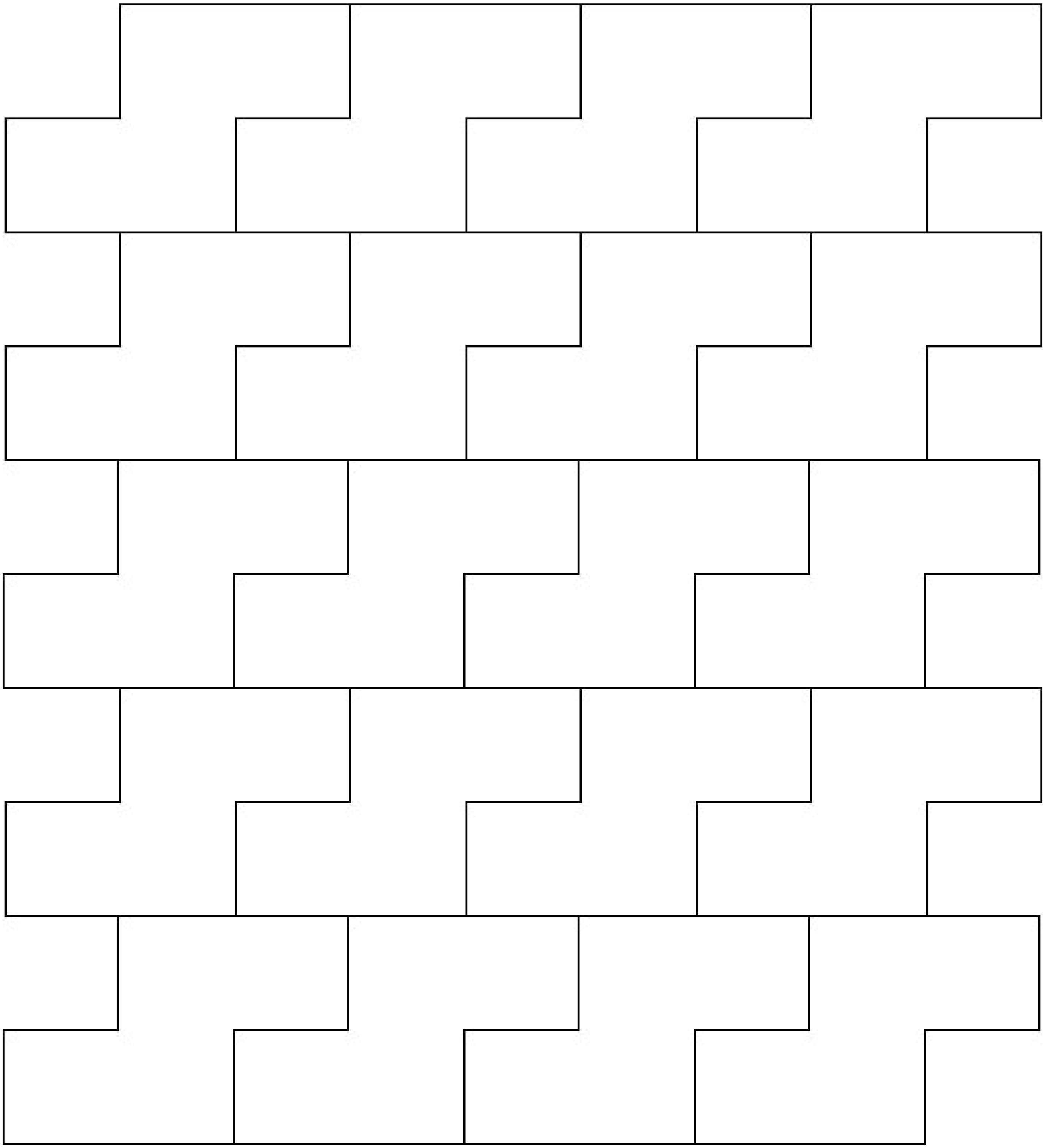
Try to solve the following:

- Remove 2 toothpicks and leave 3 triangles.
- Remove 3 toothpicks and leave 1 triangle.
- Remove 6 toothpicks and leave 1 triangle.
- Remove 4 toothpicks and leave 2 triangles.

NOTE: Each toothpick that is left must be part of a triangle. No pieces should be left sticking out!

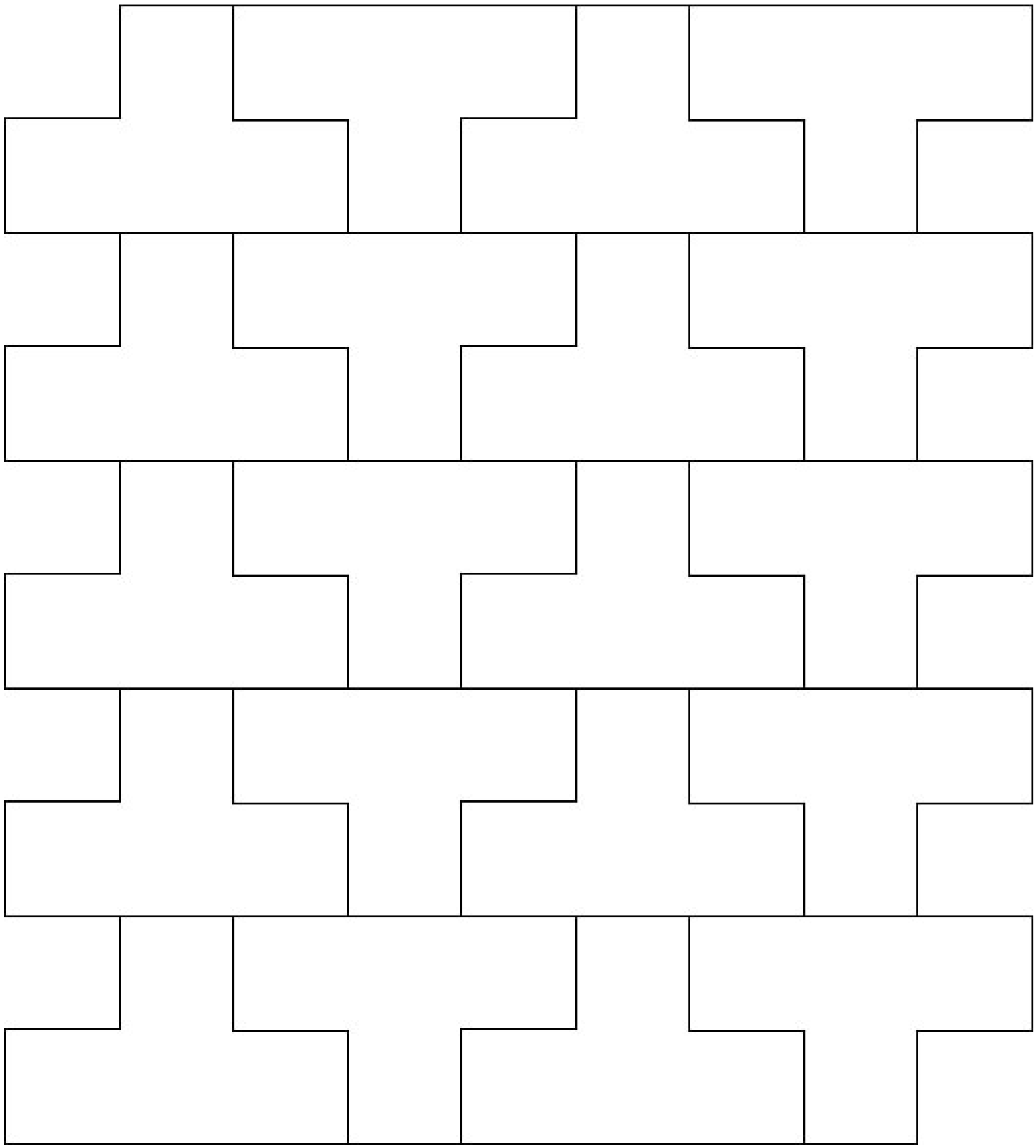
Record your solutions here:

BLM 8: Tetrominoes 1

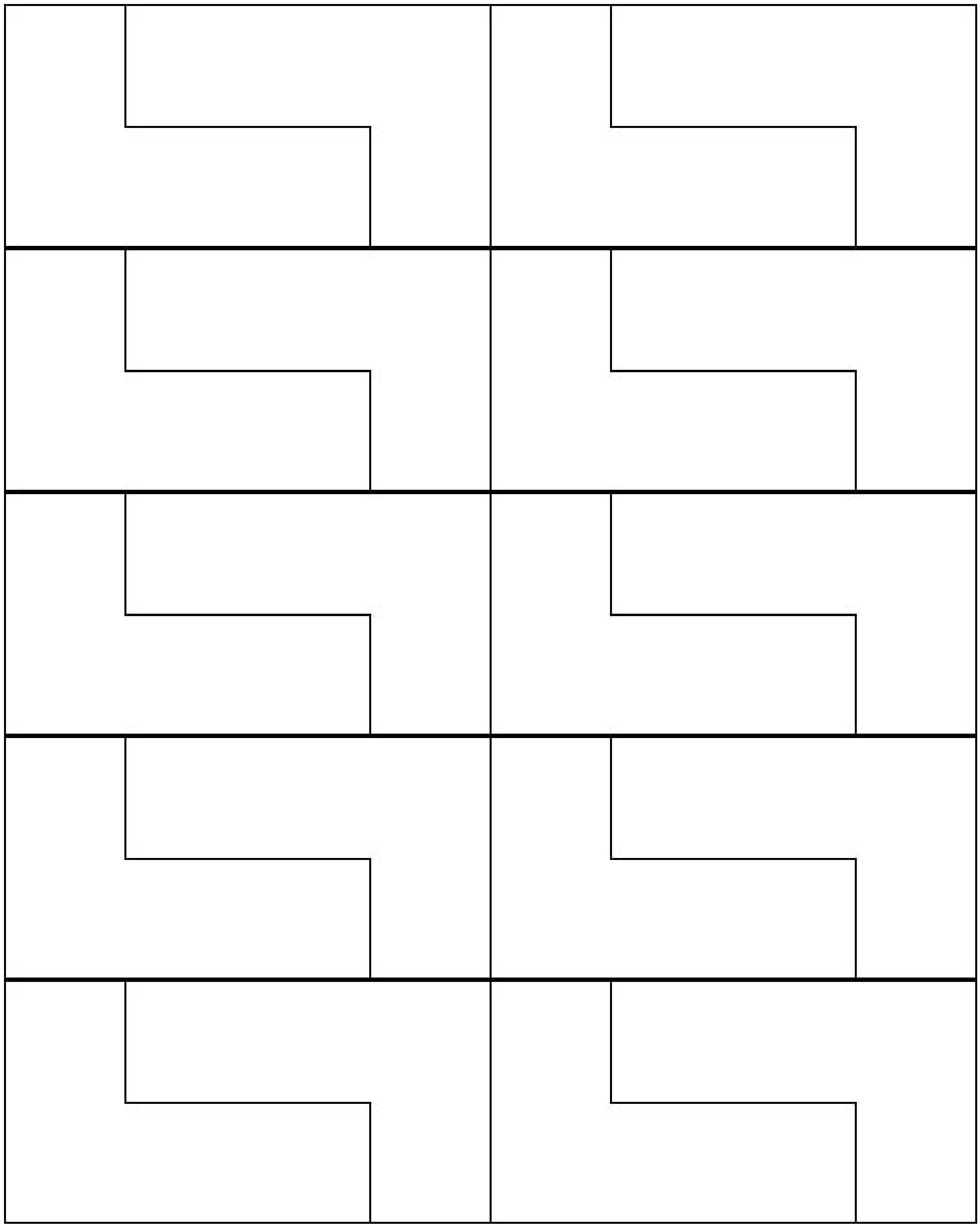


BLM 9: Tetrominoes 2

BLM 10: Tetrominoes 3



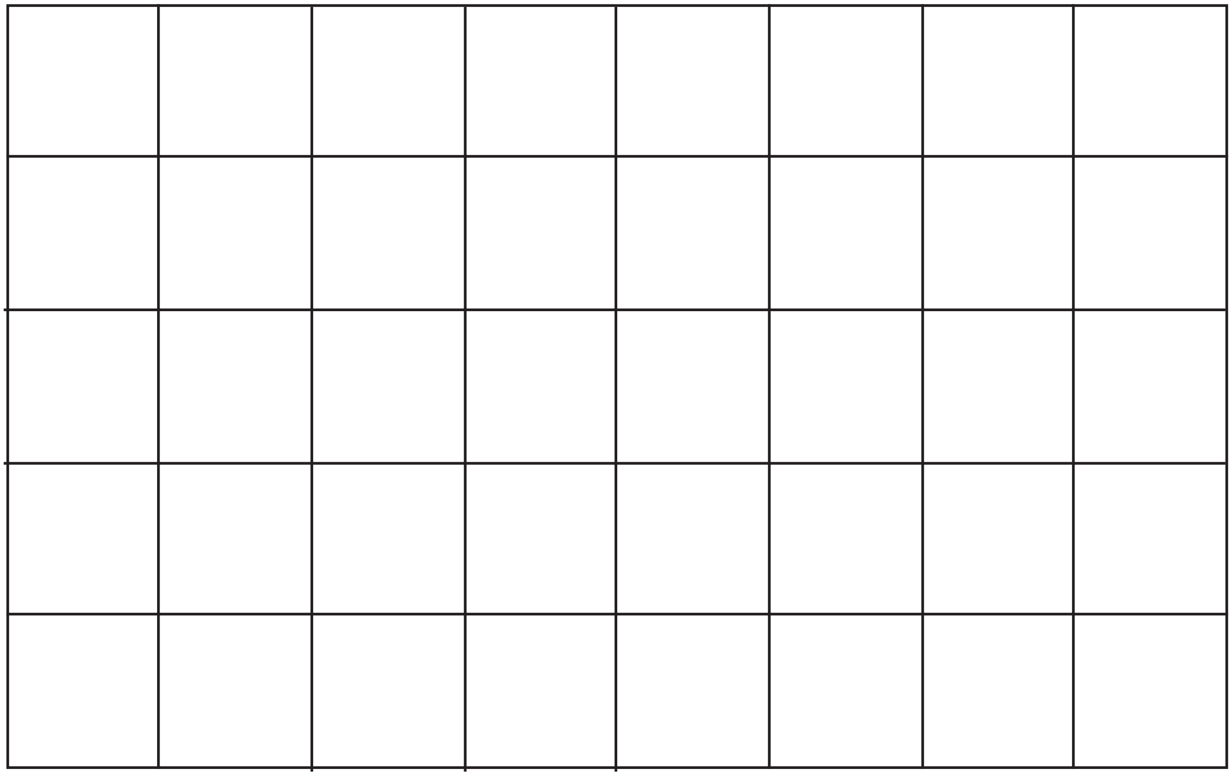
BLM 11: Tetrominoes 4



BLM 12: Tetrominoes 5

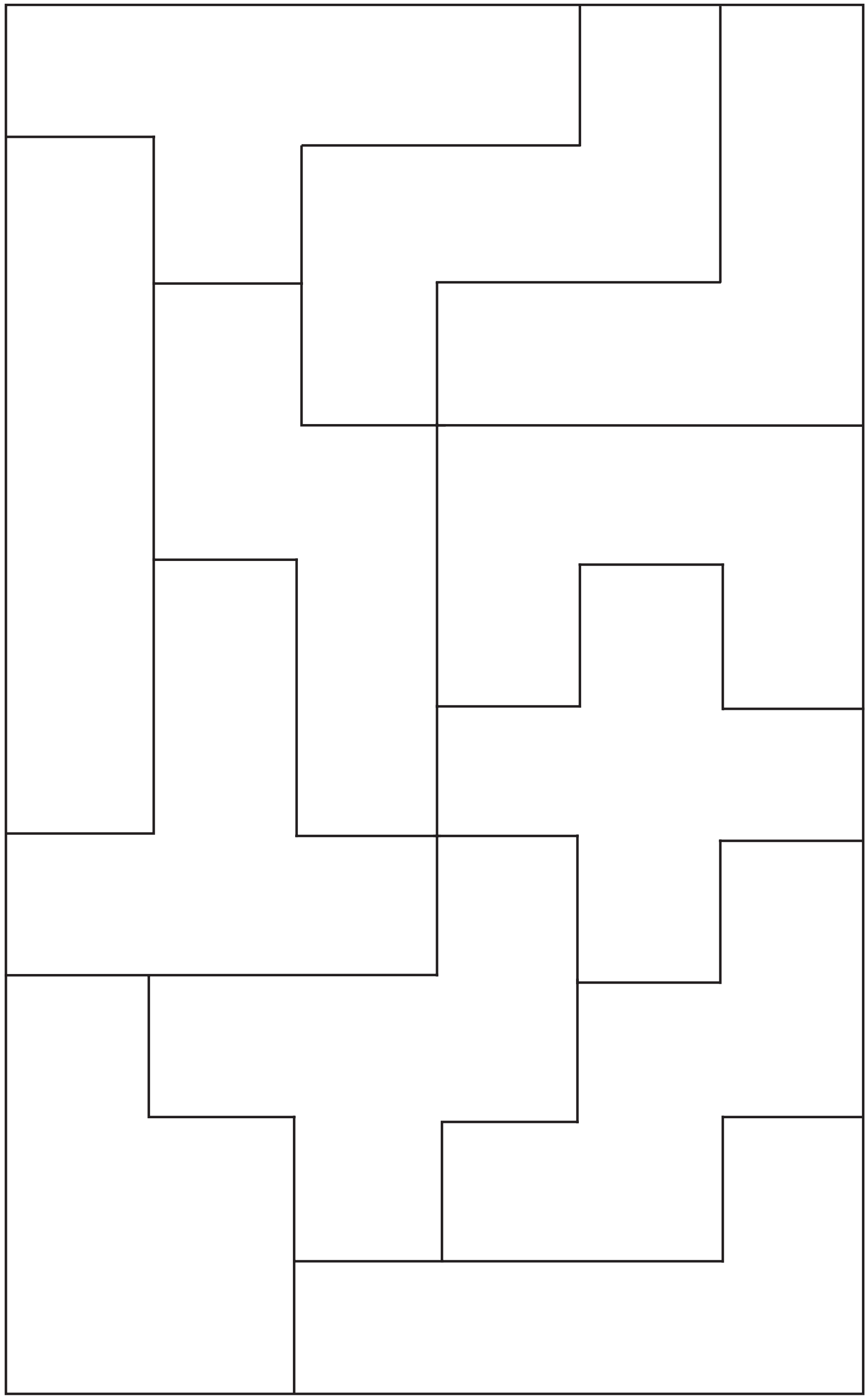
BLM 13: Tetromino Grids

Use **two** sets of tetrominoes to completely fill this grid without gaps or overlaps.



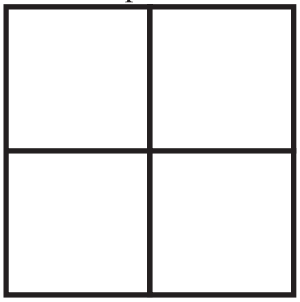
Record your solutions on the blank grid below.

BLM 14: Pentominoes

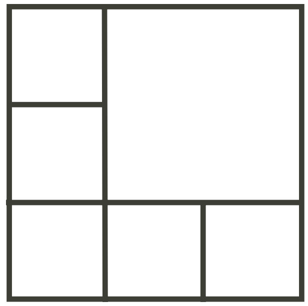


BLM 15: Dissection Squares

How can you cut a square piece of paper into each of the following number of smaller squares: 7, 8, 9, 10, 11, 12, 13, 14, 15? NOTE: Do not count overlapping squares.
For example:



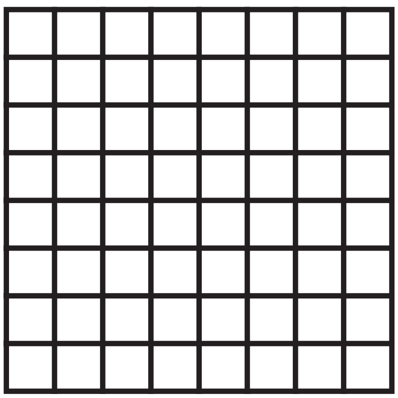
This square has been cut into 4 smaller squares.



This square has been cut into 6 smaller squares.

Record your solutions here:

7



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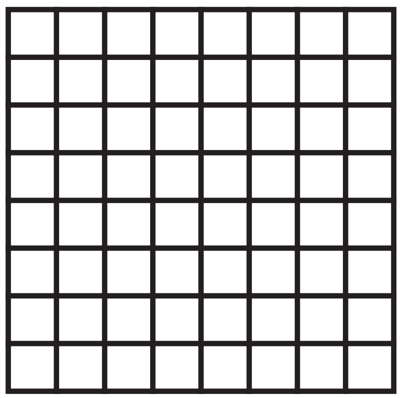

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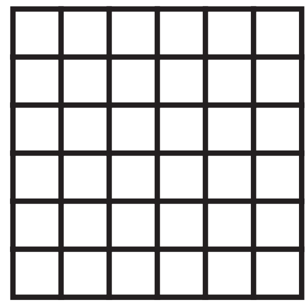

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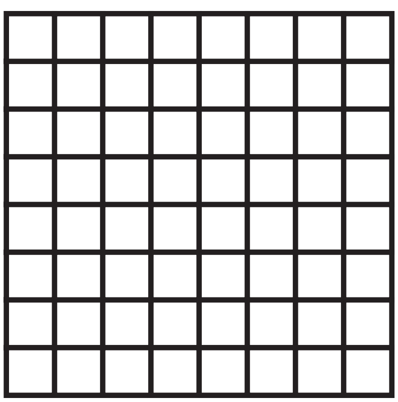

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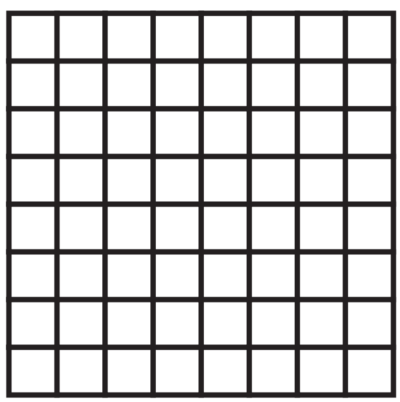

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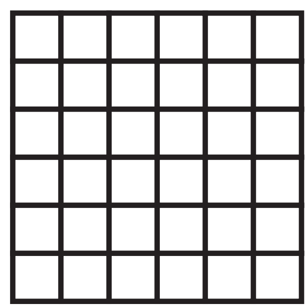

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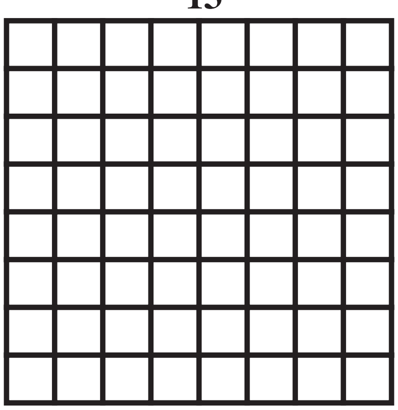

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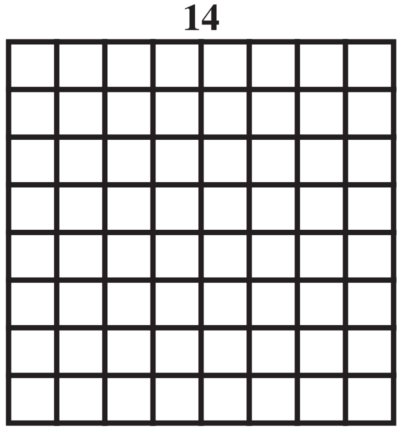

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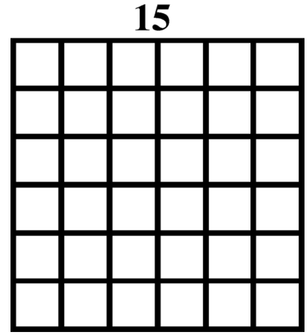
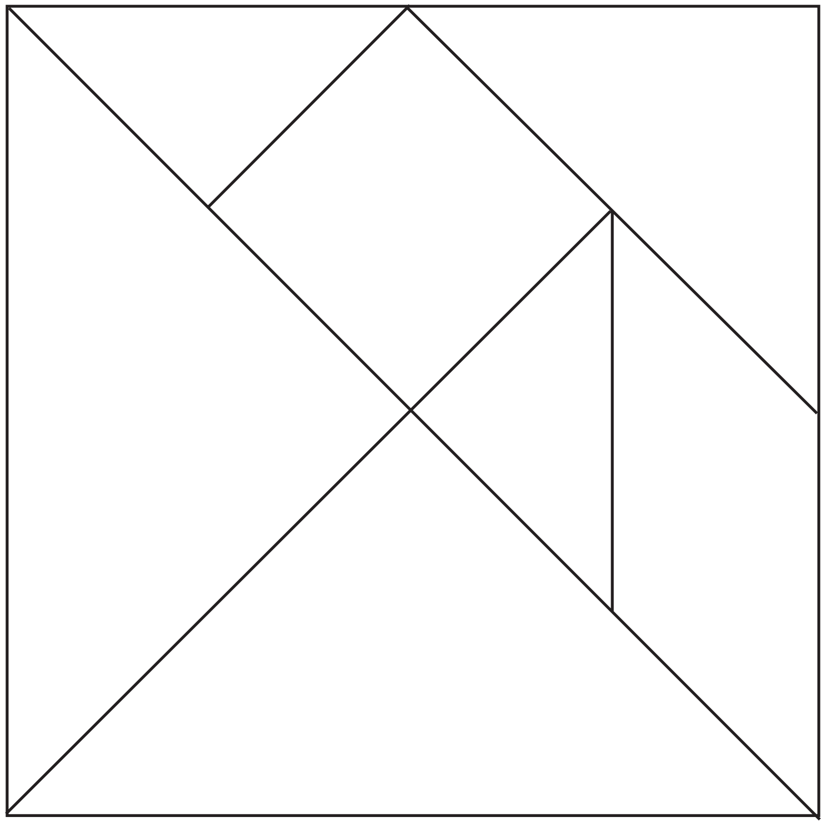


BLM 16: Tangram Templates



BLM 17: Tangram Recording Sheet

Which quadrilaterals can you make using your tangram pieces?
Draw your solution in the appropriate space. If you think there is no solution to one of the problems, explain why.

Shapes / No. of Pieces Used	Square	Rectangle	Parallelogram	Trapezoid
1				
2				
3				
4				
5				
6				
7				

Solutions & Notes

Activity 1: Sorting Polygons

- **Polygons and Non-polygons**

Regular Figures: A, D, F, I, K, O, U, W, X, Y, AA, BB, EE, LL, MM.

Triangles: B, C, H, J, K, V, CC, EE, FF, OO, SS

Quadrilaterals:
Squares: D, F, O, X
Rectangles: E, Q, GG
Parallelograms: Z, DD, HH
Rhombuses: M, II
Trapezoids: N, T
Arrowheads: TT
Kites: L, UU

Pentagons: A, G, I, P

Hexagons: R, S, W, Y, JJ, KK, QQ

Heptagons (also called Septagons): BB, LL

Octagons: U, AA

Non-polygons: NN, PP, RR, VV   Possible reasons students give: NN is not a closed figure; PP has an extra line segment; RR is not made up of line segments; VV has more than one interior space.

Activity 2: Sorting Triangles

The two charts given in the main part of the Activity are incorporated here in the chart from Extension 1.

TRIANGLES	Equilateral	Isosceles	Scalene
Acute-angled	K, EE, MM	C, V, SS	
Right-angled		J, FF	B
Obtuse-angled		OO	H, CC

Extension 2:

The geostrips in b) and e) will not make triangles. Notice that the sum of two sides in each of these sets will be less than or equal to the third side.

b) $5+10<20$ e) $5+10=15$

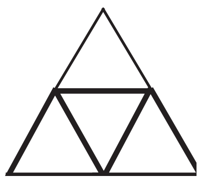
In a), c), d), and f) no matter which two sides you select, the sum of their lengths will be greater than the length of the third side.

Solutions & Notes

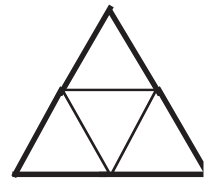
Family Activity 2:

Toothpick Triangles

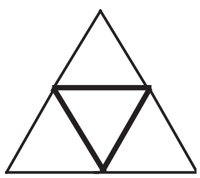
Remove 2 toothpicks and leave 3 triangles.



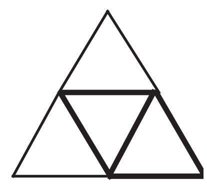
Remove 3 toothpicks and leave 1 triangle



Remove 6 toothpicks and leave 1 triangle.

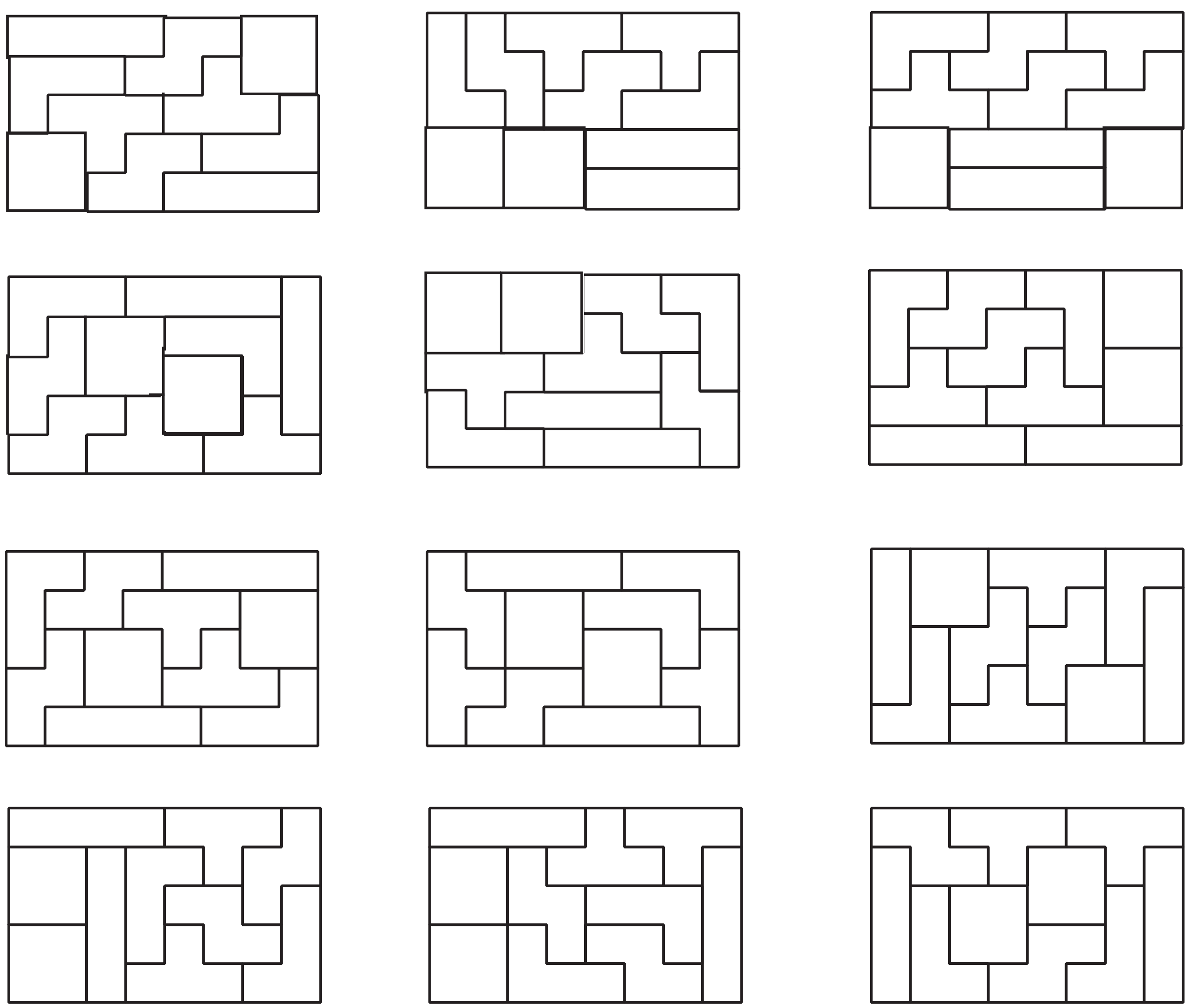


Remove 4 toothpicks and leave 2 triangles.



Activity 3: Tetrominoes

- Twelve of the approximately 800 possible solutions are shown below.

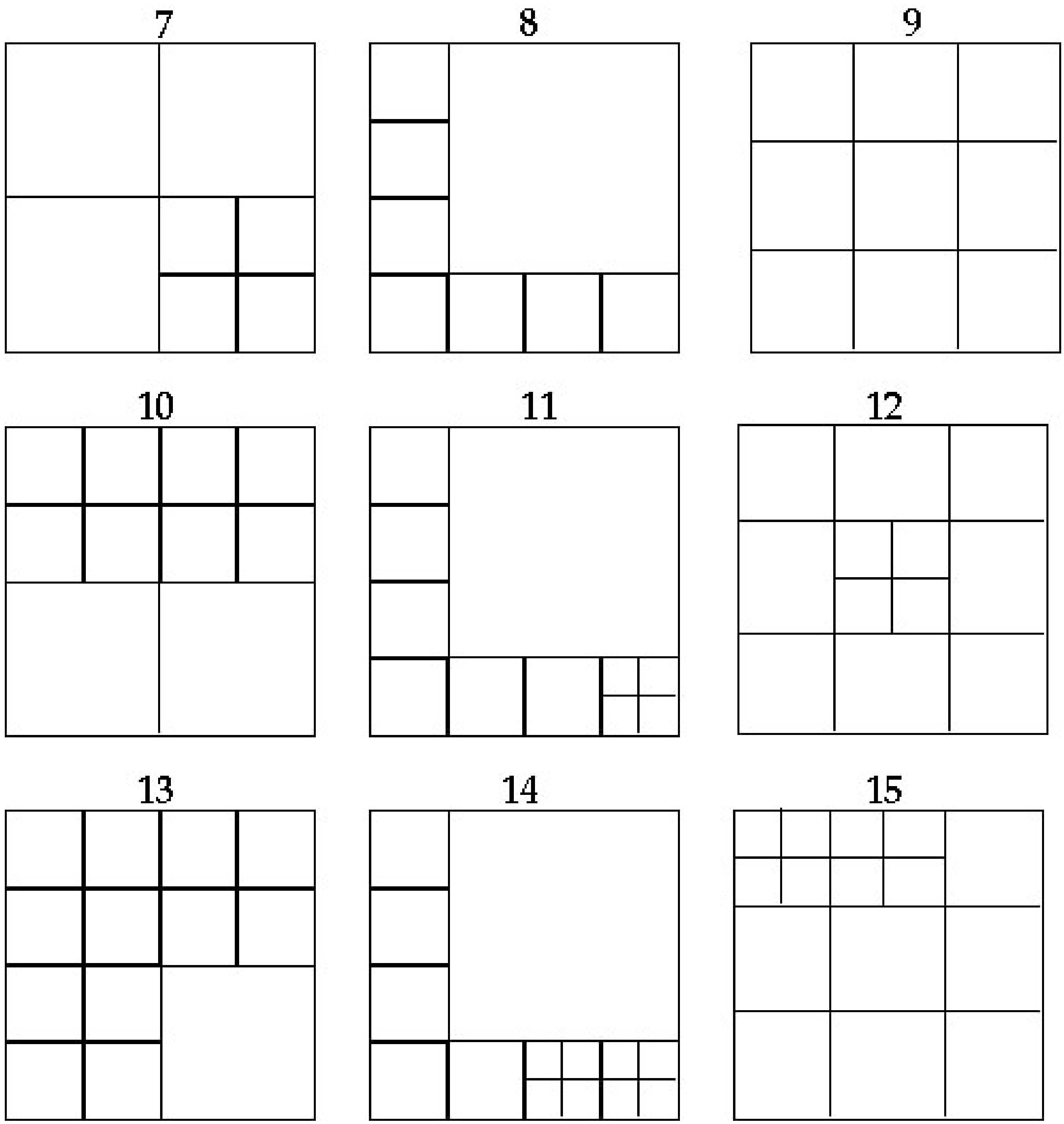


Solutions & Notes

Activity 3 (continued)

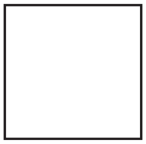
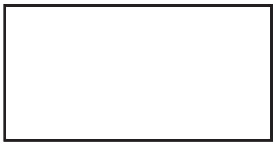
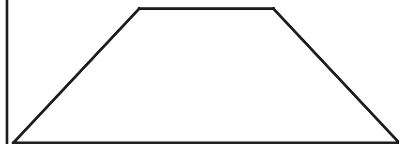
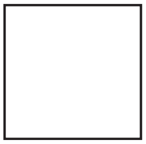
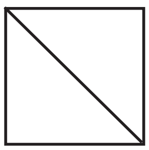
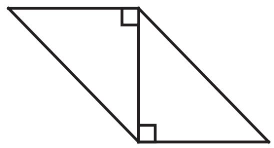
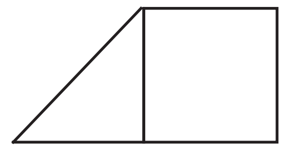
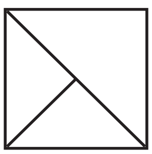
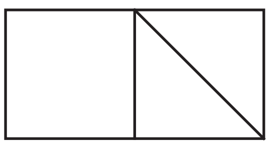
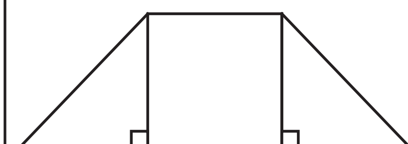
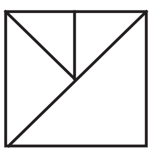
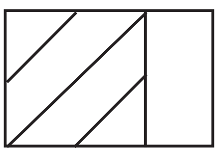
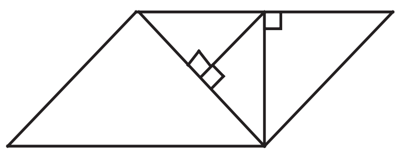
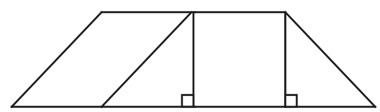
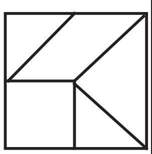
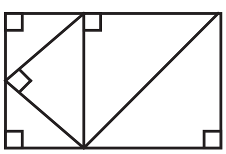
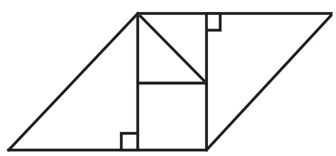
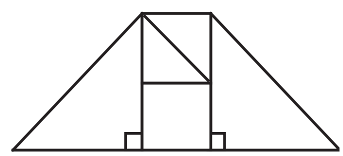
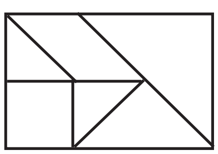
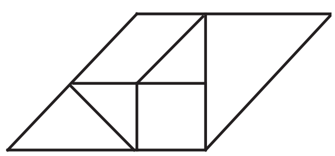
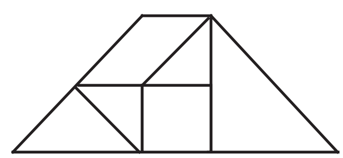
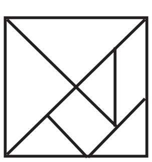
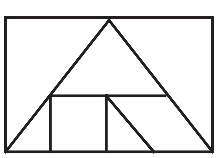
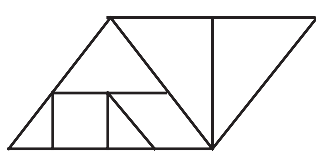
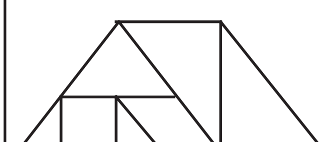
Family Activity 1:

Dissection Squares



Solutions & Notes

Activity 4: Tangram Quadrilaterals

Shapes / No. of Pieces Used	Square	Rectangle	Parallelogram	Trapezoid
				
1		Impossible		Impossible
2		Impossible		
3				
4				
5				
6	Impossible			
7				

NOTE: There may be other solutions.

Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions, by Jean Karr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels. Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. "Assessment", Arithmetic Teacher Focus Issue, February 1992, NCTM.
   This copy of NCTM's journal for elementary school addresses several issues dealing with assessment. It also includes suggested techniques and student activities.

5. How to Evaluate Progress in Problem Solving, by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Suggested Assessment Strategies

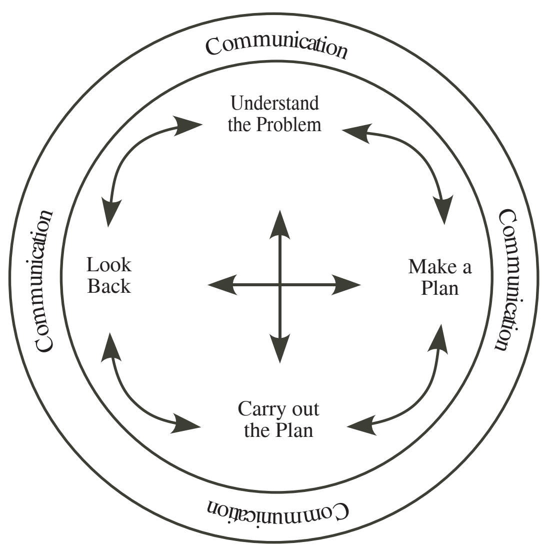
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R's"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;
Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.
Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.
Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
STRATEGIES	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
REASONING	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
RELEVANCE	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

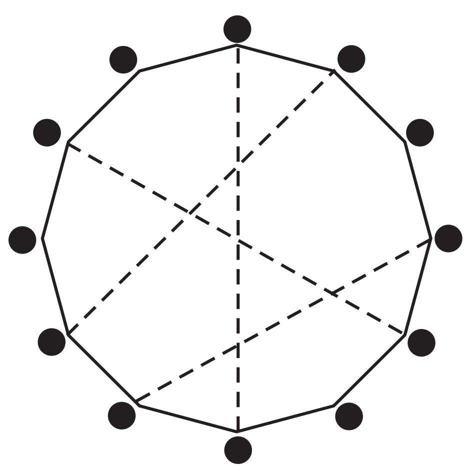
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a "what if...?" problem, such as "What if the question were reversed?", "What if we had other data?", "What if we were to show the data on a different type of graph?".

Suggested Assessment Strategies

Suggested Adaptation of Rubric for Activity 4

The criteria for this adaptation deal with using the correct number of pieces, using only the pieces given the ability to work independently (as an individual or as a group), the construction of correct figures, and the use of mathematical language to describe/explain part or all of the submitted solution.

Neatness is not considered a criterion for the task, although general legibility should be considered. If the completed work is intended for a classroom display you may wish to have students redraw their "rough work". In that case, it would be appropriate to add a criterion to the rubric to deal with appearance of the solution.

For the problem, some criteria below can be applied while students are working on the problem. For example, either observation of discussion can suggest an individual's/group's placement for the third and fifth criteria listed below.

Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
The student • places drawings at inappropriate places in the chart, and gives incomplete/incorrect solutions to several problems	The student • generally shows attention to placement in the chart but gives incomplete/incorrect solutions to some problems	The student • places drawings in correct places in the chart, and gives solutions to all problems; 1 or 2 may be incorrect	The student • places the correct solution in the correct location of the chart, and gives more than one solution to some problems
• uses pieces of more than one Tangram set or may cut pieces apart to get a solution†	• may use pieces of more than one Tangram set to get a solution	• uses Tangram pieces provided and the pieces of only one set	• uses Tangram pieces provided and the pieces of only one set
• shows a lack of application or of technique; student(s) request assistance/hints	• shows willingness to attempt the problem; may become discouraged; may ask for hints	• shows determination; may request to be told if a particular solution is possible	• shows determination and desire to achieve full solution; student(s) discourage hints
• has difficulty accepting that some problems are "impossible"; identifies some "impossible" problems as "possible" and vice versa • finds it very difficult to explain why some solutions do not exist	• identifies some "impossible" problems, but gives solutions for others; may identify some "possible" problems as "impossible" • attempts to explain why some solutions do not exist; may need directed questions to do this satisfactory	• realizes that some problems are "impossible", and will identify these after trying to solve them • can give explanations for "impossible" problems using some mathematics terms, but generally using everyday language	• identifies "impossible" problems fairly quickly • can explain why some problems are "impossible" by referring to the geometric characteristics of the pieces; uses mathematics terms correctly
• gives no evidence of using relationships among Tangram pieces to lead to successive solutions	• gives limited evidence of using relationships among Tangram pieces to work out solutions; verbal descriptions may not refer to these relationships	• gives evidence of using a partial solution to work towards a full one (e.g., replaces square with two small triangles to construct next solution across a row)	• gives evidence of using one solution to get another; of seeing relationships among the pieces and how they fit together; verbal descriptions include these relationships

† This could also be perceived as a creative response!

Other Resources

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Addenda Series, Grades K-6: Geometry and Spatial Sense" by Lorna Morrow and John Del Grande, NCTM, 1993.
   This book includes detailed lessons for each grade from K to 6, emphasizing manipulatives. BLMs are provided.

3. "Addenda Series, Grades K-6: Patterns" by John Firkins, NCTM, 1993.
   This book contains detailed lessons on various aspects of pattern and challenges students to interpret, develop, and extend patterns.

4. "Triangles: Shapes in Math, Science and Nature" by Catherine Sheldrick Ross, KidsCan Press, 1994.
   An intriguing look at the significance of triangles in math, science, and nature through problem solving activities.

5. "Grandfather Tang's Story" by Anne Tompert, Crown Publishers, 1990.
   Grandfather Tang and Little Soo arrange their tangram puzzle pieces to look like fox fairies who can change their shapes.

6. "Moving on With Tangrams" by Judy Goodnow, Shirley Hoogeboom and Ann Roper, Creative Publications, 1988.
   A collection of activities in Black-Line Master form which challenge students to cover various figures with tangrams.

7. "Cooperative Problem Solving with Tangrams" by Ann Roper, Creative Publications, 1989.
   A collection of 80 tangram problems that are to be solved by groups of four students.

8. "Making and Exploring Tangrams" by Andrejs Dunkels in Arithmetic Teacher 37(6), Feb. 1990, pp 38-42, NCTM.
   The article shows how to construct the seven tangram pieces with paper folding and cutting. Relationships of the pieces to each other are brought out. Some puzzles are included.

9. "Pentominoes Revisited" by Barry Onslow in Arithmetic Teacher 37(9), May 1990, pp 5-9, NCTM.
   The article suggests ways of introducing pentominoes, and includes jig-saw type puzzles, tessellations, and games. The 35 hexominoes are given.

10. "Tackling Tangrams" Deanna Rigdon, Jolyn Raleigh, and Shari Goodman in Teaching Children Mathematics, 6(5), January 2000, pp. 304-5, NCTM.
    Problems posed use Tangram pieces to explore topics such as area, perimeter, polygons, and selling brownies.