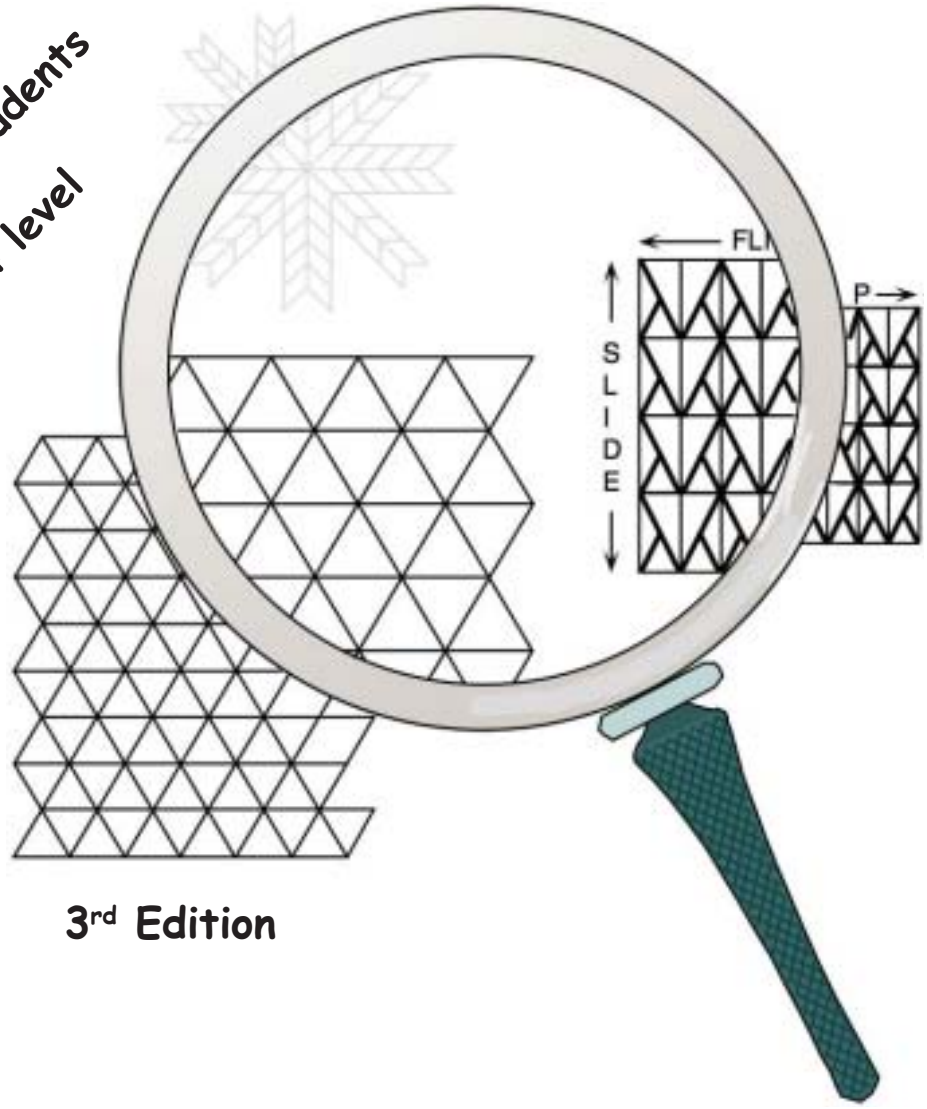
Invitations to Mathematics

Investigations in Geometry:

"Pattern and Colour"

Suggested for students at the Grade 4 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
Faculty of Mathematics, University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

The Centre for Education in Mathematics and Computing
Faculty of Mathematics
University of Waterloo
Waterloo, Ontario Canada N2L 3G1

Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics, among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will enable them to construct their own mathematical meaning and understanding. This emphasis, plus the extensions and related activities included, provide ample scope for all students' interests and ability levels. Related "Family Math" activities which may be used to involve the students' parents are also suggested.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Outcomes are related to Ministry Curricula for the province of Ontario, but are adaptable to other locales.

Investigations in Geometry is comprised of activities to enhance the students' geometry and spatial sense abilities, as well as creativity and problem-solving skills. Geometry concepts are easily integrated with other subject areas, providing ways to demonstrate that mathematics pervades everyday life.

Due to their nature, geometry activities depend heavily on manipulative materials. Every effort has been made to use materials readily available in most classrooms/schools, and a number of Black-Line Masters (BLMs) are provided to add further manipulatives.

Acknowledgements

Contributing Authors

Anne Cirillo (MSSB)
Craig Fleming (WCBE)
Jackie Harris (MSSB)
Kathy Kubota-Zarivnij (MSSB)
Kelly Lantink (WCBE)
Bev Marshman (University of Waterloo)
Brian McCudden, MSSB
Mary Morabito, WCBE
Lorna Morrow (Mathematics Consultant)
Ron Sauer, WCBE

Editors

Bev Marshman
Lorna Morrow

We wish to acknowledge the support of the **Centre for Education in Mathematics and Computing**, and in particular the efforts of Ron Scoins, Barry Ferguson, Larry Davidson, and Patty Mah, and, for the second edition, Bonnie Findlay.

Table of Contents

Overview

Common Beliefs

These activities have been developed within the context of certain beliefs and values about mathematics generally, and geometry specifically. Some of these beliefs are described below.

Geometry is best learned through a combination of active exploration and reasoning, through a cycle of concrete investigations which leads to conjecture, which can then be tested by further concrete investigations.

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Students need a rich learning environment that contains a variety of geometric objects they can manipulate in order to discover the geometric properties of objects and the relationships among them. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures.

The ability to visualize is particularly important in the study of geometry. Students need to be able to draw images in their mind of how figures look, and to be able to manipulate these images mentally.

Students need to develop and use a variety of communication skills in geometry. Geometry uses a formal language to describe objects and their interrelationships and movements in space. In grades 4, 5, 6, however, teaching students the formal language of geometry is not as important as helping them to identify geometric properties and principles and to use pictures or everyday language to explain their observations.

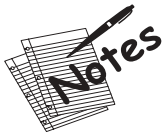
Geometry provides ample opportunity for the development of divergent thinking and creative problem solving, as well as logical thinking ability. Further, exploring problems embedded in 'real-world' settings cultivates the perception that geometry plays a critically significant role in everyday life, rather than being just a set of memorized properties and vocabulary.

Essential Content

The activities in this unit deal primarily with the ideas of symmetry, of slides, flips, and turns, and of pattern in its many manifestations. In addition, there are Extensions in Mathematics and Cross-Curricular Activities suggested for some lessons. These may be used prior to or during the activity as well as following the activity. They are intended to suggest topics for extending the mathematics of the activity and for integration with other subjects.

During this unit, the student will:

- identify a variety of types of pattern, including patterns in daily life;
- use concrete materials to create geometric patterns;
- use various materials to create other types of patterns;
- identify slides, flips, and turns;
- identify examples of symmetry;
- demonstrate slides, flips, and turns using concrete materials;
- create symmetrical designs, and examples of symmetry;
- describe a variety of patterns.



Curriculum Connections

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **What Is Pattern?**	• identifying types of pattern • creating patterns through music, drawing, use of concrete materials, etc.	• describe patterns encountered in any context (e.g., quilt patterns) • identify, extend, and create linear and non-linear geometric patterns
Activity 2 **Towers and Barriers**	• creating recipe-type patterns • following recipe-style pattern instructions • using mathematical language	• describe patterns encountered in any context (e.g., quilt patterns) • use language effectively to describe geometric concepts
Activities 3 and 4 **Patterns from Slides and Flips** **and** **Patterns From Turns**	• identifying slides and flips of a given motif in a given pattern • using slides and flips to create a two-dimensional pattern • identifying turns of a given motif in a given pattern • using slides, flips, and turns to create a two-dimensional pattern	• identify, extend, and create patterns by changing two or more attributes (e.g., orientation) • identify, extend, and create linear and non-linear geometric patterns • demonstrate an understanding of translations, reflections, and rotations • discover transformation patterns
Activity 5 **Quilt Patterns**	• identifying slides, flips, and turns in quilt patterns • creating quilt patterns using slides, flips, and turns	• identify, extend, and create linear and non-linear geometric patterns • describe patterns encountered in any context (e.g., quilt patterns) • explore transformations of geometric figures
Activity 6 **Symmetry in Patterns (optional)**	• identifying lines of symmetry in patterns • creating patterns with lines of symmetry	• understand key concepts in transformational geometry using concrete materials and drawings • draw lines of symmetry on two-dimensional shapes

Overview

ASSESSMENT

Assessment is a process of gathering evidence about a student's knowledge, skills, and values, and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned, but mathematical competence involves such characteristics as the ability to communicate, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. Because of the nature of the activities it is suggested that a variety of types of assessment be used. Suggestions include:

(i) observing students as they work to see if they are applying various concepts; to see if they are working cooperatively; to observe their committment to the tasks;

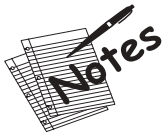
(ii) assessing the completed project to see if instructions have been followed; to see if concepts have been applied correctly; to see if the language of mathematics has been used correctly;

(iii) assessing the students' descriptions of their completed work to see if mathematical language is used correctly; to see if students understand the concepts used;

(iv) providing opportunities for student self-assessment: have students write explanations of their understanding, opinion, or feelings about an activity. One technique is to have them write under the headings What I Did, What I Learned, and How I Felt About It. Students could be asked to write a review of one day's activities or of the whole unit's work.

(v) selecting an exemplary piece of work to be included in a portfolio for assessment purposes or for sharing with parents.

See Suggested Assessment Strategies, page 41, for further discussion and sample rubrics.



Overview

Prerequisites

Although students may find more examples of slides, flips, and turns in their designs if they have had some earlier experience with these, such pre-experience is not necessary. The activities can be used as an introduction to these topics as well as an application of them. Some familiarity with symmetry is advised if you wish to examine patterns and designs for symmetry. Activity 6, Symmetry in Patterns, is an optional activity that can be used to introduce or review the concept of symmetry. This activity could be done early in the unit if materials are available.

Logos

The following logos, which are located in the margins, identify opportunities for:

Problem Solving

Communication

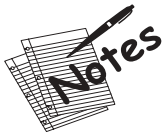
Assessment

Use of Technology

Overview

MATERIALS

ACTIVITY	MATERIALS
Activity #1 **What is Pattern?**	• Chart paper • Samples of different types of patterns (e.g., cross-stitch chart, set of knitting instructions, sewing pattern, recipe, pictures of tile patterns, wallpaper and fabric samples, plan for a bird feeder, etc.) • Rhythmic instruments (e.g., sticks, bells), or other simple musical instruments. • Colored paper, markers, crayons, scissors, glue
Activity #2 **Towers and Barriers**	• A variety of three-dimensional objects, which should include rectangular prisms, cylinders, spheres, cubes, pyramids, and cones. These can be commercial models, wooden blocks, or recyclable materials such as boxes, paper rolls, balls, etc. Each pair of students needs **two identical sets** of four or five different objects. • Self-standing barriers, such as large books or file folders
Activities #3, 4 **Patterns from Slides, Flips, & Turns**	• Copies of BLMs 1, 2, 3, 4, 5 • Scissors • Rulers (optional)
Activity #5 **Quilt Patterns**	• Copies of BLMs 6, 7, 8, 9, 10, 11, 12 • Completed projects from previous activities • Materials gathered as suggested in Cross-Curricular Activities for Activity 1 (optional) • Construction paper • Crayons, markers, glue, found materials, scissors, rulers
Activity #6 (optional) **Symmetry in Patterns**	• Pattern Blocks or BLMs 13, 14, 15, 16, 17 • Small mirrors (about 5 cm by 8 cm) • Scissors



Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week, students in our classroom will be participating in a Geometry unit titled "Pattern and Colour". The classroom activities will focus on examining, comparing, creating, and analyzing a wide variety of patterns and pattern types.

You can assist your child in understanding the relevant concepts by working together to look for patterns in your home (e.g., bathroom tiles, quilts, rhythm patterns in a favourite song, 'recipe-type' patterns such as cooking recipes or knitting patterns) and recording them in some way (e.g., make a sketch, take a photo, write a description).

Any items/patterns that you are willing to share with our class as we collect a variety of patterns of different types would be very welcome. If you work with patterns in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe the task to his/her classmates.

Sincerely,

Teacher's Signature

A Note to the Teacher:
If you make use of the suggested Family Activities, it is best to schedule class time for sharing and discussion of results.

Activity 1: What is Pattern?

Focus of Activity:

- Meanings of the term "pattern"

What to Assess:

- Basic understanding of the concept of pattern
- Ability to create and describe a pattern

Preparation:

- See the table on page 5 for materials
- Gather the particular samples desired to open the discussion
- You may wish to delegate a student to record the results of the discussion on chart paper

Activity:

INTRODUCTION

Display a variety of patterns and elicit from students descriptions of different types. Create a list on chart paper.

Types of patterns may include the following:

- repeated motif of a visual nature (could be linear, two- or three-dimensional, black and white or coloured)
- recipe-type pattern (a set of knitting instructions, a sheet of music, a cookie recipe ...)
- rhythmic forms (music, drums/percussion, heartbeat, hoofbeats of a galloping horse, the sound of someone's footsteps, ...)
- blueprint/scale drawing (houseplan, cross-stitch pattern, model airplane plan, ...)
- dance forms (square dance, line dance, circle dance, ...)
- number patterns (2, 4, 6, 8, ...; 1, , , , ...; 0, 1, 0 , 1, ...)

CREATING A PATTERN

In groups of 2 to 4, have students create a pattern of one of the types identified, first choosing which type they wish to create. Students may use any of the materials available.

Note: At this introductory stage, the patterns may be fairly simple; the main thing is to establish the concept, namely that a pattern is either an algorithm/ prescription or has repeated motifs, or possibly both.

DEMONSTRATING THE PATTERNS

Visual patterns can be posted, as completed.

Have each group demonstrate the pattern created (rhythm, dance, drawing, recipe, ...) and explain briefly why it is of the type of pattern identified. Alternatively, other students could try to identify the type of pattern and then the "authors" of the pattern can agree or disagree and explain their reasons.

Assessment

Comments in italics are explanatory, and need not be conveyed to the students.

Communication

Activity 1: What is Pattern?

Record on chart paper the names of students opposite each type of pattern originally listed. This will help to reaffirm the classification of patterns established initially.

Extensions in Mathematics:

1. Take a walk around the school (or neighbourhood), recording and identifying any patterns discovered.

Cross-curricular Activities:

1. Think about how we *respond* to patterns. Why are some motifs beautiful, others jarring? Why does some music soothe us, while other music makes us want to dance or sing? Do different people respond differently to the *same* pattern?

2. Does every pattern have a *purpose*? What are some of the purposes?

3. What do we mean by a "behaviour" pattern? Can you think of some ways in which *your* behaviour has a pattern-like predictability?

Family Activities:

1. Collect patterns from home (e.g., wallpaper, tiles, fabric, wrapping paper, recipes, ...) or from newspapers/magazines/books, and classify them.

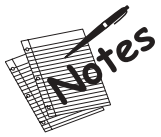
2. Bring materials to class that reflect patterns in nature (e.g., leaves, sea shells, pine cones).

Other Resources

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

3. Addenda Series, Grades K-6, Pattern
11. Invitations to Mathematics; Investigations in Patterns and Algebra, Grade 4.

Activity 2: Towers and Barriers



Focus of Activity:

- Recipe-type patterns
- Communication skills
- Sketching three-dimensional structures

What to Assess:

- Correct use of language (names/descriptions of objects)
- Accuracy of diagrams
- Ability to communicate ideas

Preparation:

- See the table on page 5 for materials
- Make, or have students make, two identical sets of three-dimensional objects for each pair of students (e.g., building blocks, small boxes)
- Provide materials for the "barrier" between students as described below.

Activity:

INTRODUCTION

Describe the following activity to students. You might choose to have two students try the activity as a demonstration. Students should work in pairs.

TOWER BUILDING

Students in each pair sit opposite each other at a table or desk with the barrier between, so they cannot see each other's work. Each child in the pair has one of the two identical sets of objects.

One student creates a tower with her/his materials.

When the tower is complete, (s)he gives a verbal description of the tower to her/his partner, who then tries to build a duplicate tower based on the verbal description. The partner may ask clarifying questions, but may not look at the completed tower.

Problem Solving

When satisfied, the students compare towers. They discuss the initial description and revise it orally until it is clear.

Together, they write down a recipe for their tower; that is, an ordered description or pattern of how to build the tower. They also complete a labelled diagram and paste it on the back of the directions.

Communication

If time permits, repeat the Tower Building activity. The student who built the first tower becomes the one who receives the instructions. The student who previously received the instructions now builds the tower.

Activity 2: Towers and Barriers

Trading Towers

Pairs should be paired for this activity. for example, Pair A attempts to build (one of) Pair B's tower(s) and Pair B attempts to build (one of) Pair A's tower(s).

Each pair exchanges recipes and sets of objects with the other pair. Each pair reads the new recipe and attempts to build a tower according to the instructions. The students then check with the diagram on the back of the directions.

Assessment

Each set of two pairs meets to discuss the towers and the clarity of the instructions. Suggestions for revisions should be considered and made if necessary.

Extensions in Mathematics:

1. The activity can be repeated in different ways:
   - instructions are given with no questions allowed;
   - instructions are given with only "yes" or "no" answers allowed.

2. The activity could be extended to a greater number of figures used. Alternatively, two-dimensional figures could be included.

3. The instructions and sets of materials could be gathered and left in a centre for further exploration.

Cross-curricular Activities:

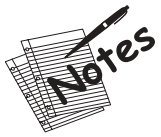
1. Technical writing (like the tower recipes) is an important skill which needs practice for competence. It can be extended into other curriculum areas. For example, write a description of how to put on your socks, write instructions for completing a specific work of art or craft, write detailed descriptions for duplicating inventions or technology. See also Activity 5 suggestions for describing a quilt square.

2. Labelled diagrams can be used in science, social studies, technology, etc. They can be detailed diagrams of completed work or planning sketches to help create the work.

3. Pop-up books can be created to illustrate towers. Students can write creative stories and/or use technical writing to explain how to create the book.

4. Structures can be built with commercial or found materials, as part of integrated curriculum — for example, building a castle, which may be related to your language arts or social studies program. Again both creative (a fairy tale or medieval adventure) and/or technical writing (how to build a castle) and labelled sketches can be used as recording devices.

Activity 2: Towers and Barriers



Family Activities:

1. Collect recipe-type patterns from home. Try to find patterns that are used in different ways (e.g., cooking, building a Lego structure). Make a list of them to bring to class.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

3. Addenda Series, Grades K-6: Patterns

Activity 3: Patterns from Slides and Flips

Focus of Activity:

- The meaning of and experience with slides and flips.
  (Previous experience with these concepts is not essential.)

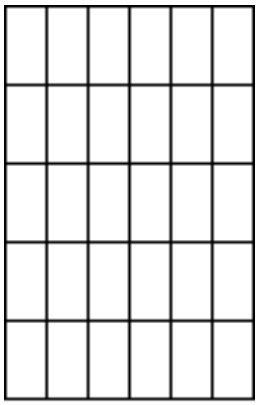
Note: Turns are usually more difficult for students to visualize. They will be dealt with in Activity Four.

What to Assess:

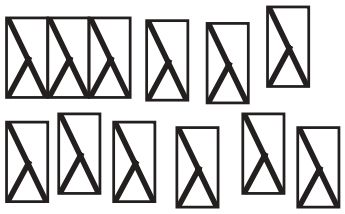
- Correct use of language (slide, flip)
- Correct demonstration of slide, flip
- Identification of visual patterns

Preparation:

- See the table on page 5 for materials
- Make a copy of BLMs 1 and 4 for each student
- Make an acetate copy of each for use on the overhead for demonstration. Cut apart the individual rectangular motifs on BLM 1. (Cut on the dotted lines)



BLM 4



Motifs from BLM 1

Activity:

INTRODUCTION

Place BLM 4 on the overhead. Place an individual motif in the upper left rectangle on the grid.

Match a second motif to the first and illustrate how to flip the motif into the neighbouring rectangle.

Continue across a row to make a pattern of flips.

Ask a student to place the motif in the first rectangle of the second row. Have students decide whether this should be in the same orientation as the first-placed motif (i.e., a slide image) or if it should be a flip image of the first. Use flips to complete the row.

Activity 3: Patterns from Slides and Flips

Repeat for the third row.

Note: The motifs are related by flips as you move across a row. They should also be related by a consistent motion as you move down a column. That is, if the students decided that the upper left motif and the one immediately under it are related by a slide, then slides should be obvious when moving down any column. (See Figure 3.1 below.) If students decided that the upper left motif should be flipped downwards to fill in the space just under it, then the overall design should illustrate flips both horizontally and vertically. (See Figure 3.2 below.) Without this consistency, it may be difficult to identify any overall pattern when the entire grid is filled in.

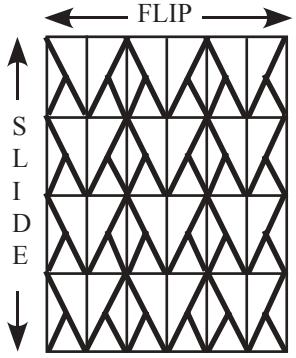


Figure 3.1

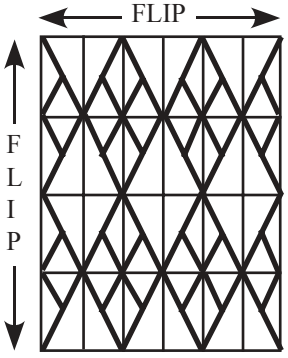
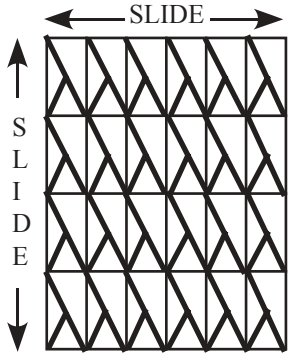
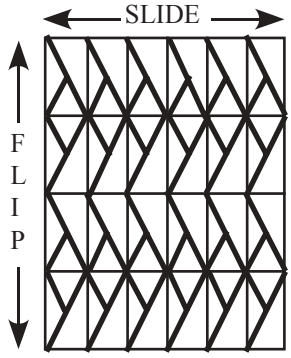


Figure 3.2

Two other examples of patterns using slides and flips are shown below.





Since the four patterns illustrated above are the only possibilities using just slides and flips, students' patterns should be identical to one of these. However, when students colour their designs to "bring out the pattern you want people to see" the finished products for the same pattern may look quite different.

Creating Patterns

Students will need to cut the motifs apart.

If they wish to illustrate a flip pattern they will need to draw the motif on the back of each motif 'card'. This can be done easily if the motif is held up to the light (against a window, for example). Alternatively, you may wish to prepare acetate copies of BLM 1 for students to cut apart. In this case, it will be easy for them to illustrate flips.

Comments in italics are explanatory, and need not be conveyed to the students.

Assessment

Problem Solving

Communication

Activity 3: Patterns from Slides and Flips

Students should be allowed time to experiment to derive patterns they like. Then either the motifs can be pasted onto the grid or students can draw the motifs on the grid using rulers. If you wish the students to keep the motifs for use in Activity Four, then they should not be pasted onto the grids at this stage. (You may wish to discourage the use of turns until Activity 4 or you may wish to see how well students can deal with turns and turn images.)

Students should colour their patterns to emphasize the pattern they want others to see. This means they should judge colours as to whether, for example, they are dominant or receding colours.

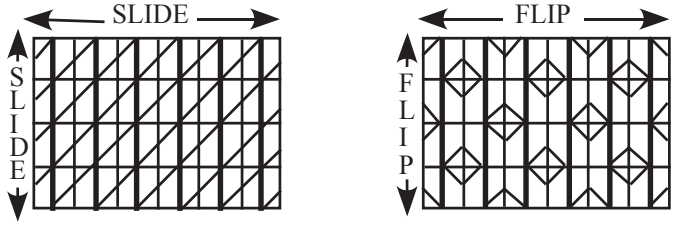
DISCUSSION

Students display their patterns. Other students try to determine whether slides or flips (or some combination) were used to generate the pattern. Students explain why they chose the colours they did. For example, they may have wanted to bring out a particular repeated figure; they may have wished to retain or eliminate symmetries that were part of the line drawings.

Keep the patterns for use in Activity 5.

Extensions in Mathematics:

1. Use the motifs from BLM 2 or 3 on the grid on BLM 5.
   Two examples using the motifs from BLM 2 shown below.



Family Activities:

1. Look for examples of slides and flips in patterns you see at home. Examine fabric, wallpaper, wrapping paper. Bring an example to school.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

2. Addenda Series, Grades K-6: Geometry and Spatial Sense

Activity 4: Patterns from Turns

Focus of Activity:

- The meaning of and experience with turns.
  (Previous experience with this concept is not essential.)

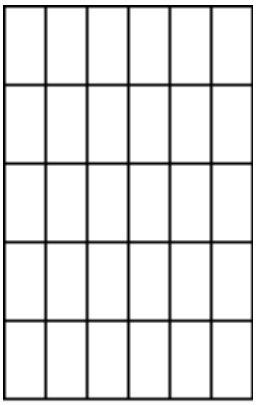
Note: Turns are usually more difficult for students to visualize. Thus, the use of manipulatives is essential.

What to Assess:

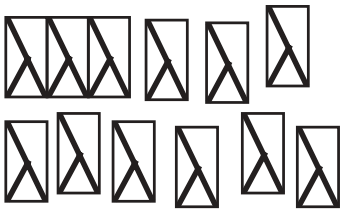
- Correct use of language (slide, flip, turn)
- Correct demonstration of slide, flip, turn
- Identification of visual patterns

Preparation:

- See the table on page 5 for materials
- Make a copy of BLMs 1 and 4 for each student. If students drew their designs in Activity 3 rather than pasting the motifs to the grid, they can use the same motifs they used for Activity 3. Otherwise they will need to cut apart another set.
- Use the acetate copy of each that you made for Activity 3 to illustrate turns for the students



BLM 4

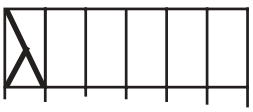


Motifs from BLM 1

Activity:

INTRODUCTION

Place BLM 4 on the overhead. Place an individual motif in the upper left rectangle on the grid.



Match a second motif to the first and illustrate a half-turn to place the motif in the neighbouring rectangle.

Continue across a row to make a pattern of half-turns.

Comments in italics are explanatory, and need not be conveyed to the students.

Activity 4: Patterns from Turns

Ask a student to place the motif in the first rectangle of the second row. Have students decide whether this should be in the same orientation as the first-placed motif (i.e., a slide image) or if it should be a flip or turn image of the first. Use half-turns to complete the horizontal row.

Repeat for the third row.

Note: If the students decided that the upper left motif and the one immediately under it are related by a slide, then slides should be obvious when moving down any column. (See Figure 4.1 below.) If students decided that the upper left motif will be flipped downwards to fill in the space just under it, then the overall design should illustrate flips vertically and turns horizontally. (See Figure 4.2 below.) If they decided to use a half-turn downwards, then the completed design should resemble Figure 4.3.

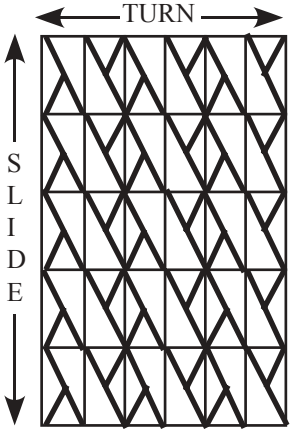


Figure 4.1

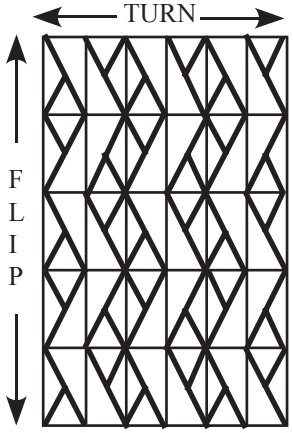


Figure 4.2

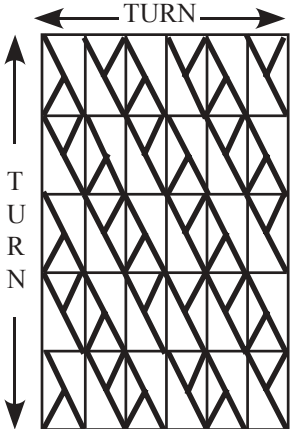


Figure 4.3

Creating Patterns

If students wish to use a flip in any direction they will need to draw the motif on the back of each motif 'card' as described in Activity 3, unless they have been given motifs printed on acetate.

Students should be allowed time to experiment to derive patterns they like. Then either the motifs can be pasted onto the grid or students can draw the motifs on the grid using rulers.

Students may be surprised to notice that the same combination (e.g., slide and turn) may give a different pattern if the two motions are switched. Compare Figure 4.4 below with Figure 4.1 above. Both use a slide and a turn but the different patterns are a result of the motions being used in different directions.

Activity 4: Patterns from Turns

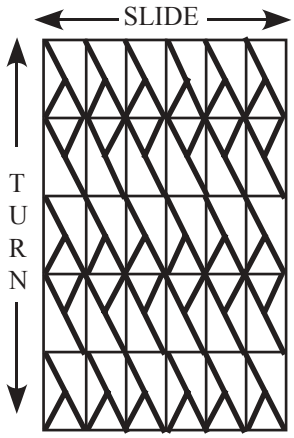


Figure 4.4

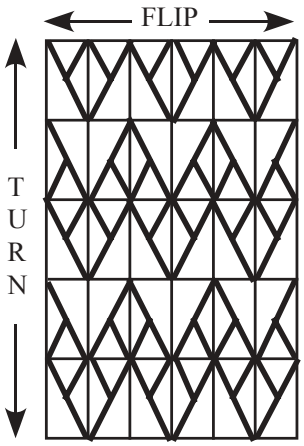


Figure 4.5

Similarly, compare Figure 4.5 with Figure 4.2. Both use a flip and a turn but they appear to be quite different.

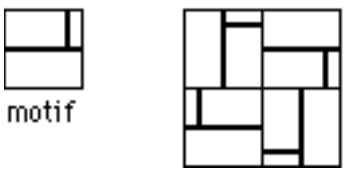
Students should colour their patterns to emphasize the pattern they want others to see. This means they should judge colours as to whether, for example, they are dominant or receding colours.

DISCUSSION

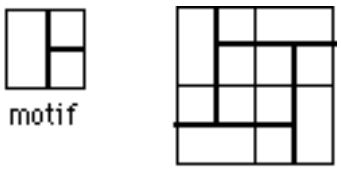
Students display their patterns. Other students try to determine whether slides, flips, or turns (or some combination) were used to generate the pattern. Students explain why they chose the colours they did. For example, they may have wanted to bring out a particular repeated figure; they may have wished to retain or eliminate symmetries that were part of the line drawings. *Keep the completed patterns for use in Activity 5.*

Extensions in Mathematics:

1. Use motifs from BLMs 2 and 3 on the grid on BLM 5.
2. Students may wish to try designing their own motifs and grids. Note that line segments on the motifs should not, when arranged on the grid, allow loose ends. (See Example 1 below.) Segments in the motif should connect with other segments. (See Example 2 below.)



Example 1



Example 2

Both Examples 1 and 2 illustrate quarter-turns which are possible on a square grid but not on a rectangular grid.

Notes

Problem Solving

Communication

Assessment

Use of Technology

Activity 4: Patterns from Turns

Cross-Curricular Activities:

1. Have students explore the web and other resources for information about tessellations in art or architecture. Perhaps they can find out about the tiling patterns in the famous Alhambra in Spain.

Family Activities:

1. Look for examples of turns in patterns you see at home. Examine fabric, wallpaper, wrapping paper. Bring an example to school.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

2. Addenda Series, Grades K-6: Geometry and Spatial Sense

Activity 5: Quilt Patterns

Focus of Activity:

- Visual patterns.
- Slides, flips, and turns.
- Symmetry (optional)

What to Assess:

- Use of the concepts of slides, flips, and turns
- Correct use of language

Preparation:

- See the table on page 5 for materials
- Make 6-8 copies of each of BLMs 6, 7, 8, 9, 10, 11, and 12
- Cut squares of construction paper about 20 cm to a side

Activity:

INTRODUCTION

Examine designs from Activities 3 and 4 for slides, flips, and turns. If students are familiar with lines of symmetry and/or turn symmetry, look for examples of these as well.

Distribute copies of the quilt patterns on BLMs 6 to 12 at random. Students should have these to examine for the first part of the discussion.

Discuss what a quilt is and how pattern is used. Note that some quilts (BLMs 11, 12) have one large overall design while others (BLMs 6, 7, 8, 9, 10) have a repeated motif within a square. Discuss ways their Activity 3 and 4 designs are like quilts and ways they are different. (Most traditional quilts are based on squares with the design made up of smaller squares and triangles.)

Look for examples of slides, flips, and turns (and symmetry, if appropriate) in the quilt designs on BLMs 6 to 12.

CREATING A QUILT

Display the available materials. Explain that students are to use their understanding of pattern, slides, flips, and turns (and symmetry, if appropriate) to create a quilt square. Make sure students understand that each of them is designing only one square, not a quilt with an overall design. The final result will be a quilt of individual, unique squares (i.e., a "sampler" quilt). In order that the quilt squares will match in size, each should be built on one of the 20 cm by 20 cm squares of construction paper.

Students should write about their quilt squares. This could be a story or a technical description. See Other Resources below for some story ideas.

Assessment

Communication

Activity 5: Quilt Patterns

DISCUSSION

Display student quilt squares in one big quilt on the bulletin board. Discuss with students how the individual squares should be arranged (e.g., side by side or in a checkerboard pattern with blank squares between) so as to achieve the most pleasing effect. Have them explain why they think a certain arrangement will be attractive.

Extensions in Mathematics:

1. Students may wish to colour one or more of the quilt patterns on BLMs 6-12 to bring out a particular part of the design. Completed efforts should be examined for slides, flips, and turns (and possibly symmetry).

Use of Technology

Cross Curricular Activities:

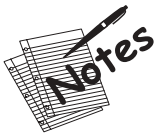
1. Research traditional quilt patterns and their names. (Use examples from BLMs 6-12). Discuss the appropriateness of the names. Have students devise descriptive names for their own quilt squares.
2. Have students explore the web and other resources for information about quilts and quilting. They could make a collection of favourite designs.

Other Resources:

For additional ideas, see annotated Other Resources list on page 47, numbered as below.

5. The Josefina Story Quilt by Eleanor Coerr
6. My Grandmother's Patchwork Quilt by Janet Bolton
7. The Patchwork Quilt by Valerie Flournoy
8. The Boy and the Quilt by Shirley Kurtz
9. Threading Mathematics into Social Studies by Jacquelin Smith
10. The Quilt Story by Tony Johnston and Toni de Paola

Activity 6: Symmetry in Patterns



Focus of Activity:

- Symmetry

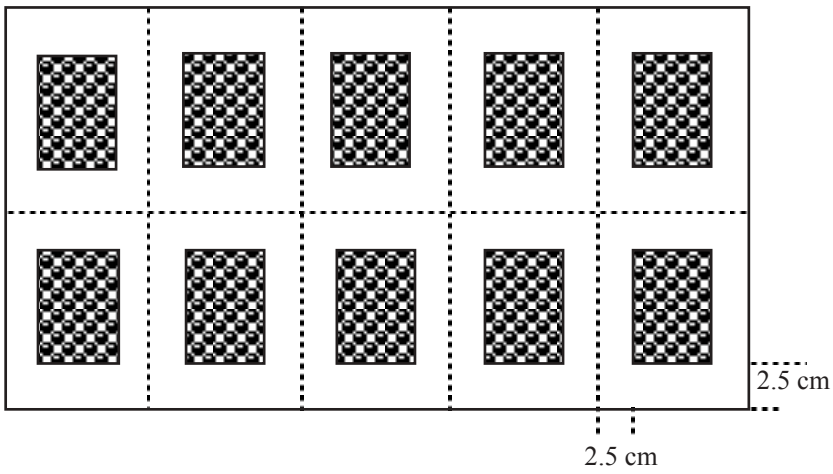
What to Assess:

- Recognition of symmetry
- Ability to produce a symmetric design

Preparation:

- See the table on page 5 for materials
- If mirrors are not available, you may wish to construct some by pasting aluminum foil onto small pieces of heavy cardboard. Take care not to wrinkle the foil. These mirrors will be sufficiently good to enable students to see images of the pattern blocks.

  One way of making these 10 at a time is to cut the back (or front) of a standard 500 gm cereal box into 10 rectangles, each 5 cm by 8 cm. (The 'mirrors' or reflecting cards fit nicely in two rows of five.) Then cut a 50 cm length of heavy guage aluminum foil 30 cm wide, and carefully smooth it into a flat surface, shiny side down. Dot the four corners of each card with a bit of glue, and spread them, glue side downward, on the foil. They will fit into two rows of five with borders of 2.5 cm. Cut half-way between the cards on all sides (as shown by dotted lines in the diagram below), and fold the foil around the edges to the back; it is stiff enough to hold without glue, although you may wish to cut away some of the thickness at the corners. (Mylar film, though more expensive, reflects better. Mylar is available at art supply stores.)



- Make copies of the pattern blocks on BLMs 13, 14, 15, 16 and 17 and have students cut them apart. Each type of pattern block should be a different colour.

Activity 6: Symmetry in Patterns

Activity:

INTRODUCTION

Have students work in pairs. Each pair should have several of each type of pattern blocks and two mirrors taped together along one edge to make a pair of hinged mirrors.

Instruct students to place the hinged mirrors in front of them so they stand upright and to fit any pattern block into the angle between the mirrors. Ask what they see. *The number of reflections that they see will depend on the angle between the mirrors.* Students may recognize the results as being similar to kaleidoscopic images.

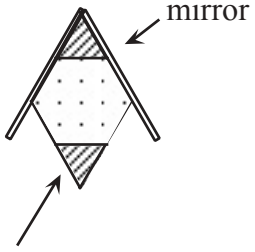
ALTERNATE

To introduce the concept of symmetry with just two mirrors, set the mirrors on a central table with 3 or 4 pattern blocks in place between them. Invite students to look at the result.

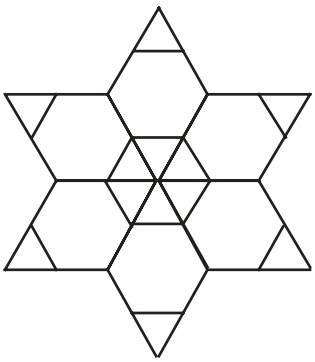
Problem Solving

Have students add two or three more pattern blocks to the one between the mirrors (see diagram below). Instruct them to study the design formed by the pattern blocks and their reflections, then remove the mirrors and build the entire design using pattern blocks.
Some children may need to replace the mirrors from time to time to check.

Ask students to try to make a design at their desks with pattern blocks that might have been made the same way, (i.e., with several reflections).



Pattern Blocks

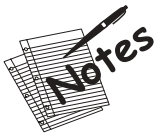

Full design as seen in the mirror

DISCUSSION

Ask students what is special about the designs they made. Ask what there was about the design that helped them construct the entire design when the mirrors were removed. Elicit the idea of symmetry as a type of balance. If students are familiar with the idea of symmetry you may wish to discuss both line symmetry and turn symmetry.

Examine the patterns from Activities 3 and 4, in particular, for symmetry. Ask students if colouring the patterns destroyed any of the symmetries that were present in the line drawings. Patterns from Activity 1 may also be analyzed if time permits.

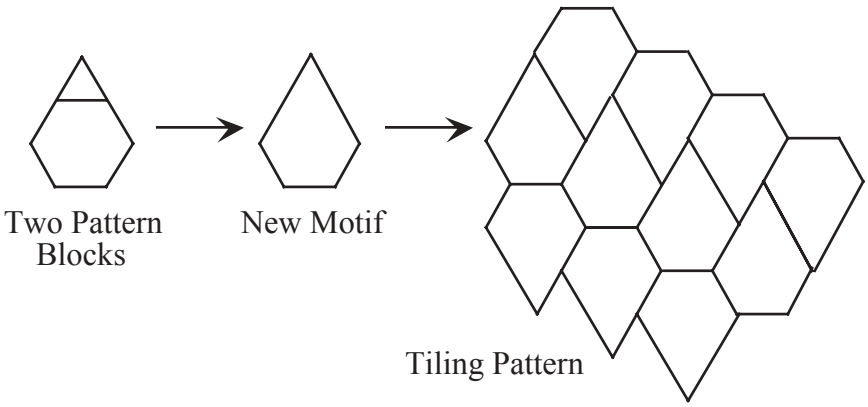
Activity 6: Symmetry in Patterns



Extensions in Mathematics:

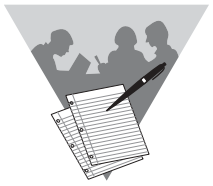
1. Experiment with different angles between the hinged mirrors. What angle gives 4 images in total? What angle gives 6?

2. Tape two pattern blocks together to make a new figure. Make several copies of this figure. Create a design with these new figures. Examine the design for symmetry. For example, the tiling pattern below shows half-turn symmetry.



These designs can become permanent if the cut-out pattern blocks from BLMs 13, 14, 15, 16, and 17 are used. Students can then colour their designs and write about the symmetries (or lack of) in them.

Discuss with students why they chose the pattern blocks they did. Did they plan ahead? Did the designs/patterns "just emerge"? Try to assess students' ability to visualize.



Cross Curricular Activities:

1. Make symmetric designs by placing blobs of paint or ink on a piece of paper and folding the paper.

2. Illustrate a form of symmetry by cutting a free-form figure with one straight side from construction paper. Cut parallel "slices" off the figure. Flip alternate slices along the straight side. Paste all pieces down on a second piece of construction paper. This creates a positive/negative effect.

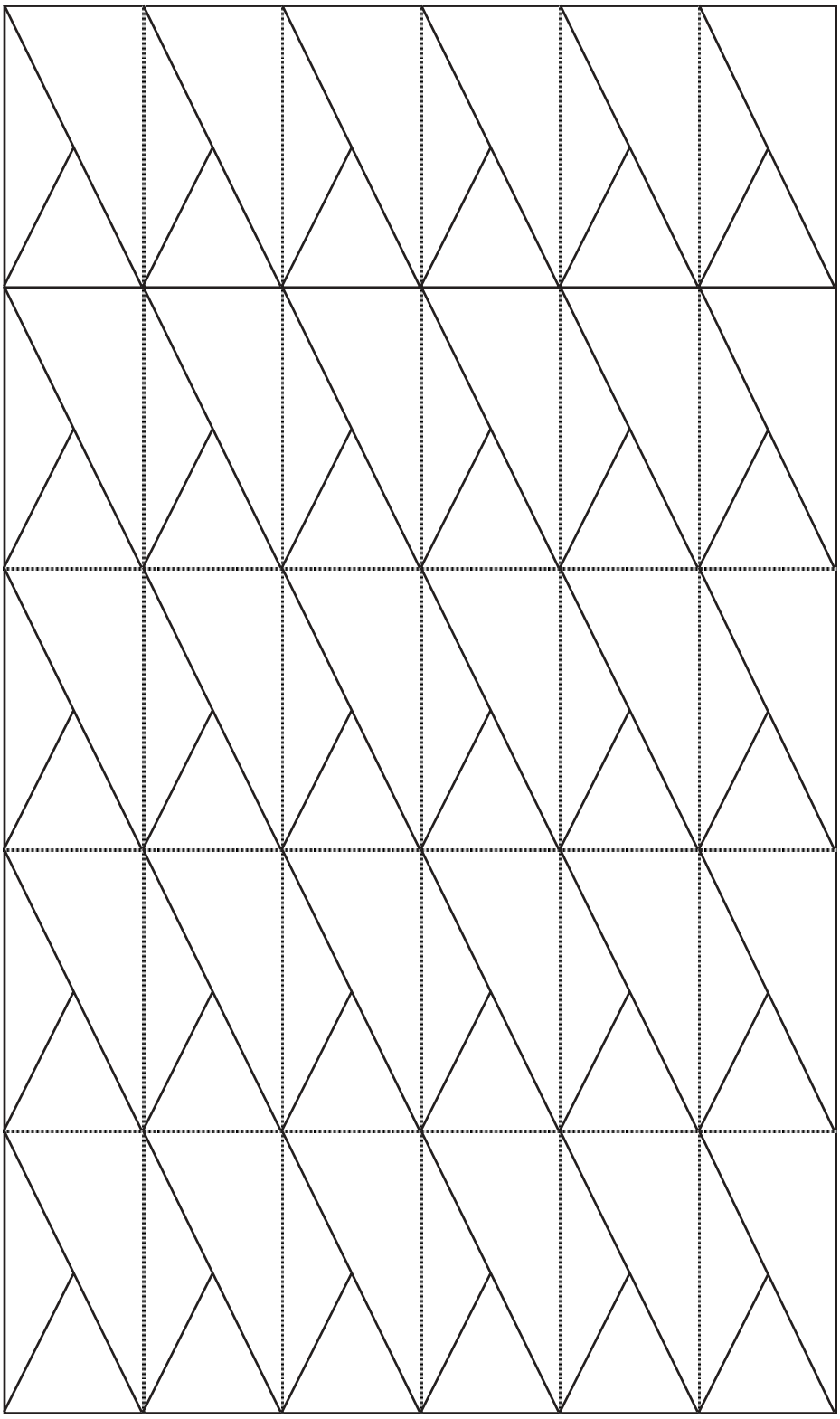
3. Look for symmetry in nature. Are we symmetrical? Place a mirror at the midline of a full-face photo to see how one's appearance would differ if one's face were truly symmetrical.

Other Resources:

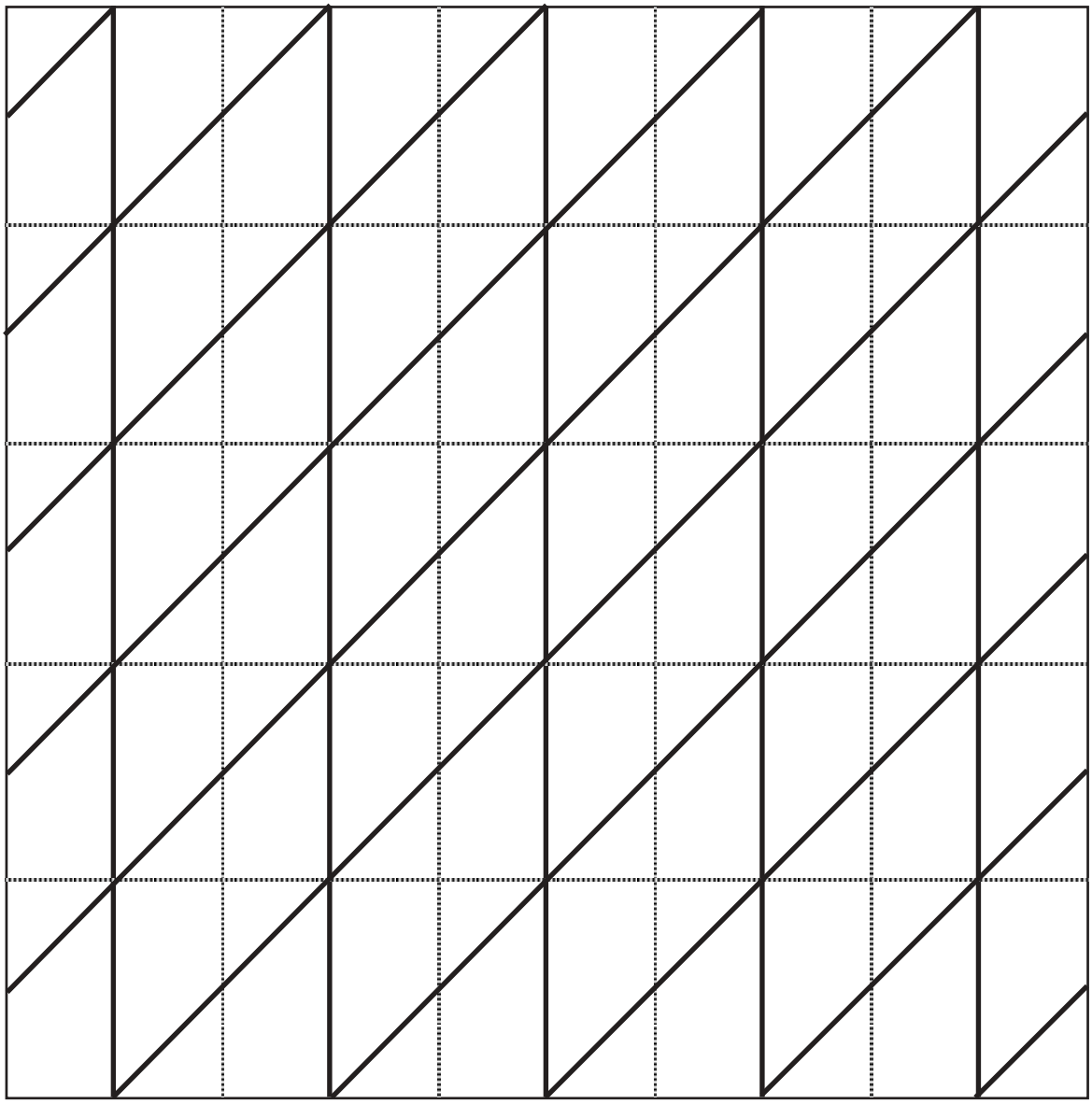
For additional ideas, see annotated Other Resources list on page 47, numbered as below.

2. Addenda Series, Grades K-6: Geometry and Spatial Sense

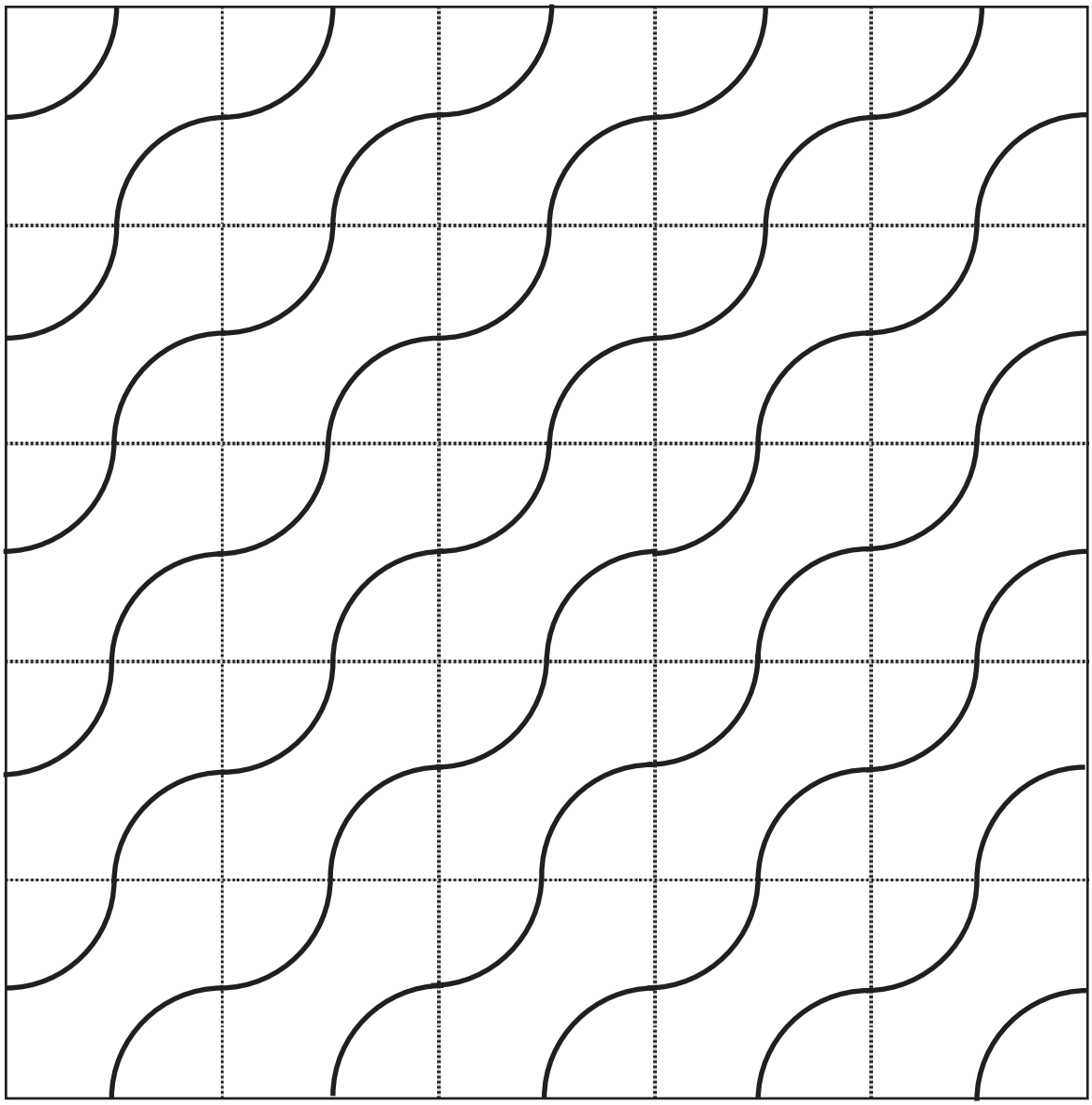
BLM 1: Motif 1



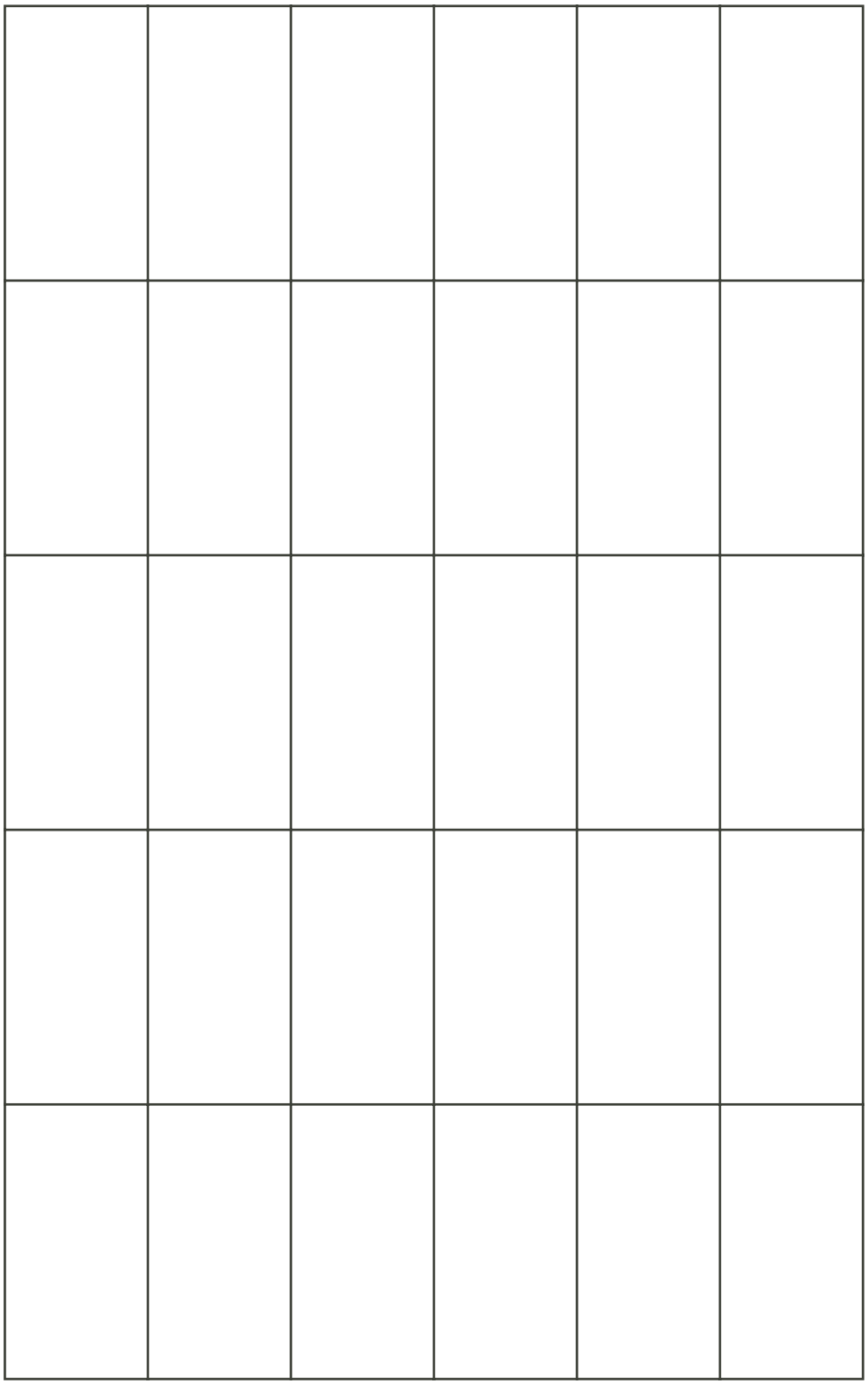
BLM 2: Motif 2



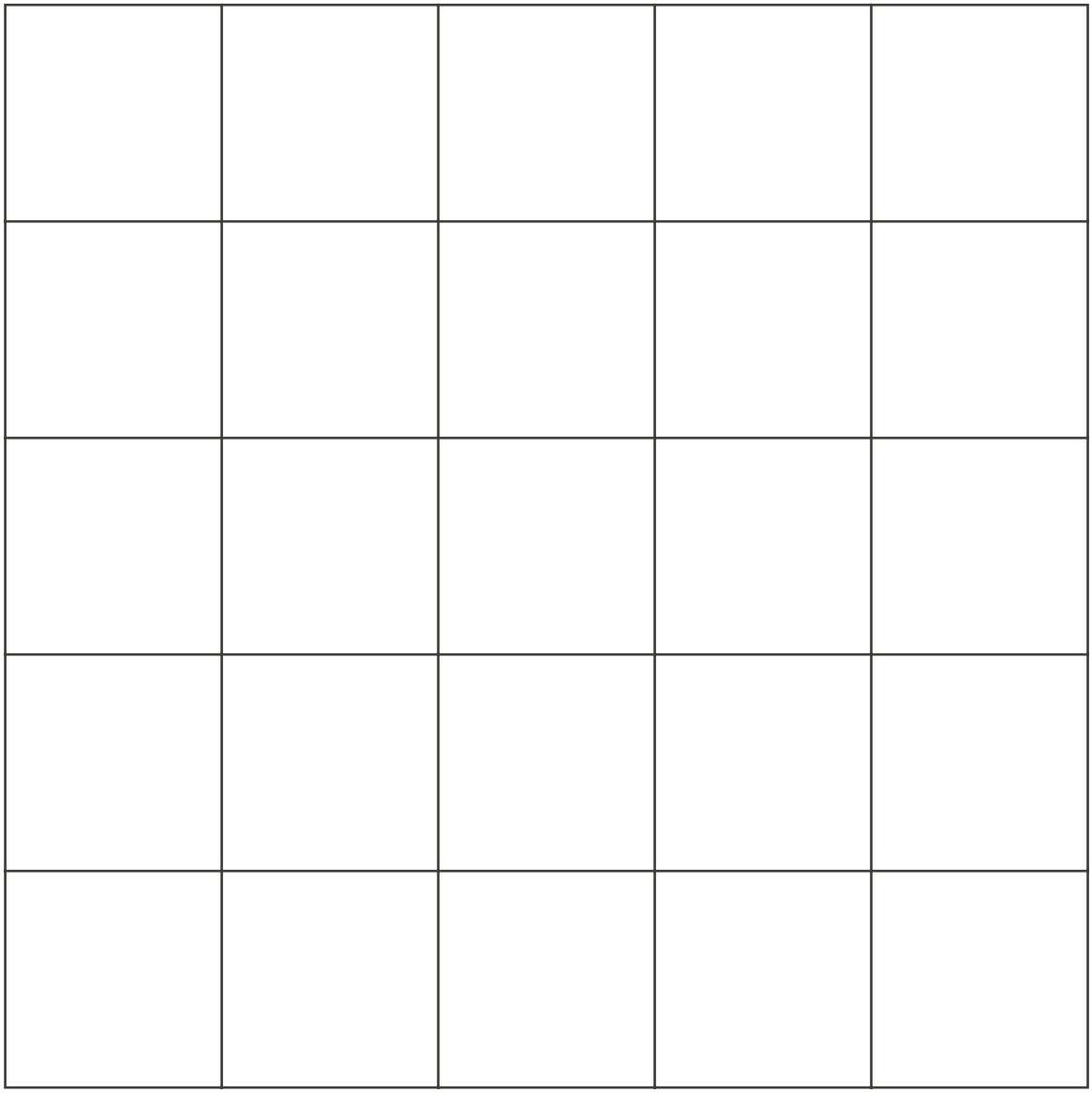
BLM 3: Motif 3



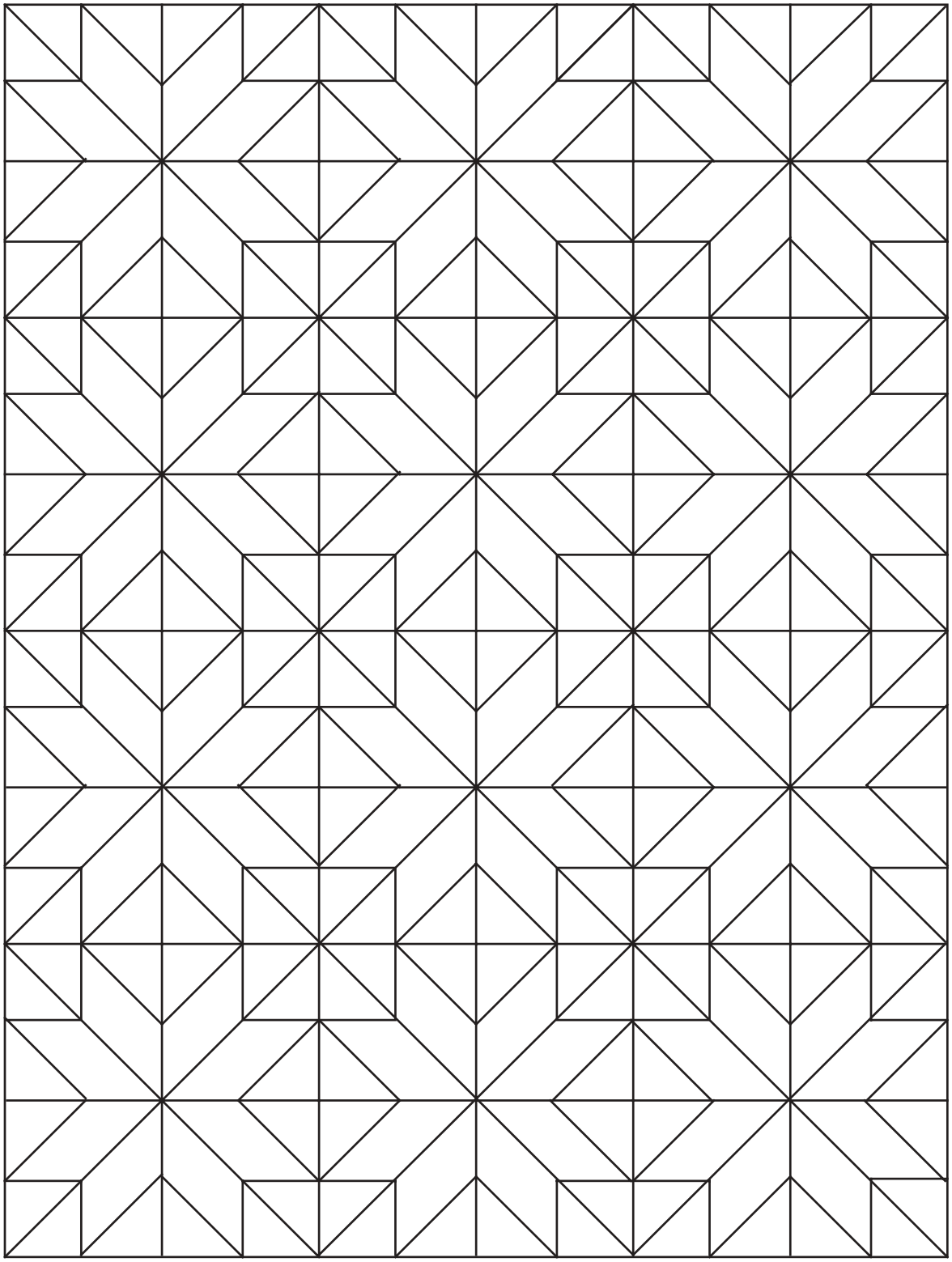
BLM 4: Grid 1



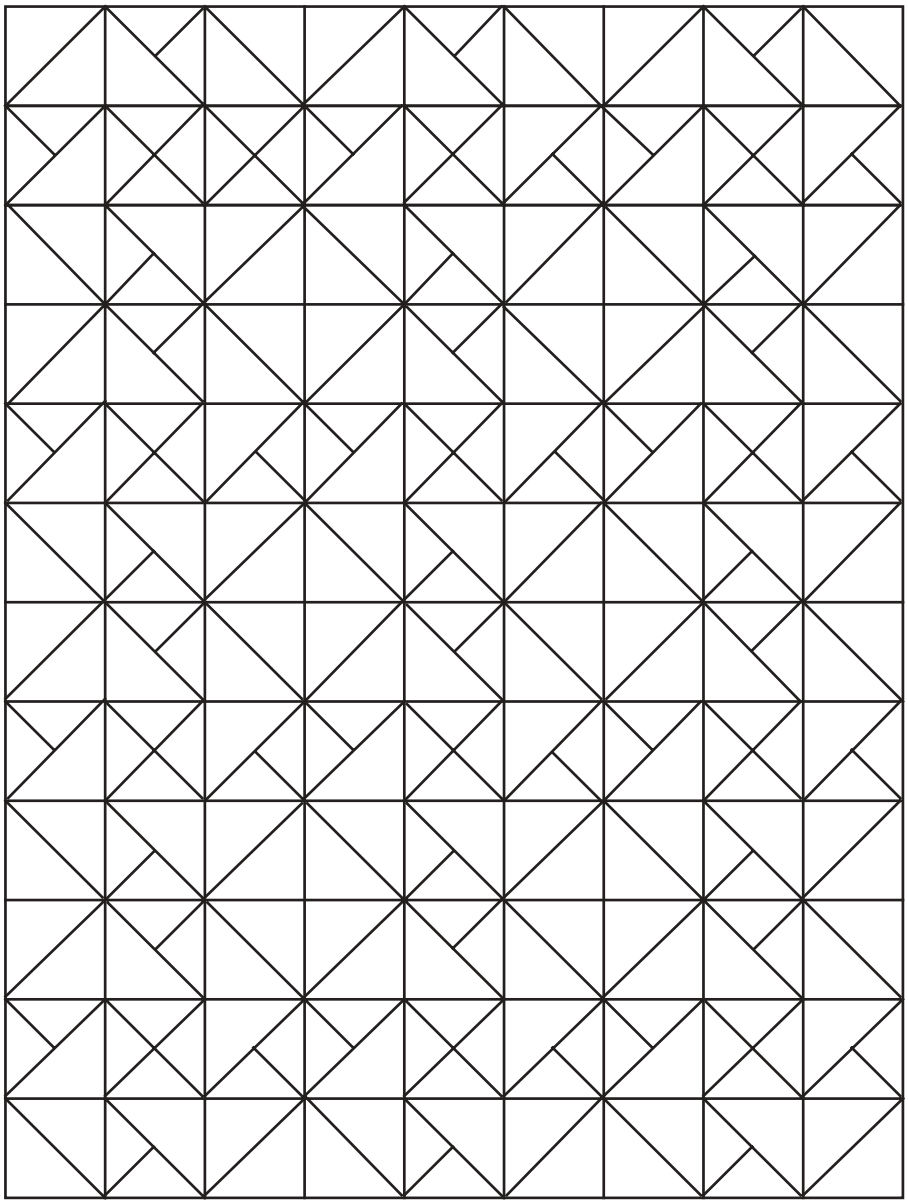
BLM 5: Grid 2



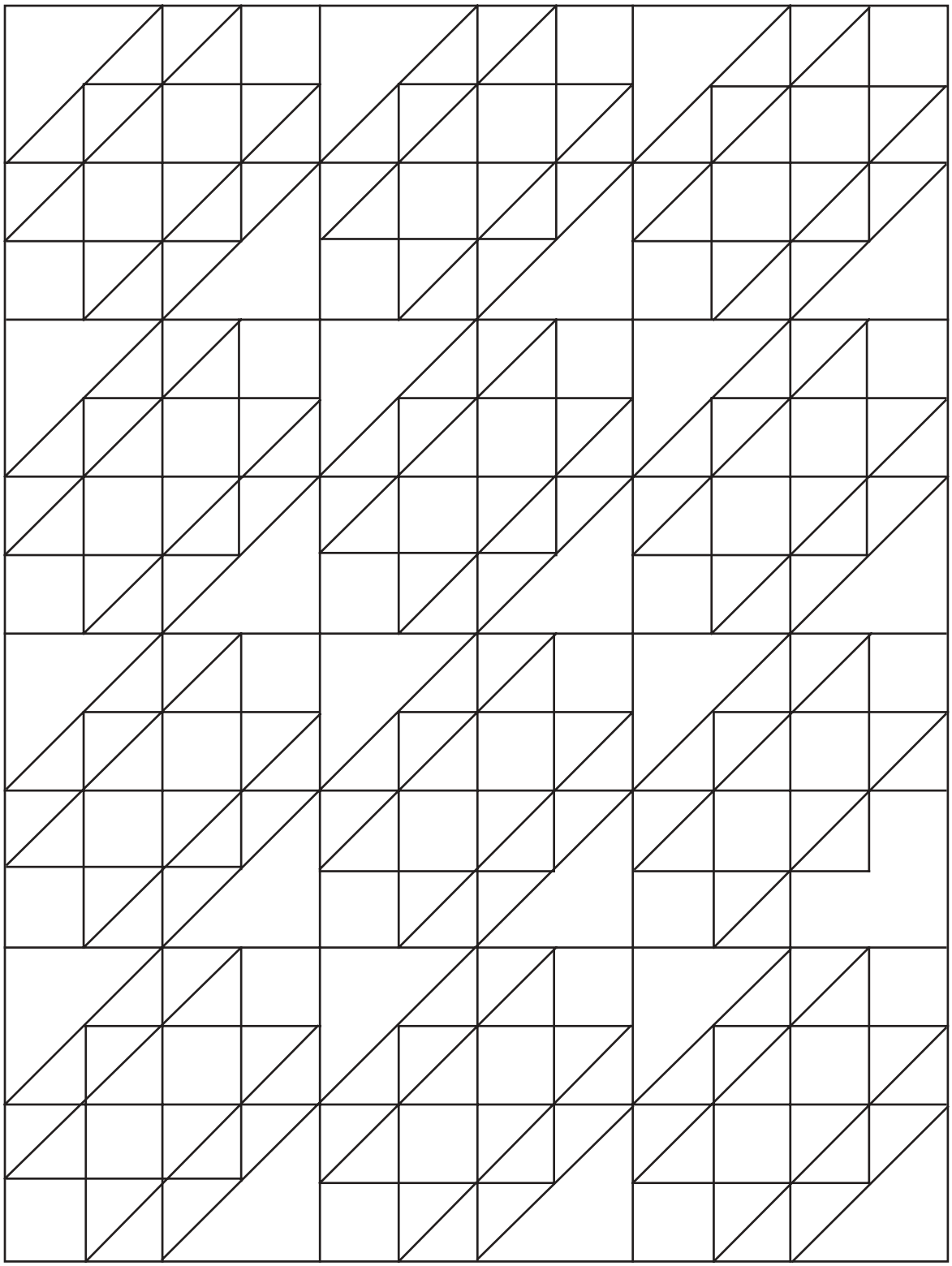
BLM 6: Quilt 1: Columns



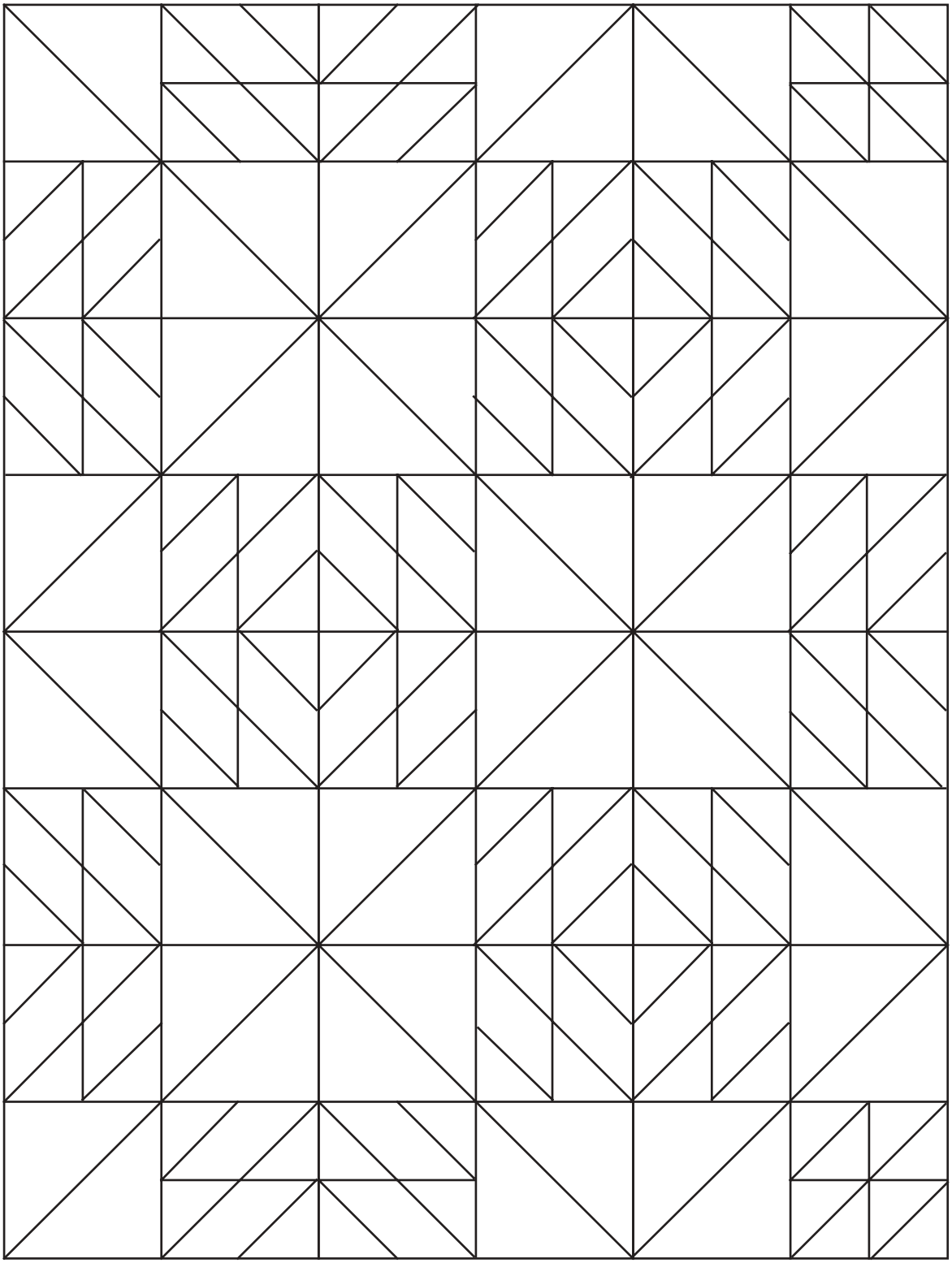
BLM 7: Quilt 2: Card Trick



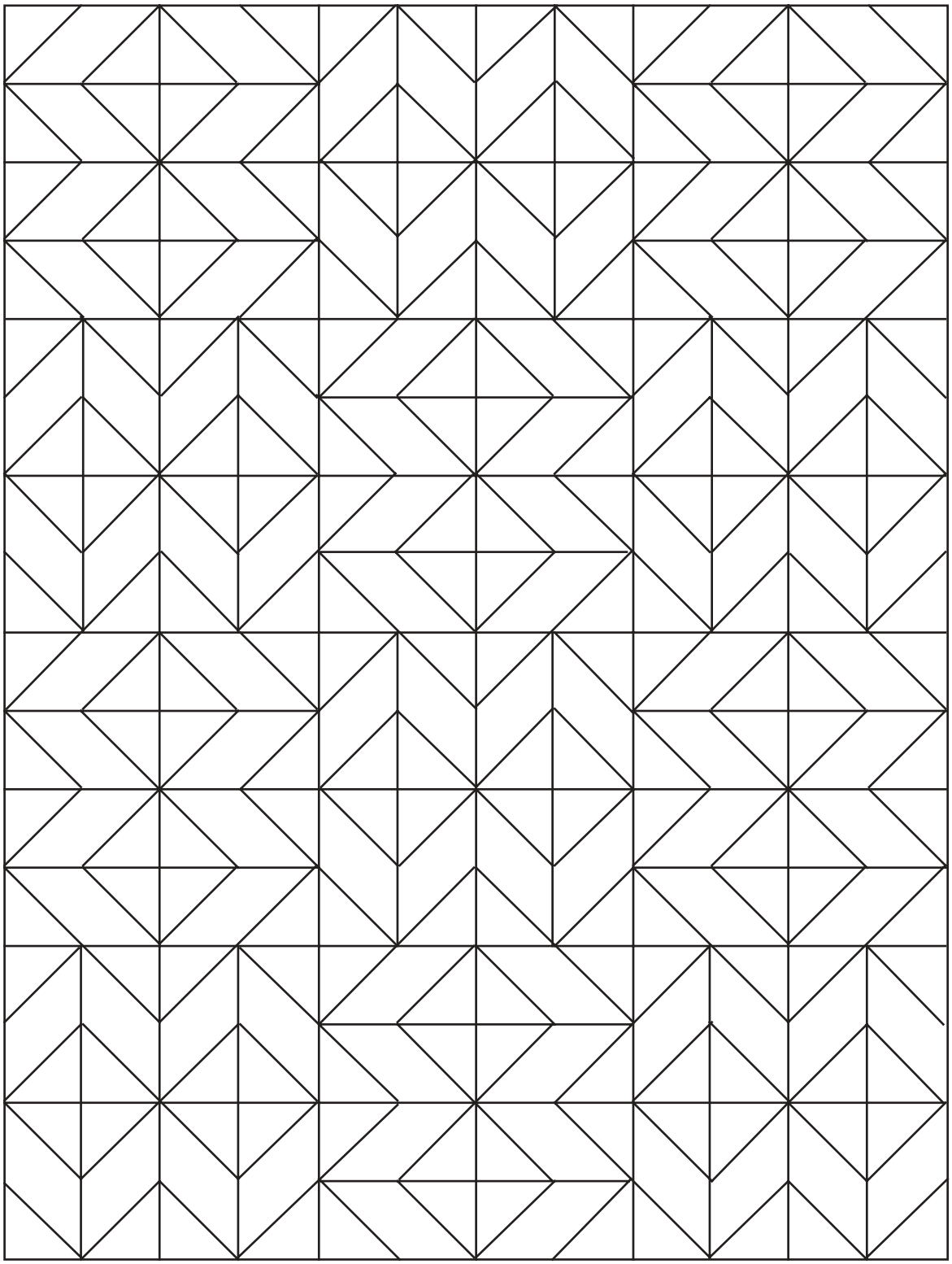
BLM 8: Quilt 3: Hovering Hawks



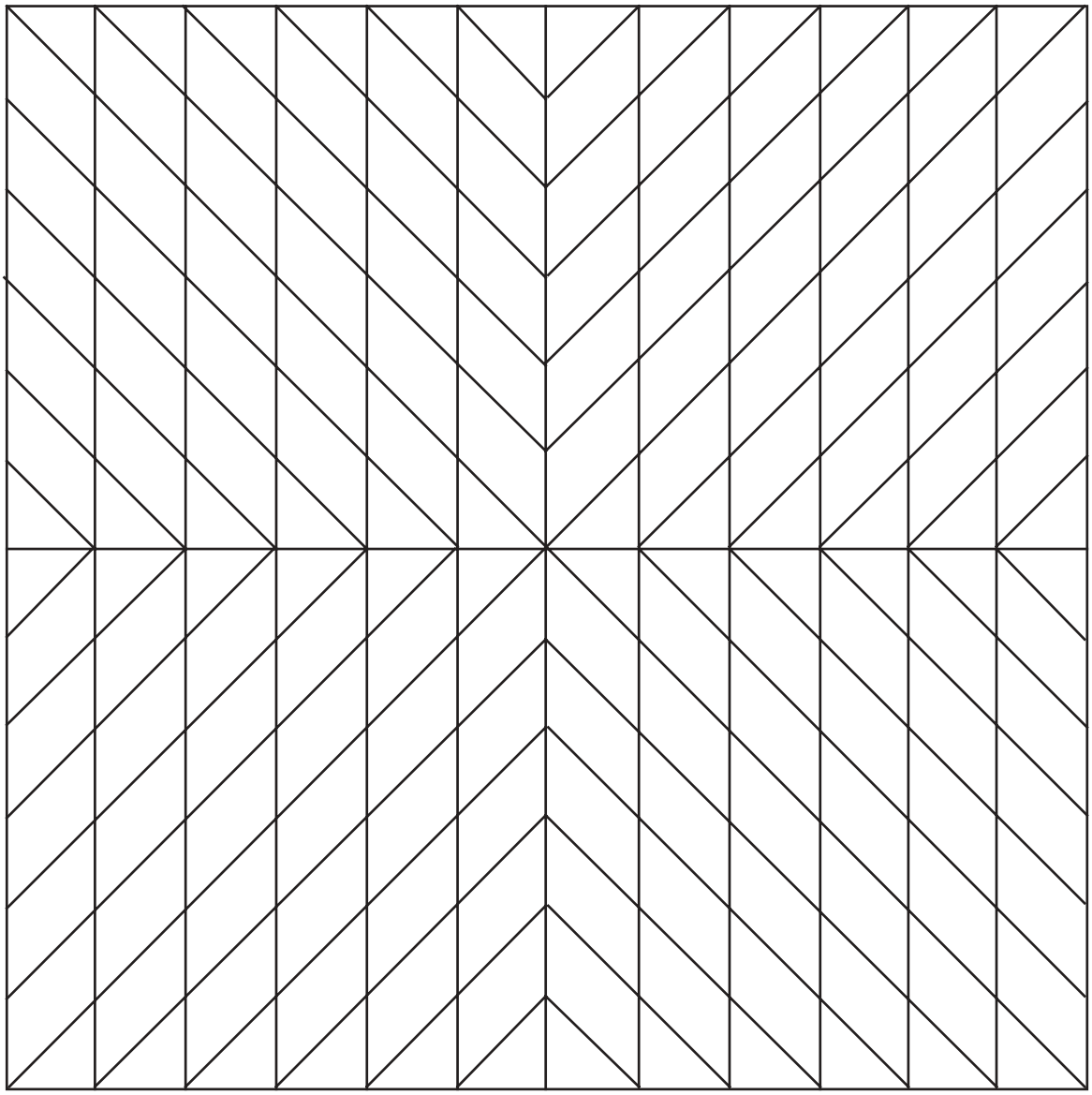
BLM 9: Quilt 4: Flock-Of-Geese



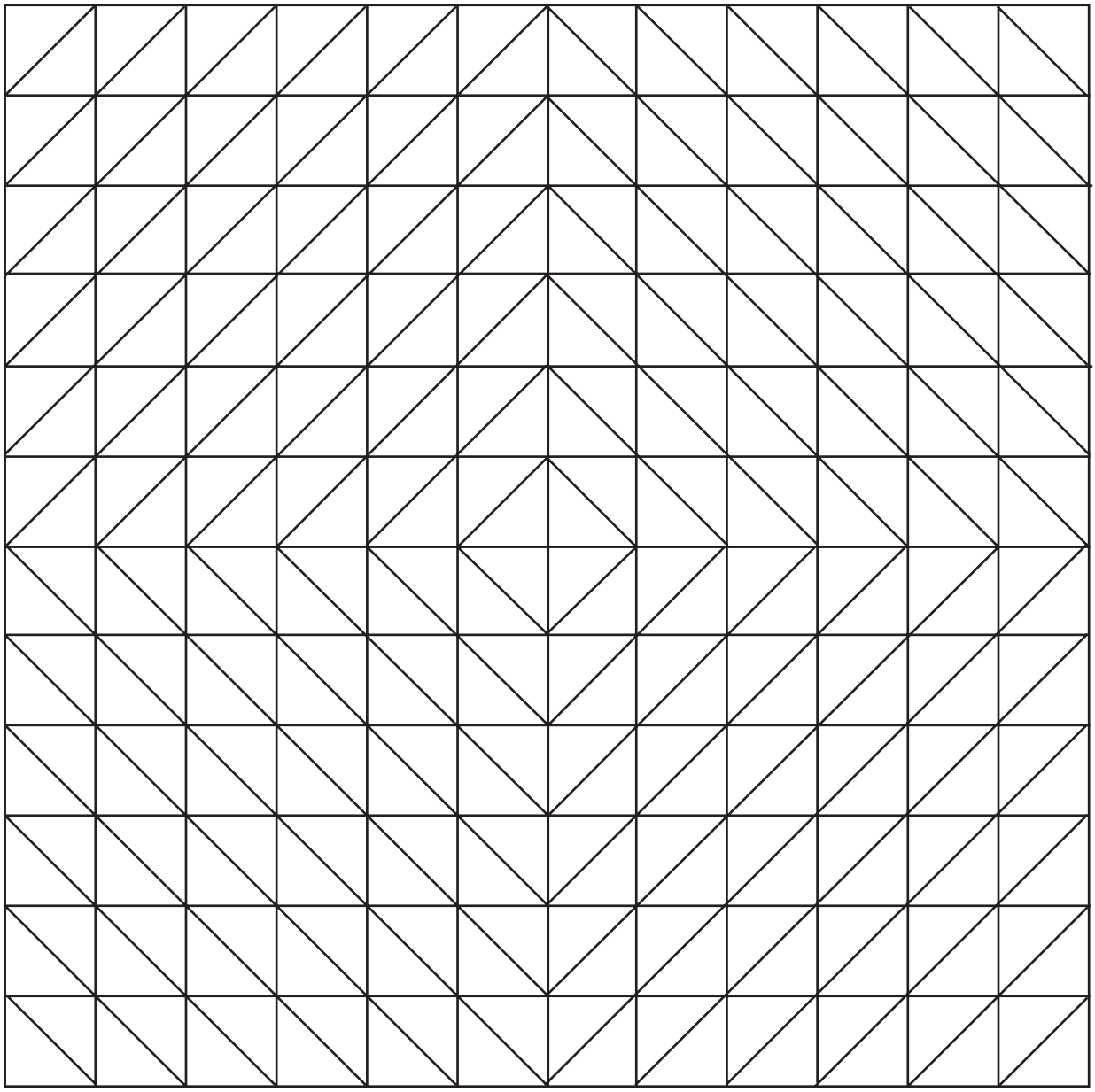
BLM 10: Quilt 5: Ribbons



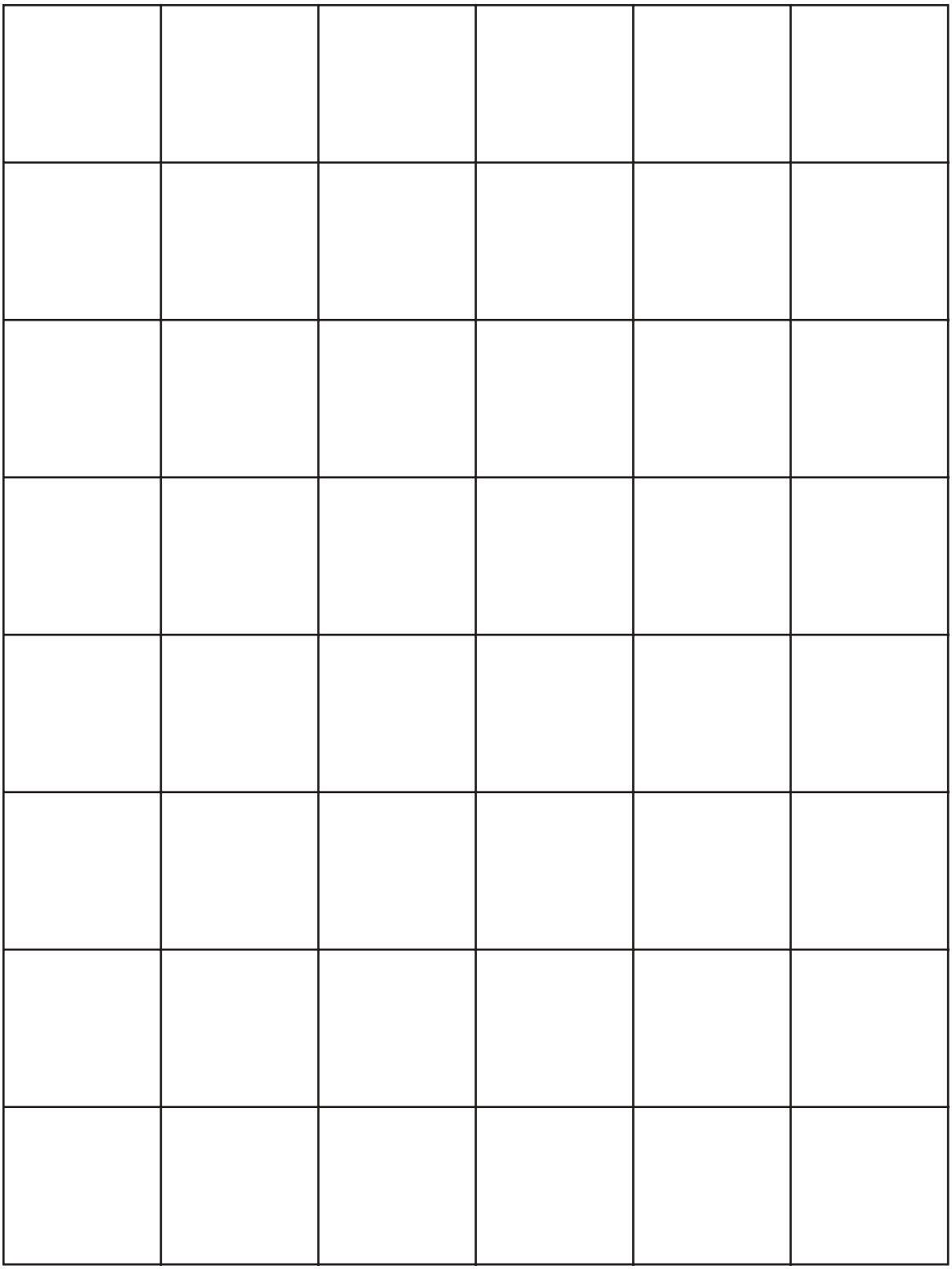
BLM 11: Quilt 6: Sunburst



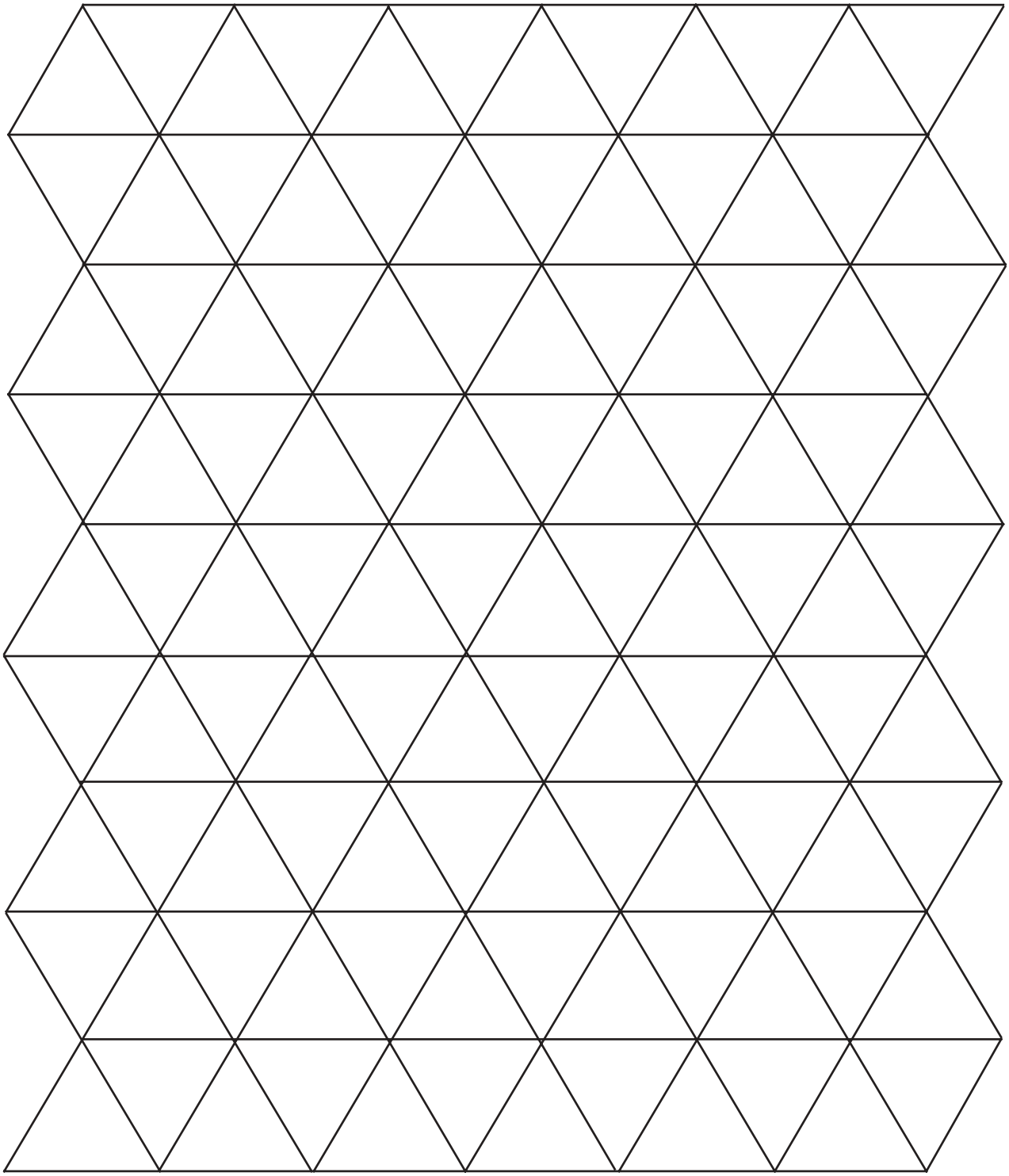
BLM 12: Quilt 7: Barn Raising



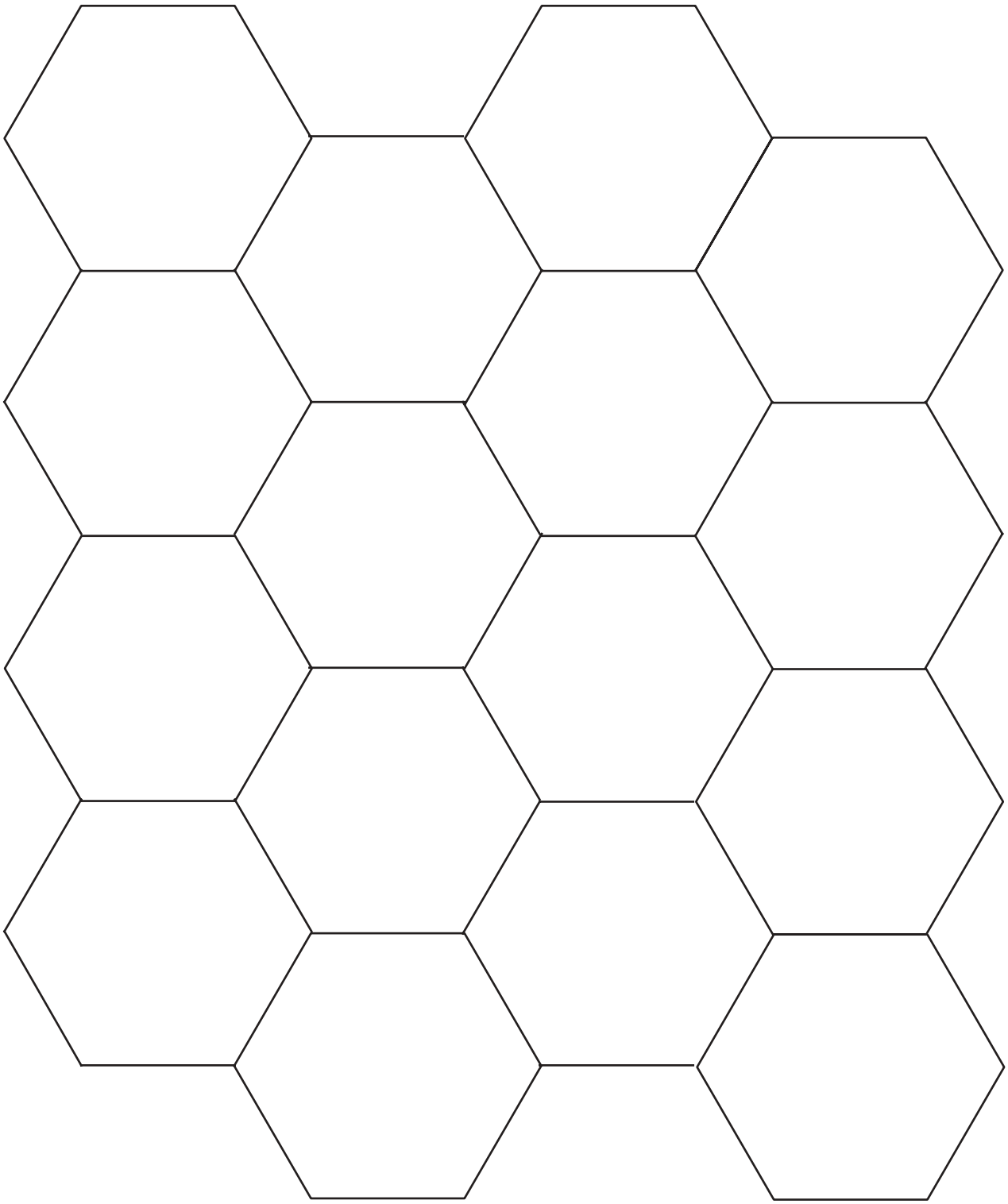
BLM 13: Pattern Blocks 1: Squares



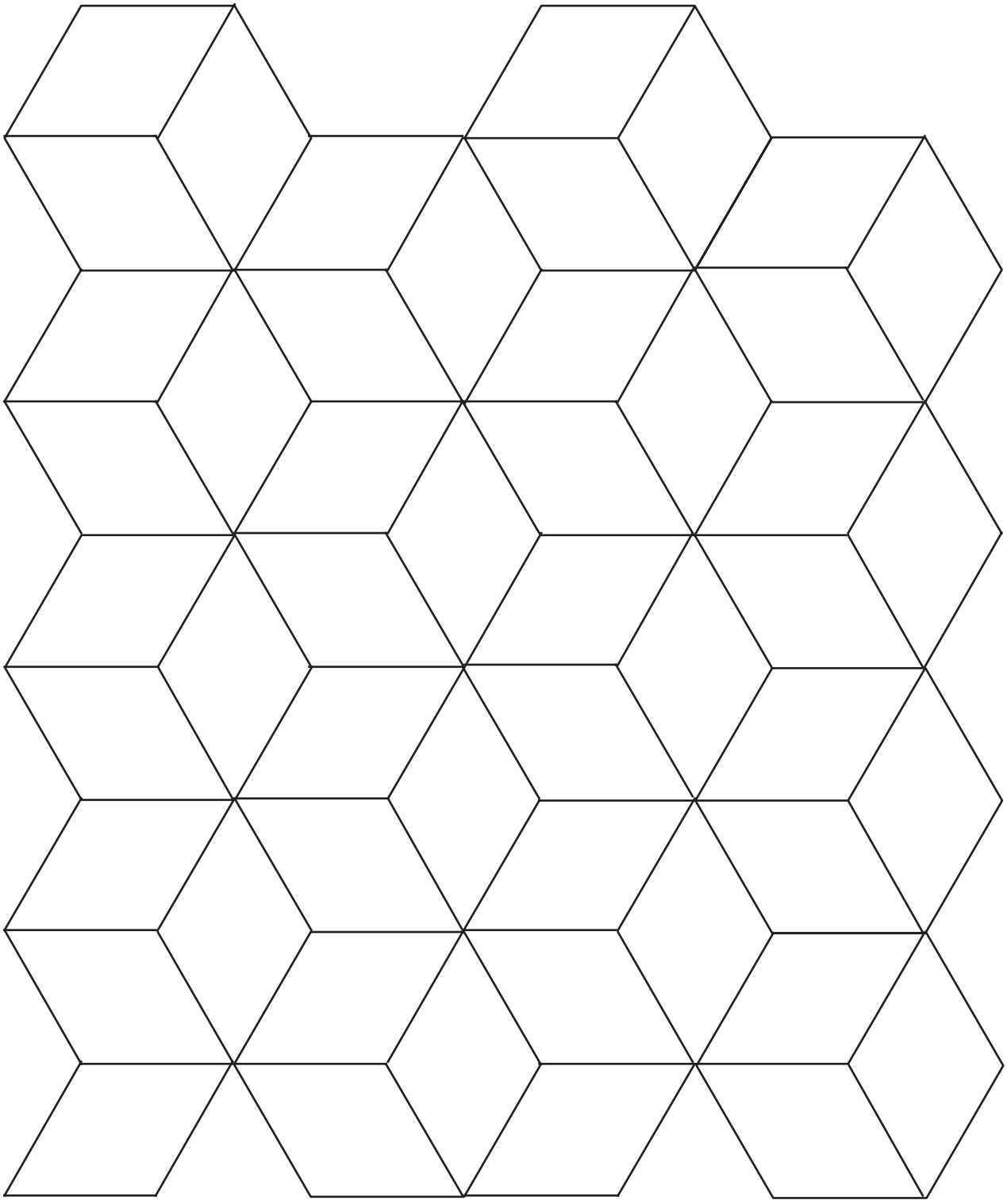
BLM 14: Pattern Blocks 2: Triangles



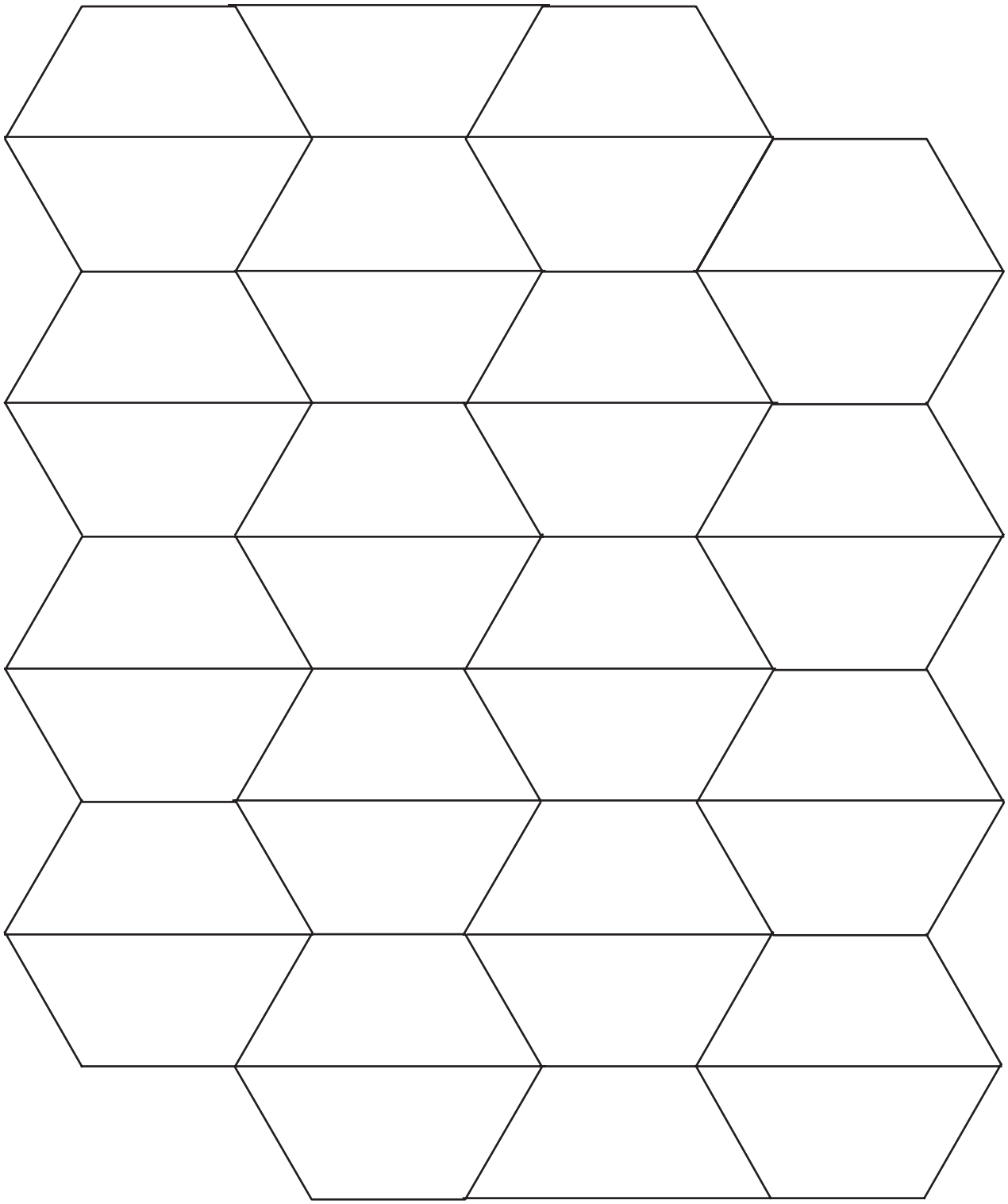
BLM 15: Pattern Blocks 3: Hexagons



BLM 16: Pattern Blocks 4: Rhombuses



BLM 17: Pattern Blocks 5: Trapezoids



Suggested Assessment Strategies

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. "Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions", by Jean Karr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels. Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. "Assessment", Arithmetic Teacher Focus Issue, February 1992, NCTM.
   This copy of NCTM's journal for elementary school addresses several issues dealing with assessment. It also includes suggested techniques and student activities.

5. "How to Evaluate Progress in Problem Solving", by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

Suggested Assessment Strategies

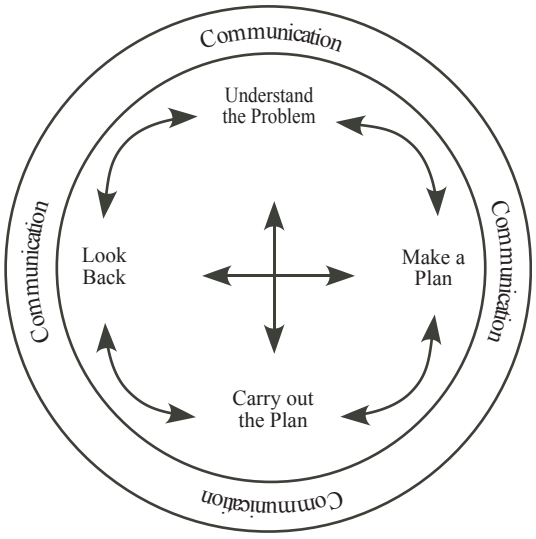
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R's"

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Level 1: Limited identifies students who are in need of much assistance;

Level 2: Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

Level 3: Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.

Level 4: Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
STRATEGIES	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strateges with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Suggested Assessment Strategies

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

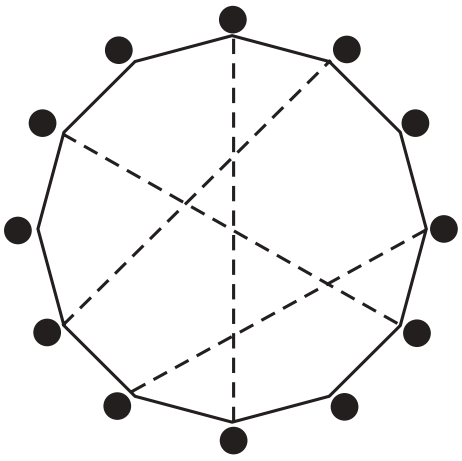
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a "what if...?" problem, such as "What if the question were reversed?", "What if we had other data?", "What if we were to show the data on a different type of graph?".

Suggested Adaption of General Rubric for Activity 5

The adapted rubric takes into account the instructions the students have been given (e.g., use slides, flips, turns), related topics (e.g., the nature of a quilt square), and the description of or instructions for the quilt square designed. Use some or all of the criteria described below, depending on how the problem was posed to the students.

Notice that none of the criteria deal with the neatness of the final product. If you are assessing the result at the stage of "first draft" this will be appropriate. If, however, you are assessing the final piece of work partially as a classroom display item, it would be appropriate to add an additional criterion to the rubric dealing with appearance.

Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
The student • shows a random placement of figures with no apparent plan or understanding	The student • shows some repetition of a motif; may use only 1 figure	The student • gives evidence of two or more figures incorporated into the quilt square	The student • shows a complex motif, which can be dissected and component parts identified by the student(s)
• shows no conscious effort to include slides, flips, or turns or uses these incorrectly	• may include slides, flips, or turns, but these may not be identified by the student(s)	• correctly incorporates at least one of slides, flips, or turns; these are correctly identified by the student(s)	• incorporates slides, flips, or turns to construct a complex design; student(s) identify the motions
• shows a lack of any form of symmetry in the design	• may use line symmetry, but will not identify it without prompting	• uses line symmetry, and can identify this; may or may not use turn symmetry	• uses both line and turn symmetry and can identify both
• includes an incomplete description with inaccurate use of mathematical terms	• includes a few math terms (e.g., square, triangle), but generally uses everyday language	• uses mathematical language as necessary to describe the quilt square	• uses appropriate mathematical language correctly
• gives incomplete instructions for square construction; shows little/incorrect use of mathematical language	• gives haphazard instructions for square construction but the main components are included; shows some attempt to use correct mathematical language	• gives a coherent and understandable description of square construction; minor points may be omitted; shows good use of mathematical language	• gives a clear, concise, accurate description of the construction of the square given in correct, appropriate, mathematical language

Other Resources

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Addenda Series, Grades K - 6: Geometry and Spatial Sense" by Lorna Morrow and John Del Grande, NCTM, 1993.
This book includes detailed lessons for each grade from K to 6, emphasizing manipulatives. BLMs are provided.

3. "Addenda Series, Grades K-6: Patterns" by John Firkins, NCTM, 1993.
This book contains detailed lessons on various aspects of pattern and challenges students to interpret, develop, and extend patterns.

4. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.
The document provides a selection of open-ended problems tested at the Transition Years Level. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level.

5. "The Josefina Story Quilt" by Eleanor Coerr, Harper and Row Publishers, 1989.
A historical background of the use of the quilt to relate and remember events.

6. "My Grandmother's Patchwork Quilt" by Janet Bolton, Doubleday Books for young readers, published by Delacorte Press, 1993/94.
A wonderful book for explaining a patchwork quilt. Designs are found in the book for children to make their own quilts.

7. "The Patchwork Quilt" by Valerie Flournoy, Dial Books for Young Readers, E.P. Dutton, Inc., 1985.
A young girl's grandmother introduces her to what a quilt represents by making one with her.

8. "The Boy and the Quilt" by Shirley Kurtz, Good Books, 1991.
A boy sets out to make a quilt for himself.

9. "Threading Mathematics into Social Studies" by Jacquelin Smith in Teaching Children Mathematics 1(7), March 1995, pp 438-444, NCTM.
A teacher describes activities for students in the classroom and in the community that developed around the theme of quilts. Several books are referenced.

10. "The Quilt Story" by Tony Johnston and Toni dePaola, Scholastic Inc., 1992.
The Quilt Story is about two girls, one from pioneer days and one from modern times, who share the same quilt.

11. Invitations to Mathematics; Investigations in Patterns and Algebra, Grade 4.
The Centre for Education in Mathematics and Computing, University of Waterloo, 2001.
The booklet contains several activities dealing with pattern in number and geometry.