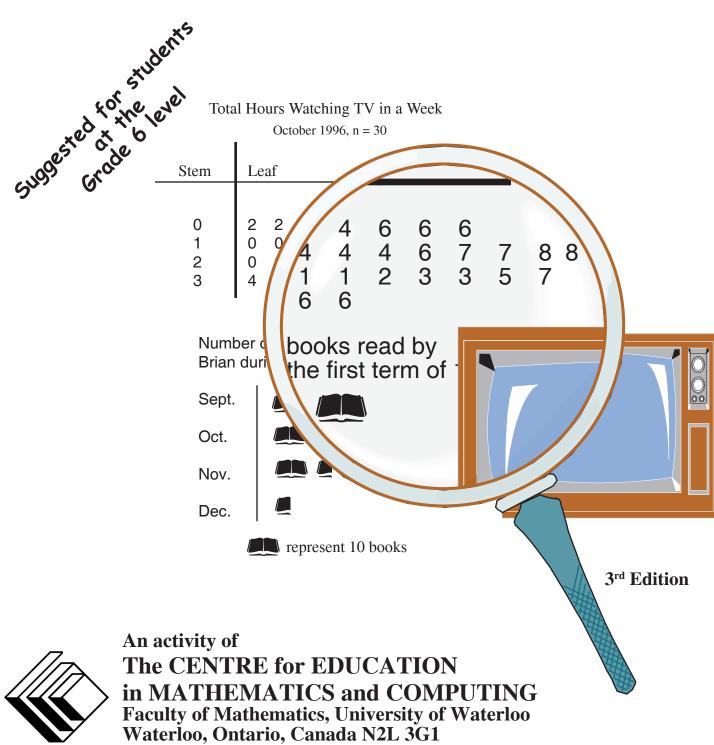
# Invitations to Mathematics

# Investigations in Data Management "TV - Vice or Virtue?"



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### Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics, among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the extensions and related activities included with individual activities, provide ample scope for all students' interests and ability levels. Related "Family Math" activities to involve the students' parents/care givers are also suggested.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

Investigations in Data Management is comprised of activities to enhance the students' abilities to collect, represent, and interpret data, frequently in a problem-solving mode. Since today's media make constant use of data presented in various forms (often in order to sway opinion), it is especially important to help students learn to read and interpret these graphs, charts, or tables. Data management is equally important in the depth and variety of its connections to other subjects such as environmental studies.

Information on all the available units in the *Invitations to Mathematics* series can be found at the end of this booklet.

Preface

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### COMMON BELIEFS

These activities have been developed within the context of certain beliefs and values about mathematics generally, and data management specifically. Some of these beliefs are described below.

### Importance of Statistics

In an information-rich society such as ours, statistics are an increasingly important aspect of daily life. "Educators and mathematicians now stress the importance of incorporating data analysis in the elementary mathematics curriculum to prepare students for living and working in a world filled with information based on data."

Corwin and Russell

Connections to Other Curriculum Subjects and Mathematical Strands

Activities which involve data management can provide a meaningful link to other content areas such as environmental studies and science. These activities help students to develop critical-thinking and problem-solving skills, and can reinforce communication skills as students discuss and write about their conclusions. Within mathematics, these activities provide opportunities for students to represent, interpret, discuss information, as well as estimate, measure and develop number sense.

### Connections to the Real World

Through collecting and analyzing real data, students encounter the uncertainty and intrigue of real mathematics. "We are living in a world of information. Stop and think a moment about the number of facts, figures, and other data that confront us each day. What do we do with all this information? We ignore some of it, we organize some of it to fit what we already know, or we summarize it by using shorter descriptions or other numbers. How is this information presented to us? It may be presented in written descriptions, in graphs or tables or in summary numbers such as averages. How do we learn to make sense of all this information? This is where we, the K-6 teachers and teacher educators, enter the picture; we need to help our students - from the time they first enter school - to make sense of data."

National Council of Teachers of Mathemtics (NCTM)

# Importance of Language

A central activity in data analysis is dialogue and discussion. In a classroom setting, a significant amount of time should be devoted to reflection, discussion, and writing about the meaning of the data.

### Importance of Real Data

In data analysis, students use numbers to describe, compare, predict, and make decisions. Because real data are used, there are no predetermined "answers". "Not only do you not know the answer [to the investigation] in advance, but, without seeing the data, you may not even know what the most interesting questions are going to be!"

Corwin and Russell



GOOD TASKS ARE ONES THAT DO NOT SEPARATE METHEMATICAL THINKING FROM MATHEMATICAL CONCEPTS OR SKILLS, THAT CAPTURE STUDENTS' CURIOSITY AND INVITE THEM TO SPECULATE AND TO PURSUE THEIR HUNCHES.

NCTM

Overview Page 1



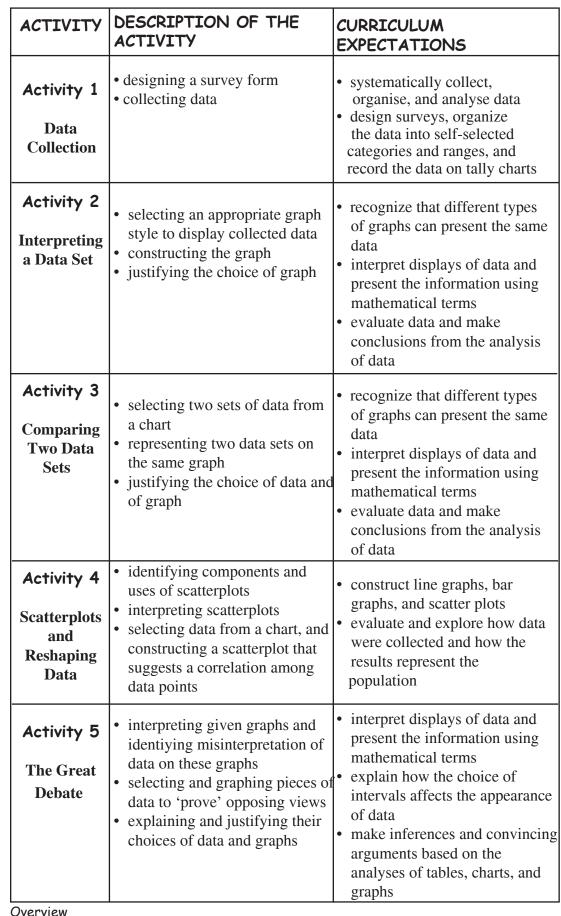
# ESSENTIAL CONTENT

The activities in this unit focus on concepts in data management. During this unit, the student will:

- collect, organize, graph, manipulate, read, describe, interpret, and analyse data:
- draw comparative graphs;
- apply data management skills in a real life investigation;
- logically support an argument by selecting/reshaping data.

Page 2 Overview

### CURRICULUM CONNECTIONS





Overview

Notes

Overview

### **A**SSESSMENT

Assessment is a process of gathering evidence about a student's knowledge, skills, and values, and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned, but mathematical competence involves such characteristics as the ability to communicate, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. Because of the nature of the activities it is suggested that a variety of types of assessment be used. Suggestions include:

- (i) observing students as they work to see if they are applying various concepts; to see if they are working cooperatively; to observe their committment to the tasks;
- (ii) assessing the completed project to see if instructions have been followed; to see if concepts have been applied correctly; to see if the language of mathematics has been used correctly;
- (iii) assessing the students' descriptions of their completed work to see if mathematical language is used correctly; to see if students understand the concepts used;
- (iv) providing opportunities for student self-assessment: have students write explanations of their understanding, opinion, or feelings about an activity. One technique is to have them write under the headings What I Did, What I Learned, and How I Felt About It. Students could be asked to write a review of one day's activities or of the whole unit's work.
- (v) selecting an exemplary piece of work to be included in a portfolio for assessment purposes or for sharing with parents.

See Suggested Assessment Strategies, page 41, for further discussion and sample rubrics.

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### PREREQUISITES

Students need some prior experience (not necessarily extensive) in the following:

- collecting different types of data: quantitative (e.g., lengths, weights, time) and qualitative (e.g., eye colour, food preferences, rating scales);
- organizing data (e.g., using charts or tables);
- constructing a variety of graphs (e.g., pictographs, bar and line graphs);
- describing, reading, and interpreting data using a variety of methods and tools.

# Logos

The following logos, which are located in the margins, identify segments related respectively to:

Problem Solving (





Use of Technology









Overview Page 5



# MATERIALS

ACTIVITY	MATERIALS
Activity 1 Data Collection	<ul> <li>Copies of BLMs 1 and 2</li> <li>Materials for students to make scrapbooks (markers, construction paper, etc.)</li> <li>Chart paper or blackboard</li> <li>Markers</li> </ul>
Activity 2 Interpreting A Data Set	<ul> <li>Copies of BLMs 3, 4, and 5</li> <li>Chart paper</li> <li>Markers</li> <li>Computer software such as Claris Works<sup>TM</sup>, Microsoft Works<sup>TM</sup>, or WordPerfect 6.0a<sup>TM</sup> (optional)</li> </ul>
Activity 3 Comparing Two Data Sets	<ul> <li>Copies of BLMs 3, 4, and 5</li> <li>Copies of BLMs 6 and 7</li> <li>Chart paper</li> <li>Markers</li> <li>Blank acetate sheets for overhead projector</li> <li>Copies of BLM 8 (optional)</li> <li>Copies of BLM 15 (optional)</li> <li>Copies of BLM 12 (optional)</li> <li>Computer software as above</li> </ul>
Activity 4 Scatterplots and Reshaping Data	<ul> <li>Copies of BLM 6</li> <li>Copies of BLMs 9 and 10</li> <li>Copies of BLM 11 (optional)</li> <li>Copies of BLM 15 (optional)</li> </ul>
Activity 5 The Great Debate	<ul> <li>Copies of BLM 13</li> <li>Copies of BLM 14</li> <li>Copies of BLM 15 (optional)</li> </ul>

Page 6 Overview

### LETTER TO THE PARENTS



### SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week, students in our classroom will be participating in a data management unit called "TV - Vice or Virtue"? The classroom activities will focus on collecting and organizing data, and constructing and interpreting a variety of graphs.

You can assist your child in understanding the relevant concepts by working together to look for situations where graphs might be used, and by helping in the collection of data for the purpose of creating graphs.

Various family activities have been planned for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with graphs in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher's Signature

### A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Overview Page 7



### Activity 1: Data Collection

### Focus of Activity

Designing and using a questionnaire/data collection record sheet for a chosen topic

### What to Assess

- Relevance of questions for inquiry
- Organization of data collection sheet
- Collaboration with other students

### Preparation

- See the table on page 4 for materials
- Make copies of BLM 1 and BLM 2 for each student.
- Have students set up a scrapbook or bulletin board space which they will use for data and graphs found in media text (e.g., newspaper, magazines, flyers), journal entries, and work related to this unit.
- Provide chart paper and markers.

### **A**CTIVITY

Have students read the article, "TV-Chip Ensures School Success" on BLM 1 and generate ideas about the relationship between television and school success. For example, the students might suggest that the article implies a relationship among: academic grades, the number of hours watching TV, the type of TV programs viewed, and the number of hours doing homework. Students can record their thoughts on BLM 1.

Have students develop questions for investigation (from among their suggestions), such as, "How is the number of hours watching television and/or playing video games related to student achievement in school?

It is important to note that, although both collected and given data sets used in this unit seem to indicate some kind of relationship, this does not mean that there is a cause-and-effect relationship between, for example, TV watching and school grades.

Have the students discuss ways to collect data. During the large group discussion, record the students' ideas on chart paper or the blackboard, as shown below.

- personal (face to face) surveyswritten questionnairestelephone survey

- chat rooms on line

Once this list has several suggestions, have students suggest questions that would help them collect data to answer one of the questions for investigation that were mentioned earlier. Record these questions for easy reference as shown below.

Activity 1 Page 8

### Activity 1: Data Collection

### Questions To Ask:

- How many hours of television do you watch Monday? Tuesday?... Sunday?
- How many hours of video games do you play at home, Monday? Tuesday? ... Sunday? At an arcade?
- When do you watch TV or play video games throughout the day? In the morning...?
- What are your academic grades (i.e., from your previous report card, or evaluations of current work samples)?
- How many hours of homework do you do on Monday? Tuesday? ... Sunday?
- What are your favourite TV shows?
- What type of TV show do you like best?
- · What is your favourite video game?

To stimulate thinking about questionnaires, have the students complete BLM 2, working in small groups. Discuss, as a whole class, characteristics of a well-constructed survey/data collection sheet. [See Solutions and Notes]

Next, have each small group select one of the questions from the chart paper/chalkboard to investigate. Then have students create ways to record their data from questionnaires using tables, such as the examples below.

# How Many Hours of TV Do You Watch Each Day?

Sunday	Monday	Tuesday	Wednesday	minut Soldy	Friday	Satuday
4	1	2	1	4	2	0
2	0	0	2	4	0	2
1	0	0	0	3	0	1



Problem Solving



Activity 1 Page 9



Comments in italics are explanatory, and need not be conveyed to the students.

# Communication

# Use of Technology



Problem Solving



### Activity 1: Data Collection

# When do You Watch TV the Most During the Week?

Morning	Afternoon	Evening	Why?
		J	I watch hockey a lot.
		J	My favourite shows are on.
		1	Only time when I am home.

Encourage the students to investigate a variety of questions, so that different data sets are collected. Include the number of hours spent watching TV, the number of hours spent playing video games, and the number of hours spent doing homework. (Data for these three sets will be used for Activity 2 - Interpreting A Data Set.) Some of the data (e.g., number of hours per day spent watching TV) will, of necessity, be estimates.

Have each group of students collect and record their data using the questionnaires they have designed.

To help keep the class organized during this phase, you may choose to have the questionnaires circulate through the class, while the students themselves stay in their groups.

Upon completion of the data collection, have students assess whether the data they collected provides the information expected. During a large group discussion, students can then reflect on the quality of their questions and their process of data collection. For example, students might make suggestions about how to improve the quality of their questions or think of other ways to organize their collection of data.

### Writing About Their Work

What are some characteristics of a good questionnaire? How does the purpose of a survey determine the questions you might ask?

### Extensions in Mathematics

- 1. Students develop more questions for further investigations. For example, the students might ask, "What type of TV shows do you generally watch?" or, "In what extra-curricular activities do you participate?" Students might also collect data from students of different ages (i.e., primary and intermediate level students) and make comparisons among the data.
- 2. Students could investigate questions such as "How many hours do you spend on line? in chat rooms? dealing with email?" by doing surveys on line.

Page 10 Activity 1

### Activity 1: Data Collection

### Cross-Curricular Activities

- 1. Students examine the class collection of graphs from newspapers, magazines, and poster advertisements and think about ways that data was collected and the types of questions that the authors asked.
- 2. Select graphs from the collection that are used to convey persuasive messages. Students could identify ways the design and layout of the graph/advertisement are used to support the message.

### Family Activities

1. Students discuss the article, "TV-Chips Ensure School Success" (BLM 1), with their family members and suggest ways to investigate any possible relationship between TV watching and student learning. Students might summarize their family's reactions and ideas in a journal entry.

### Other Resources

For further details, see annotated Other Resources list on page 52, numbered as below.

- 1. "Making Sense of Data: Addenda Series, Grades K 6", Mary Lindquist et al.
- 3. "Developing Graph Comprehension: Elementary and Middle School Activities", Frances R. Curcio



Activity 1 Page 11



See Solutions and Notes for suggestions in discussing the various graphs on BLMs 3, 4, and 5.

### Use of Technology



### **Assessment**



### Activity 2: Interpreting A Data Set

### Focus of the Activity

- Selecting and constructing an appropriate graph for a set of data
- Interpreting data graphs

### What to Assess

- Appropriateness of graph for the data
- Accuracy of construction of graph, inclusion of necessary components
- Accuracy and justification of interpretations of data graphs
- Quality of both mathematical and language skills in presentation and in journal entry
- Collaboration with other students

### Preparation

- Make copies of BLMs 3, 4, and 5 for each student plus one of each for the overhead projector.
- Provide for access to database and spreadsheet software (e.g., Claris Works<sup>TM</sup>, Microsoft Works<sup>TM</sup> WordPerfect <sup>TM</sup>) (optional).
- Have chart paper and markers available.

### ACTIVITY

### INTRODUCTION

Brainstorm ways of organizing and representing data with the students. As ideas arise, list the suggestions on the chalkboard; the list may include conventional items such as bar graphs, or more imaginative suggestions. Distribute BLMs 3, 4, 5, and review or introduce the characteristics of various types of graphs.

### GRAPHING DATA

Each student or group should select and complete a type of graph that can be used to display meaningfully the data that the student/group has collected in Activity 1. Provide chart paper and markers so that students can make a graph large enough to be seen by the whole class during their presentation. You may wish to have one or two groups generate graphs using available computer software.

Encourage students to refer to BLMs 3, 4, and 5 to ensure that their graphs include all necessary components (e.g., labels on the axes for bar/broken-line graphs; title for any graph; legend or key for picto-graphs or glyphs).

In the large group, students from each group present their graphs on chart paper or overhead and explain:

- why they chose a particular way of presenting their data
- how they constructed the graph
- what conclusions they drew from the analysis and interpretation of their graph

Page 12 Activity 2

### Activity 2: Interpreting A Data Set

Following each small group presentation, ask questions like those listed below. Record some responses to act as a guide for a large group discussion following the presentation.

- What is the range of data values?
- Describe the shape of the graph? What does that suggest?
- What does the graph not tell you about the data that you might want to know?
- What questions do you have, if any, about this graph? How would you find answers to those questions?
- How would your interpretation change if the scale were different? If it began at a value other than 0?

### Writing About Their Work

After the large group discussion, you may wish to provide time for students to share and summarize their ideas about data graph construction and interpretation in pairs or in small groups. Have them record their ideas in a written journal entry. Some additional suggested student journal questions are:

- What did you learn about collecting and representing data?
- What questions, if any, do you still have about constructing or interpreting data graphs?
- Do you still feel you chose the best type of graph to display your data? Explain why, or why not, and suggest improvements.

### Extensions in Mathematics

1. Students construct a similar graph for their data with one or more of the following changes: use a different scale on one or both of the axes; omit two or three pieces of data that don't seem to fit the "shape" of the graph. In small and large groups, students discuss the effect of these graph construction changes in relation to the interpretation of the data graph. *This provides useful preparation for Activities 4 and 5*.

### Cross-Curricular Activities

1. Students, working in small groups, examine the class collection of graphs from newspapers, magazines, and poster advertisements and think about ways that data can be interpreted and presented in a non-visual medium. Their results are discussed by the class as a whole.

### Family Activities

1. During a Family Math Night or Curriculum Night, design different ways to collect data from the family visitors. For example, visitors might be asked to identify how many hours they watch TV or play video games on specific days of the week (or some completely different type of data, such as how many hours per week they spend on hobbies or gardening). Later in the evening (whenever a large group is formed), students and parents could construct graphs, manually or using the computer, and, during a guided discussion, make interpretive statements about the data collected.









Activity 2 Page 13



# Activity 2: Interpreting A Data Set

### Other Resources

For further details, see annotated Other Resources list on page 52, numbered as below.

- 2. "Dealing with Data and Chance: Addenda Series, Grades 5-8", Judith S. Zawojewski et al.
- 3. "Developing Graph Comprehension: Elementary and Middle School Activities" by Frances R. Curcio.

Page 14 Activity 2

### Activity 3: Comparing Two Data Sets

### Focus of Activity

- Representing two sets of data on various types of comparative graphs
- Comparing two sets of data and interpreting the results

### What to Assess

- Construction of data graph
- Appropriateness of type of graph for given data
- Relevance of interpretation of data graph
- Ability to justify conclusions drawn from graph
- Use of mathematical language both orally and in writing
- Collaboration with other students

### Preparation

- Make copies of BLMs 6 and 7, one for each group of students.
- Ensure that students have their copies of BLMs 3, 4, and 5.
- Have chart paper and markers available.
- Make copies of BLM 8 for all students (optional).
- Have copies of BLMs 12 and 15 available (optional).
- Provide for access to database and spreadsheet software (e.g., ClarisWorks<sup>TM</sup>, Microsoft Works<sup>TM</sup>, WordPerfect <sup>TM</sup>) (optional).

### **A**CTIVITY

Distribute copies of BLM 6 and allow students time to examine it. To assist students in reading the chart/table ask such questions as:

- What sets of data are given?
- How many students participated?
- What is the total number of hours spent by student #1 on homework for the whole week? [Answer: 14]
- Which of Students 6 or 10 spent more time playing video games? [Student 6: 0 h; student 10: 9 h]
- What is the greatest number of hours spent watching TV on any one day? Which student(s) spent this much time watching TV? [Students 14, 19, and 24 each spent 7 h watching TV on Saturday].
- What average grade did each of these students have? [E for each student]
- What conclusion might you draw about TV watching and grades? [Students may suggest that too much TV means poor grades, or they may suggest that a conclusion cannot be drawn with only three pieces of data. Discuss with students that, while it appears that TV and grades may be related somehow, this is not to be considered as cause-and-effect. For example, the students may be watching TV because they have difficulty with school work and are trying to avoid it.]

Explain that students will be asked to construct a double bar graph (or other type of comparative graph) using two sets of data. Referring to the examples on BLMs 3 and 4, discuss the differences and similarities between:

- (a) bar graphs and double bar graphs
- (b) stem-and-leaf plots and back-to-back stem-and-leaf plots



Activity 3 Page 15



### Problem Solving



### Use of Technology



### Assessment



### Activity 3: Comparing Two Data Sets

You may wish to discuss the appropriateness or inappropriateness of other types of graphs that appear on BLM 5. For example, picto-graphs might be used with two different symbols - one for each set of data. Broken-line graphs are not appropriate since the data sets do not indicate change. If circle graphs were used, one circle graph would be needed for each set of data.

In small groups, have students choose two data sets (e.g., Hours Doing Homework and Student Grades; Hours Watching TV and Student Grades) and select a way to organize, represent, and analyse the data. Students who previously collected and graphed the number of hours spent on homework, TV, or video games may compare their own data with similar data from BLM 6. Other students may choose two sets of data from BLM 6.

There are two types of bar graphs possible from these data. One type will compare the same trait for different populations (e.g., hours spent on homework for two sets of people). The other will compare different traits of members of the same population (e.g., time spent on homework and time spent on TV for one set of people).

Have students construct graphs that compare their two sets of data (e.g., Double Bar Graphs or Back-to-Back Stem-and-Leaf Plots; see BLMs 3 and 4); they may do so manually or using computer software. If students are generating graphs manually, provide them with chart paper and markers or overheads, so that each graph is large enough to be seen by the other students during large group discussions. If you have access to licensed software such as ClarisWorks<sup>TM</sup>, Microsoft Works<sup>TM</sup> or WordPerfect<sup>TM</sup>, you may wish to have some students create a graph using a computer.

As students are completing their comparative graphs, distribute copies of BLM 7. During their presentations to the class, students should include answers to questions 1 to 4 on BLM 7. They will be responding to questions 5, 6, and 7 in their journals later on.

### DISCUSSION

Each group of students posts their complete graph, and presents their conclusions to the class, including responses to questions 1 to 4 on BLM 7.

### Writing About Their Work

Have students respond to questions 5, 6, and 7 on BLM 7, referring to the posted graphs.

### Extensions in Mathematics

1. Referring to BLM 8, explore a three-dimensional bar graph. Computer software (as previously mentioned) could be used.

Page 16 Activity 3

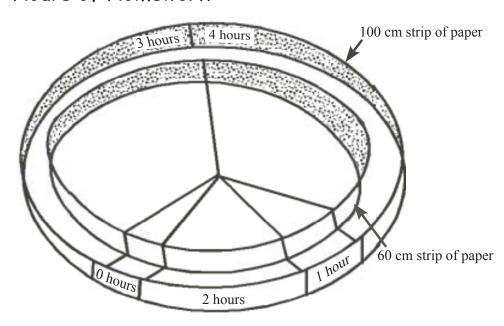
### Activity 3: Comparing Two Data Sets

- 2. Have students individually create glyphs which might represent some or all of the following:
  - the number of hours of television they watch each night of the week;
  - the number of hours of video games they play each night at home, at the arcade;
  - when they watch TV or play video games during the day;
  - their academic grades;
  - the number of hours they do homework each night of the week;
  - the number of hours they spend on line for recreational/school purposes;
  - other related ideas that the students generate.
- 3. Ask students how they think Grade 6 students spend their time in addition to TV, video games, and homework. Have them write a prediction. Then have them prepare a survey, collect data. and compare with their predictions. They should identify both agreements and disagreements of collected data with the prediction.
- 4. Have students represent, say, their data on hours of homework per week on a strip of graph paper (BLM 15). For example, if 2 students do 0 hours, then 2 squares will be coloured red. If 8 students do 1 hour each, then 8 squares will be coloured blue. Similarly, 3 squares will be coloured to represent the 3 students who spent 3 hours each on homework, and so on.



The strip is then formed into a circle as shown by the inner circle. By using a 100 cm strip joined into a circle, the divisions can be extended to delineate the percentage divisions.

# Hours of Homework





The templates on BLM 12 may help students in recording the human-circlegraph data on paper.

Activity 3 Page 17



# Activity 3: Comparing Two Data Sets

### Cross-Curricular Activities

1. Construct a circle graph from the following information. (Use BLM 5 as reference. One of the templates on BLM 12 could be used to help construct an accurate circle graph.)

In a typical 100 kg collection of garbage from a household, the garbage can be classed as follows:

paper: 35 kg yard waste: 25 kg plastic: 5 kg metal: 5 kg food: 9 kg other: 12 kg glass: 9 kg

Because the total is 100 kg, the actual masses of types of garbage can be easily converted to percentages, if you wish.

### Family Activities

1. The whole family can become involved in the following: Compare one week's garbage collection for the whole family (Estimate where necessary), and compare with the data given above. Explain any differences.

### Other Resources

For further details, see annotated Other Resources list on page 52, numbered as below.

- 5. "Gender, Ninja Turtles, and Pizza: Using a Classroom Database for Problem Solving", Judith Day Siegel
- 6. "The Environmental Math Workbook", by Evannah Sakamoto
- 9. "Power Over Trash", Robert N. Ronau and Karen S. Karp

Page 18 Activity 3

### Focus of Activity

- Construction and interpretation of scatterplots
- Selecting/reshaping data to create a bias

### What to Assess

- Construction of scatterplot
- Accurate and meaningful interpretation of graphs
- Correct use of mathematical terms
- Collaboration with other students

### Preparation

- Ensure that students have their copies of BLM 6.
- Make copies of BLMs 9 and 10 for all students or groups of students.
- Make copies of BLM 11 for all students or groups of students. (Optional)
- Have copies of BLM 15 available. (Optional)

### ACTIVITY

### INTRODUCTION

Distribute copies of BLM 9 and BLM 10.

Focus students' attention on the scatterplots themselves. Discuss with the students ways in which a scatterplot is different from and similar to the graphs they have seen earlier. Ask why they think it is called a "scatterplot".

Students should see that each student is graphed separately in the scatterplots, which compare one measurement from BLM 6 against another. In the earlier graphs, one measurement was graphed against a number of students. Like all graphs, a scatterplot is a picture of some specified data.

Scatterplot 1 uses data from BLM 6. Since each dot represents one student, (with the exceptions noted below), it should be possible to identify each student on this graph by placing the student's number beside the dot representing time and grade for that student. Ask students to identify a particular student on Scatterplot 1 by referring to BLM 6. For example, Student 29 has a grade of B and spends 3 h/week playing video games. Locate the point representing Student 29. Both student 1 and student 28 have grades of A and spend 9 h/week playing video games. Why is there only one dot for the two of them?

Ask what they think the smaller numbers on the graph mean. These numbers show that the dots beside the numbers represent 2 or 3 students. For example, students #9 and #21 both had 'C' grades and spent 5 hours playing video games.

Using the statements and questions beside each graph on BLM 9, help students learn to interpret the data illustrated. You may choose to do this in discussion with the whole class or first assign it to small groups for discussion, and then work with the whole class.



Activity 4 Page 19



Discuss the difference between Scatterplot 1 and Scatterplot 2. Scatterplot 2 shows a distinct trend along an imaginary line sloping upward to the right. Question 3 on BLM 9 provides an intuitive introduction to the meaning of the line which is called the "line of best fit".

Even on a scatterplot that does seem to show a trend, there are often points lying well away from the line of best fit. These points are called "outliers" because they "lie outside" the general area of the graph. If you used the suggested Extension in Activity 2, students have already identified some outliers — "pieces of data that did not fit the shape of the graph". A deletion of outliers may lead to a different interpretation of the graph as some students will have found from that Extension.

Have students identify one or two outliers on Scatterplot 2. Ask them to describe the person identified by each point. Discuss how removing outliers can appear to change the meaning of a graph.

If students need more help with the interpretation of a scatterplot, you might consider using Extension number 1 below that describes "correlations".

### CONSTRUCTING SCATTERPLOTS

Distribute copies of BLM 10. Students could work in small groups to compile the data (i.e., calculate totals of hours per week), graph the data, and answer the questions given. Discuss.

Then ask students to select a set of 5 to 8 students from BLM 6 for a scatterplot so that it seems to show either

- (a) that the number of hours spent watching TV seems to be related to the number of hours spent on homework.
- (b) that the number of hours spent watching TV does not seem to be related to the number of hours spent on homework.

Tell students that they may be using this or similar data sets and graphs next day to support their positions in a debate.

Stress with students that even though data might show that two data sets are related, this does not mean that one is the result of the other.

### WRITING ABOUT SCATTERPLOTS

When would it be advantageous to use a scatterplot? Suggest data for which a scatterplot would be suitable. Suggest data for which a scatterplot would not be particularly helpful. Tell why.

### Extensions in Mathematics

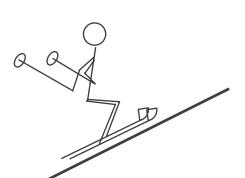
1. A scatterplot such as #2 on BLM 9 shows a "positive correlation" between height and leg length. That is, as one measurement increases (or decreases), the other also increases (or decreases). If one measurement decreases as the other

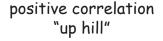
Problem Solving

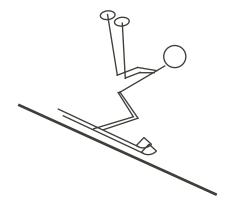


Page 20

increases, the graph is said to show a "negative correlation". Notice that







negative correlation "down hill"

in a positive correlation, the line of best fit goes upward, while in a negative correlation the line of best fit goes downward. Where no line of best fit can be drawn there is no correlation. BLM 11 can be used if you wish to deal with correlation.

### CORRELATION

Distribute copies of BLM 11. Examine the examples at the top of the page, and read the first three questions. Encourage students to describe the graphs and to try to explain the terms "positive correlation" and "negative correlation." Ask students if their scatterplots of data from BLM 6 show either positive, negative, or no correlation. Encourage them to explain what this means — i.e., "Scatterplot 2, BLM 9, shows a positive correlation because, as the length of the leg increases, the height increases."

Students should work in small groups to complete BLM 11. They should be able to suggest some data that will support their position, and sketch a graph of this. You may wish to extend this activity to the actual collection of data where possible and the construction of scatterplots to test students' predictions. Data from all family members can be collected as a family activity.

Students may have difficulty with the idea of a "negative" correlation being a good thing in certain circumstances. For example, the more times you brush your teeth, the fewer cavities you will get, according to the Dental Association. Since one of these measures decreases as the other increases, this is a negative correlation, but it is not a negative situation. Help students with this rather tricky idea.

2. Students could collect data from their own class and construct a scatterplot to compare with Scatterplot 2 on BLM 9.







Problem Solving



Activity 4 Page 21



### Cross-Curricular Activities

1. Scatterplots of geographical data might help students see whether or not there is any correlation between pieces of data. For example, plot latitude of cities in the Northern Hemisphere against average temperature. Or plot average temperature of such cities against precipitation. Or height above sea level against temperature. Students might suggest other sets of data they would be interested in comparing.

### Other References

For further details, see annotated Other Resources list on page 52, numbered as below.

- 4. "A Television Programming Challenge: A Cooperative Group Activity that Uses Mathematics", David L. Pagni
- 7. "Ideas", Sharon L. Young
- 12. "Students' Interpretations of Misleading Graphs", Suzanne R. Harper

Page 22 Activity 4

### Activity 5: The Great Debate

### Focus of Activity

• Manipulation of data to support a chosen position

### What to Assess

- Interpretation of given graphs
- Identification of misinterpretation of graphs
- Coherent argument
- Collaboration with others

### Preparation

- Make copies of BLM 13 for all students.
- Make copies of BLM 14 for all students.
- Have copies of BLM 15 available.

### ACTIVITY

Tell students that they will be taking part in debates. Discuss with students the idea that a debate is a formal argument in which two people or groups argue on two sides of an issue. For example, they constructed graphs the previous day that contradicted each other, since one group selected a set to try to show that there appeared to be a relationship between TV watching and grades, while others tried to show that there was no relationship. You may wish to distribute copies of BLM 13 to the students and discuss them briefly. If this is the students' first experience with a debate you may wish to highlight the major points only.

Distribute copies of BLM 14 to students in pairs or small groups. They could record their conclusions on the back of the sheet or in their notebooks.

In problem 1, students should recognize that Graph B has an expanded vertical scale that makes the growth of the audience appear greater. Ask students for other ways to achieve the same result. For example, if the horizontal scale is shrunk as well, the slope of the line will be steeper and it will appear that the growth is very fast indeed.

In problem 2, graph B shows the data accurately. In Graph A, both the height and the width of the bars have been doubled. This increases the area by a factor of four. Ask students how many "Elaine bars" would fit inside the "Roger bar". They should see that Graph A seems to be showing that Roger ate four times as much popcorn as Elaine and 16 times as much as Chris.

*One simple way to solve problem 3 is to use only the data from the last three years.* 

There are three different techniques used on BLM 14 to attempt to prove false conclusions. The third of these three--careful selection of pieces of data--is probably the method used by students for the final scatterplot of Activity 4.

Discuss these techniques with students and tell them that they will be asked to select pieces of data to "prove" opposing views. For example, 'playing video games is bad for your grades' versus 'playing video games is not bad for your grades'. Refer to the data they selected in the previous activity. (See Constructing Scatterplots).



If all students participate in the debates, two to three periods will be necessary. Consider this time constraint when planning the activity.

### Problem Solving



Activity 5 Page 23



### Communication



### Assessment



### Activity 5: The Great Debate

With students' help, list other affirmative and negative positions that can be taken.

Distribute copies of BLM 13 and give students time to read through it before discussion. Once students understand the basic format of presentations and rebuttals, they will need some planning time.

Students, in pairs, should choose either the affirmative or negative side of one position. Some teacher intervention may be necessary to make sure that both sides of a position have defenders. Time should be allowed for student pairs to select data and construct a graph that they feel will support their position. Students should make some notes about the points they wish to make during their debate.

Some questions that might help them in this could be written on chart paper or the blackboard. For example, How and why did you choose your data set?

Depending on the noise level and/or space available, you might have two debates (or more) going on simultaneously in different corners of the classroom. Each debate should have an audience as well as debaters. One member of the audience can act as timer, another as moderator. Debater #1 of each pair should give the opening argument, and debater #2 give the rebuttal. The teacher or other adult should act as judge (see BLM 13).

After all students have participated in a debate, debrief them with such questions as:

What were some good arguments that you heard during the debates? Why were they good?

What were some poor arguments? Why were they poor? How could they be improved?

If you were to plan another debate, what things would you do differently? What things would you do the same? Why?

What do you know now about graph construction and interpretation that you didn't know before we started these activities?

Alternatively, the questions above could be used to prompt Journal entries.

Page 24 Activity 5

### Activity 5: The Great Debate

### Extensions in Mathematics

1. Students may wish to try selecting pieces of data for other debates. They may suggest collecting data from the classroom and trying to manipulate it to give a false picture.

### Cross-Curricular Activities

1. Have students explore their scrapbook graph collections (See Activity 1, Preparation) for cases of data manipulation that give false positions. They could be encouraged to look for graphs and data used this way in the media.

### Other References

For further details, see annotated Other Resources list on page 52, numbered as below.

- 10. "Graphing in the Information Age: Using Data from the World Wide Web", Juli K. Dixon and Christy J. Falba
- 11. "Interdisciplinary Projects Enhance Teaching and Learning", Jo Clay Olson



Activity 5 Page 25

# BLM 1: News

# **TV-Chips Ensure School Success**

Reduce the negative influence of too much TV and video games in your home. The use of TV-Chip, the television and video game time manager leads to better reading skills, better grades, and improved concentration during study time. TV-Chip enables parents to pre-set the TV and video game viewing time periods and hours, weekly. Within these preset time periods, children are encouraged to evaluate and plan their television and video game playing time.

Up to six children can use different personal codes to turn on the TV. When their weekly TV allotment is used up, the TV shuts off until the TV time allotment is renewed each Sunday at midnight.

Parents may customize their programming of the TV-Chip using optional functions, such as programming: particular types of TV shows, specific days for TV viewing, certain times of the day, or even overriding the system.

This life-time investment is only \$149.95, not including federal and provincial taxes and shipping and handling costs.

Write your thoughts about the relationship between TV watching and school success.

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# BLM 2: Data Collection Sheets

The data collections sheets/questionnaires given below were designed by some grade six students to collect data to make graphs.

How many hours of homework do you do each night?							
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						
Name:	Hours:						

1. These are poor examples of data collection sheets. Why?

How many hours of TV do you watch each day?									
Mo	n	Tues	Wed	Thurs	Fri				
Morning									
Afternoon									
Evening									

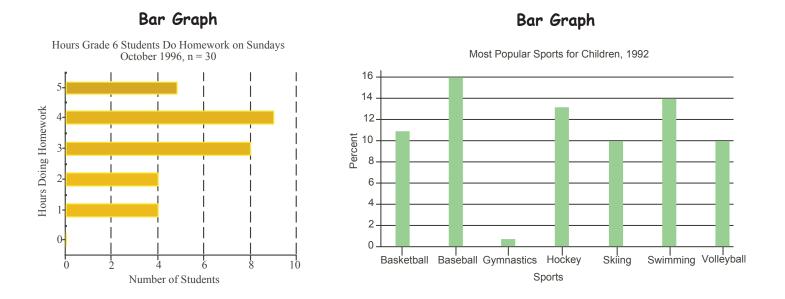
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2. On the back of this sheet, design a better data collection sheet for one of the questions given above, and explain why it is better.

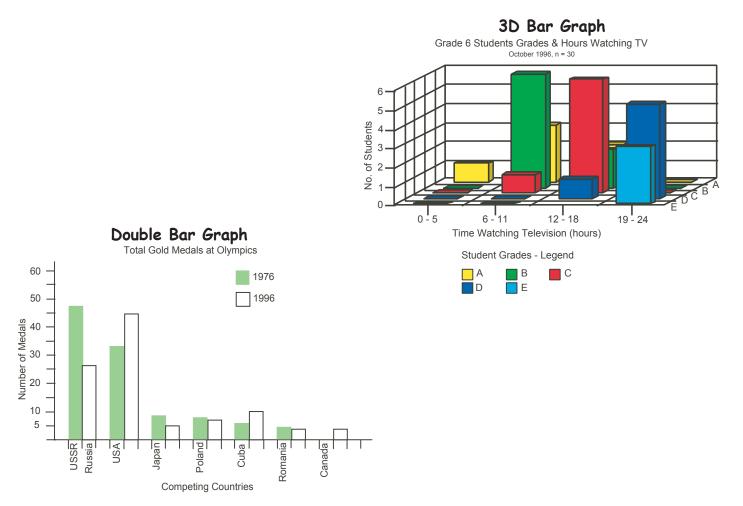
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# BLM 3: Sample Graphs - 1

Bar graphs are used to compare different pieces of data.



If there are two sets of data that are to be compared, a double bar graph might be used (see below at left). Computer programs can be used to produce three-dimensional bar graphs (see below at right.)

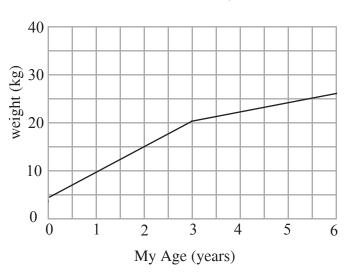


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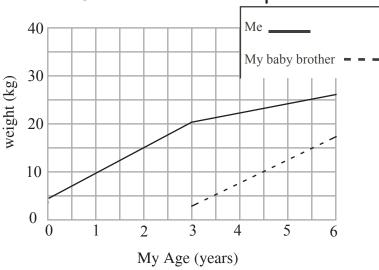
# BLM 4: Sample Graphs - 2

A broken-line graph is used to indicate a trend. Two or more line on one graph can be used to show how two or more things have changed.

# Broken-Line Graph



### Double Broken-Line Graph



### Stem-and-Leaf Plot

A "Stem-and-Leaf" plot is often used with numerical data.

The stem-and-leaf plot on the left below shows the number of hours each person in a class watched TV for one week.

Total Hours Watching TV in a Week October 1996, n = 30

Stem										
0 1 2 3	2	2	4	4	6	6	6			
1	0	0	4	4	4	6	7	7	8	8
2	0	0	1	1	2	3	3	5	7	
3	4	5	6	6						

For example, the third line of the graph shows 2 0 0 1 1 2 3 3 5 7

This tells us that there were two students who watched 20 hours, two who watched 21 hours, one who watched 22 hours, two who watched 23 hours, and so on.

### Back-to-Back Stem-and-Leaf Plot

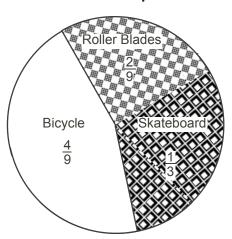
The back-to-back stem-and-leaf below shows the hours of TV watching for two different classes.

Mr. McCudden's Class, n = 26  7 5 5 4 3 3 3 2 2 2 1  8 7 3 2 1 0 0 0  1 1 0  8 7 6 5																					
	7	5	5	4	3	3	3	2	2	2	1	0	1	1	2	3	4	4	6	9	
				8	7	3	2	1	0	0	0	1	0	0	0	2	4	4	8		
									1	1	0	2	3	3	4	4	5				
								8	7	6	5	3	1	1	2						

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# BLM 5: Sample Graphs - 3

### Circle Graph



Our Favourite Wheels

The circle graph is used to show parts of a whole. If you knew how many students were in the class surveyed, you could figure out how many preferred each type of wheel.

# "Glyph" Graph

An unusual graph, called a "glyph" graph can be used to illustrate several pieces of data at once.

### A Weather Graph

Shape of head: the cloudier, the less round the head
Mouth the hotter, the longer the mouth
Nose the higher the wind, the larger the nose
Eyes the larger the eyes, the more rain





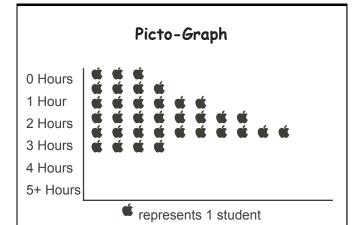






Monday Tuesday Wednesday Thursday Friday

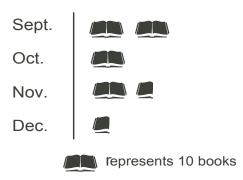
Check your school library for information about ancient peoples who used glyphs to record their history (e.g., Incas, Maya)



The "legend" in a picto-graph may represent one item or person, as in the graph above, or several items or people as in the picto-graph below.

### Picto-Graph

Number of books read by Brian during the first term of 1997



What can you say about Brian's reading habits from September to December?

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# BLM 6: Sample Raw Data Set

"How Does the Number of Hours Watching Television and Video Game Playing Affect Student Achievement in School?" Grade 6 Students in Room 17; Octover, 1996; n = 30

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# BLM 7: Interpreting Comparative Graphs

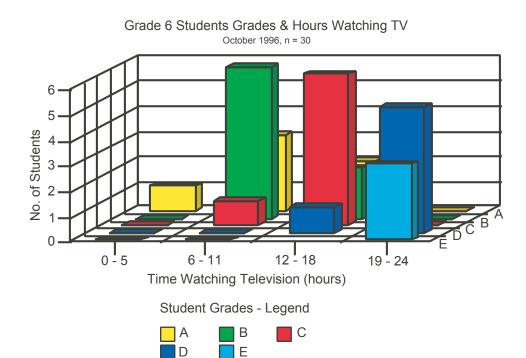
1.	When you graphed your date why did you choose that particular comparative graph?								
2.	What two sets of data are you comparing? Why?								
3.	What does your graph tell you about the data used?								
4.	What did you expect the graph to tell you? Does your graph meet your expectations? Explain.								
•									
An	swer the following on the back of this page or in your journal.								
5.	Describe the shape of one (or more) of the graphs. What does this tell you?								
6.	Select one of the posted graphs. What does this graph not tell you about the data collected? Why?								
7.	Do you have any further questions about a particular graph? How could you find the answers?								

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# BLM 8: Three Dimensional Bar Graphs

Number of Hours Grade 6 Students Watch Television and Their Average Grades; October 1996; n = 30

Grades	0 - 5 Hours	6⊶11:Hours	::12:::18:Hours::	:19 - 24 Hours
А	1	3	2	0
В	0	6	2	0
С	0	1	6	0
D	0	0	0	5
E	0	0	1	3



Answer the following questions on the back of this sheet:

- 1. Look at the table above. Why do you think the data on the number of hours of TV watching are grouped?
- 2. What does "n = 30" tell you?
- 3. Now look at the graph. Moving from front to back, what is the order of the grades being plotted?
- 4. What part of the graph would you select if you were trying to convince someone that too much TV watching seems to be linked to lower grades?
- 5. How could you improve on the way this graph presents the data?

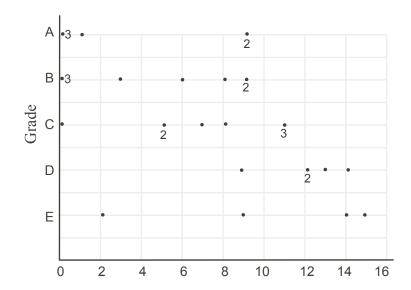
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# **BLM 9: Scatterplots**

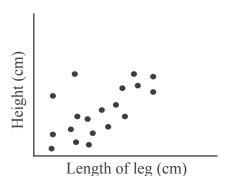
Answer the questions on the back of this sheet or in your note books or journals.

- 1. This "scatterplot" compares the number of hours spent playing video games with students' grades.
- a) The graph shows four students whose grade is E. How many hours per week did each spend playing video games?
- b) How many students had C grades? How do you know?
- c) There are several students who did not play video games at all. What are the most common grades for these students?
- d) What conclusions might you draw from this graph? Explain why.

Grade 6 Students Video Game Playing and Grades



- 2. This is an incomplete scatterplot comparing height with leg length for a class of grade 6 students.
- a) What does the data appear to show?
- b) As the height increases, what happens to leg length generally? Would you expect this to occur? Why or why not?
- c) Would your interpretation of the graph differ if the actual measurements were given? Why or why not?



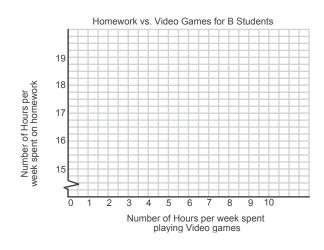
3. Sometimes, when a scatterplot seems to show a trend, a statistician might draw something called a "line of best fit." What do you think this means? Could you draw a line of best fit for either of the scatterplots above? If so, draw it. If not, tell why not.

# BLM 10: Reshaping Data

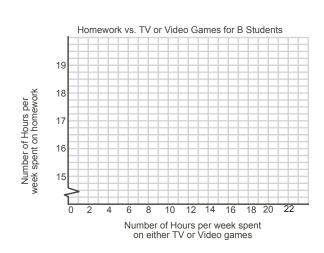
1. Complete the chart for all students with a "B" grade, using data from BLM 6.

Student	Average Grade		lours per Wee		Ttl # of hrs/wk watching TV or playing video games
4	В	16	11	9	20
6	В	18	11	0	11
10	В			9	
11	В			0	
12	В			0	
20	В				
26	В				
29	В				

- 2. a) Complete the scatterplot to the right.
  - b) Does your graph have a line of best fit? Explain.

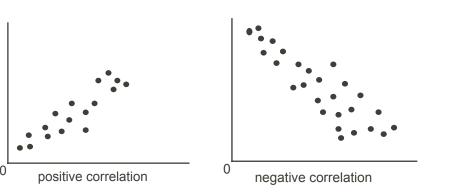


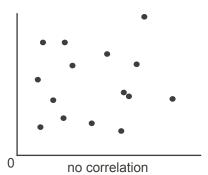
- 3. a) Complete the scatterplot to the right.
  - b) Compare this with the graph above.
  - c) Does this scatterplot have a line of best fit? Does this surprise you? Explain.



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- 1. Describe the graph above that shows a positive correlation between the two measurements graphed. Why do you think this is called a positive correlation?
- Describe the graph above that shows a negative correlation. Compare this with a positive correlation.
- Why do you think the third graph above shows no correlation?
- From among the following data sets, select two that you think would show:
  - a) a positive correlation;
  - b) a negative correlation;
  - c) no correlation.

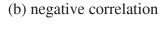
Data Sets:

- Age in months (A)
- (B) Height in centimetres
- Number of years since starting school (C)
- Number of times per day of brushing teeth (D)
- Number of blocks from school (E)
- Speed of cars passing the school (F)
- (G)Number of cavities in teeth
- Number of cars parked in front of the school (H)
- Time of school day on a 24 hr clock (I)

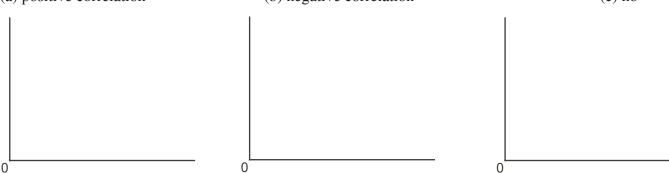
\_\_\_\_\_ (b) \_\_\_\_\_ (c)

5. Make up or collect some reasonable data for your choices for question 4. Sketch a graph for each.

(a) positive correlation

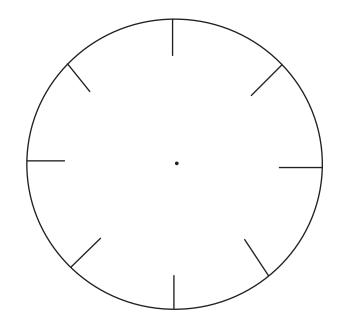


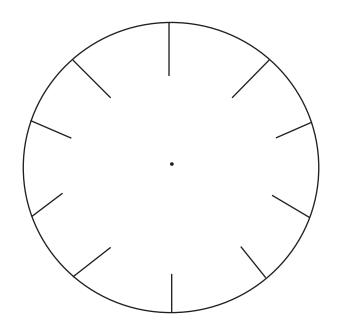
(c) no



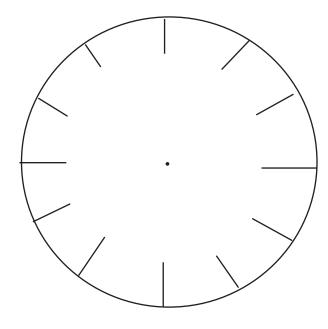
# BLM 12: Templates for Circle Graphs

1. 2.

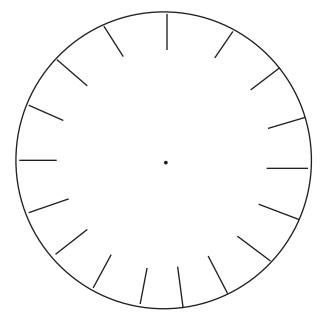




3.



4.



# BLM 13: Notes on Debating

A debate is basically an argument between two teams, usually of two persons each. Each person has a turn to try to convince the audience that his/her argument is valid. The arguments illustrate two sides of an issue. One team takes the "affirmative" position and the other takes the "negative" position.

For example, suppose one side tries to prove that the more you watch TV, the poorer your grades will be. Then the two positions would be given as:

\*Affirmative: TV watching is bad for your grades.

\*Negative: TV watching is not bad for your grades.

You have been trying to select data to show that there are relationships between hours of homework, hours spent watching TV, hours spent playing video games, and grades. The graphs that you have constructed could be used to try to prove an affirmative or a negative position.

Once you and your partner have chosen a position and found another pair who will argue the opposite position, you are ready to plan your debate. Spend some time organizing what you want to say, preparing any graphs you think will help, and deciding which partner will give which arguments.

When you are ready for the debate, you need a timer, a moderator, a judge, and an audience.

The following are the steps in the debate:

- 1. The "first affirmative" presents his/her arguments (3 minutes).
- 2. The "first negative" presents his/her arguments (3 minutes).
- 3. The "second affirmative" presents his/her rebuttal (This is a chance to answer criticisms given by the other team) (3 minutes).
- 4. The "second negative" presents his/her rebuttal (3 minutes).
- 5. The "first affirmative" has an opportunity to present a short rebuttal (1 minute).
- 6. The moderator allows members of the audience to give their opinions (optional).
- 7. The audience has a chance to vote for the better debating team. This is not a vote for what you believe, but for which team you think had the better arguments. The moderator takes the vote which may be a show of hands.

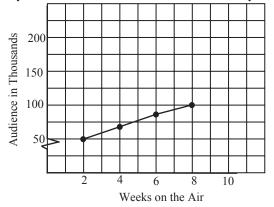
Page 38 Black Line Masters

# BLM 14: Manipulating Data

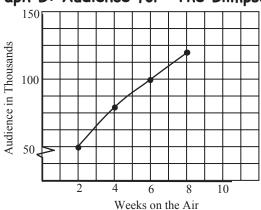
1. The sponsors of the Blimpsons kept track of their viewing audience for the first few months of the new show The results are shown in Graph A.

The star of the show wanted to keep the show on the air so he redrew the graph (see Graph B) and said the graph showed a rapidly growing audience. How did he make it appear as if the audience were growing faster? Was it really increasing faster? Explain.

Graph A: Audience for "The Blimpsons"



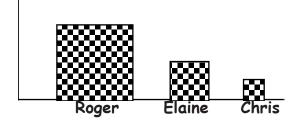
Graph B: Audience for "The Blimpsons"



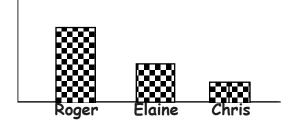
2. Roger, Elaine, and Chris all enjoyed eating popcorn while watching TV movies. Roger ate twice as much as Elaine, and four times as much as Chris.

Which graph below shows this accurately? How is the other graph distorted? What does it seem to show?

Graph A: Popcorn Consumption



Graph B: Popcorn Consumption



3. The sales for Ezy-Win Komputer Game for several years are given below:

Year	1992	1993	1994	1995	1996
Sales:	2345	4200	2150	2080	3000

Construct a graph using data from the chart to make Ezy-Win look like a reasonably successful company.

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# BLM 15: 1 cm × 1cm

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#### Solutions and Notes

## Activity 1: Data Collection

Characteristics of a well-constructed survey/data collection sheet:

- Questions should be clear, with little possibility of misinterpretation.
- Questions should elicit concise answers.
- Questions should be easy to answer.
- Questions should elicit accurate answers.
- The answers to questions should be relevant.
- The language used in questions should be easy to interpret.

# Activity 2: Interpreting a Data Set

Questions for comparisons between types of graphs are included in Activity 3 and on BLM 8. The following questions are included here as suggestions should you wish to focus in detail on graph characteristics here rather than in Activity 3.

# BLM 3: Sample Graphs-1

- 1. Bar graphs are used to compare different pieces of data. What different pieces of data are being compared in the first two graphs? For the first graph, numbers of students are being compared; for the second graph, measures of popularity of sports are being compared.
- 2. Select one or two "Questions to Ask" from Activity 1 and ask students if bar graphs would be appropriate for displaying these data. Ask why or why not. Since the data is comparative, bar graphs are quite appropriate. So are pictographs or "Glyph" graphs.
- 3. Ask students how a double-bar graph is like and is different from a single-bar graph. A double bar graph illustrates two sets of data that one wishes to compare.

The three-dimensional graph will be explored further with BLM 8 in Activity 3. Alternatively, you may wish to use BLM 8 in Activity 2.

# BLM 4: Sample Graphs-2

- 4. Ask students to list possible types of data for which a broken-line or double-broken-line graph would be suitable. For example, one's height over several months, growth of money in a savings account. Such data show change over time.
- 5. Referring to the back-to-back stem-and-leaf plot, ask how many students in each class watched more than 30 hours of TV per week. There are 4 in McCudden's class and 3 in Kubota's. What was the greatest number of hours watched? One student watched 38 hours. Why is it easy to compare the TV watching of the two classes using this type of graph? Answers will vary but should indicate that placing the data sets opposite each other that is, with the 'leaves' on opposite sides of the 'stem' makes it easy to compare.

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## BLM 5: Sample Graphs-3

- 6. Referring to the "Glyph" graph, ask students to describe the weather on Monday (Sunny, very hot, little wind, possible showers). Which day was the windiest? (Friday) Which day had the most rain? (Thursday)
- 7. Have students suggest glyphs that might be used to show TV watching, hours of homework, and grades.

# Activity 3: Comparing Two Data Sets

### BLM 8: Three Dimensional Graphs

- 1. Answers will vary. Students should recognize that if the data were not grouped, there might be only one student in each category, and therefore comparison would be meaningless.
- 2. The number of students included in the data is represented by 'n'. In this case there are 30 students (n = 30).
- 3. From front to back the grades are E, D, C, B, A.
- 4. Answers will vary. One possible choice is the set of data for 19-24 hours, since this shows that all these students have grades of D and E.
- 5. Answers will vary. Students may suggest different groupings, use of colour, use of double- or triple-bargraphs, etc.

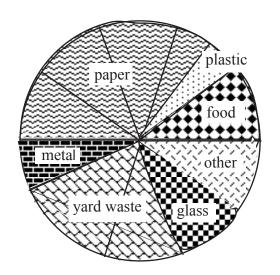
#### Cross-Curricular Activites

The ten-division template from BLM 12 was used for this graph. Lines were drawn to divide the circle into ten equal pieces.

Thus, paper (35 kg out of 100 kg) uses three and one-half sections; plastic uses one-half section; food uses 9/10 of a section; etc.

Divisions for 1/2 or 9/10 were estimated.

The graph is suitable for comparing the amount of household garbage in each category.



#### Solutions and Notes

# Activity 4: Scatterplots and Reshaping Data

# BLM 9: Scatterplots

- 1. a) The "E" students played 2, 9, 14, and 15 hours.
  - b) Seven students had C grades.
  - c) The most common grades for these students are A and B.
  - d) Answers will vary, but students should realize that the number of hours playing video games does not seem to be related to grades. Ask students to explain their conclusions. Reasons could include (i) there are not enough students; (ii) some students never play video games and if we graphed only those that do play, the graph may seem to show a relationship. Stress with the students that even though a graph seems to show a relationship this is not proof that one phenomonon causes the other.
- 2. The data appear to show that, as height increases, leg length increases (or vice versa). This should be seen as reasonable whether or not measurements are given, since our growth increases primarily because of growth in our long bones, three of which arein the leg.
- 3. A "line of best fit" usually shows that two sets of data are related in some way. Therefore a line of best fit is possible for the growth graph but not the other.

## BLM 10: Reshaping Data

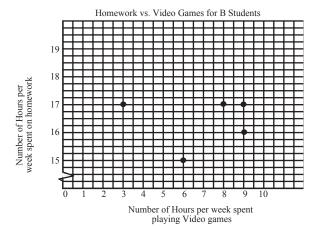
Make sure students understand that "hours per week spent on either TV or Video games" means the total time spent on both as in the last coloumn of the chart.

Student	Average Grade	Homework	lours per Wee TV	Video	Tti# of hrs/wk watching:TV or
				Games	playing video games
4	В	16	11	9	20
6	В	18	11	0	11
10	В	17	10	9	19
11	В	18	8	0	8
12	В	17	8	0	8
20	В	17	13	8	21
26	В	15	9	6	15
29	В	17	10	3	13

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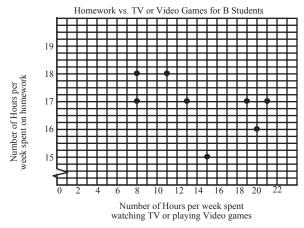
Solutions and Notes

2. a)



b) The graph does not have a line of best fit. Students should be aware that (i) the points on the graph do not suggest any relationship between hours of homework and hours playing video games, and (ii) there are too few points to show any relationship.

3. a)



- b) The points seem to have moved further to the right, but no relationship is suggested.
- c) There is no line of best fit. This may or may not surprise students. They should recognize the reasons as being similar to those given above for question 2.

# BLM 11: Correlation (optional)

- 1. This is a positive correlation since, as one variable increases, so does the other. A line of best fit would slope up to the right.
- 2. A negative correlation shows that one factor decreases as the other increases. A line of best fit would slope down to the right.
- 3. There is no line of best fit.
- 4. Answers will vary. Students should explain why they made the choices they did. Possible answers are:
  - a) positive correlation: A and B; A and C
  - b) negative correlation: D and G; possibly F and H (cars drive more slowly when there are several cars <sup>TM</sup>1 parked in front of the school);
  - c) no correlation: E and G; E and F
- 5. Answers will vary.

# Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a studetn's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

#### Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and repsond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

#### Observations

Research has consistently shown that the most relaiable method of evaluation is the eon-going, inclass observation of students by tachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics:
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits individual and collaboartive;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, as a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

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#### Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assits students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a cooperative activity or project using a questionnaire which asks how well the group worked together, student evaluative comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the studnet's attitudes, mathematics understanding, achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectiations and criteria for evaluation.

#### Resources for Assessment

- 1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.
- 2. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.

The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., POC 1J0. Phone/Fax 705-732-1990.

- 3. "Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions", by Jean Karr Stenmark (Ed.), NCTM, 1991.
  - This book contains a variety of assessment techniques and gives samples of student work at different levels. Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca
- 4. "Assessment", Arithmetic Teacher Focus Issue, February 1992, NCTM.

  This copy of NCTM's journal for elementary school addresses several issues dealing with assessment. It also includes suggested techniques and student activities.
- 5. "How to Evaluate Progress in Problem Solving", by Randall Charles et al., NCTM, 1987. Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

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### A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

#### "US and the 3 R's"

There are five criteria by which each response is judged:

Understanding of the problem,

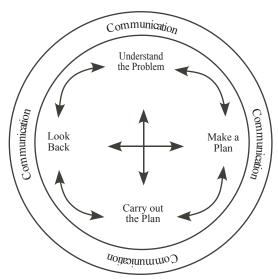
Strategies chosen and used,

Reasoning during the process of solving the problem,

Reflection or looking back at both the solution and the solving, and

Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Limited identifies students who are in need of much assistance;

**Acceptable** identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

**Capable** students may occasionally need assistance, but show more confidence and can work well alone or in a group.

**Proficient** students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

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# LEVEL OF RESPONSE

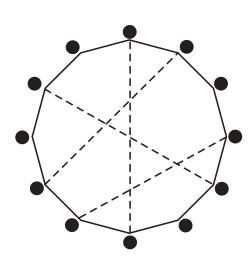
		Limited	Acceptable	Capable	Proficient
CRIL	U N D E R S T	requires teacher assistance to interpret the problem     fails to recognize all essential elements of the task	shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
E	s N	needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
RIAF	T I R N G F F F F F F F F F F F F F F F F F F	<ul> <li>applies strategies randomly or incorrectly</li> <li>does not show clear understanding of a strategy¹</li> <li>shows no evidence of attempting other strategies</li> </ul>	<ul> <li>attempts an appropriate strategy, but may not complete it correctly<sup>2</sup></li> <li>tries alternate strateges with prompting</li> </ul>	<ul> <li>uses strategies effectively</li> <li>may attempt an inappropriate strategy, but eventually discards it and tries another without prompting</li> </ul>	<ul> <li>chooses and uses strategies effectively<sup>3</sup></li> <li>recognizes an inappropriate strategy quickly and attempts others without prompting</li> </ul>
.OR	R S E A S O N	makes major mathematical errors     uses faulty reasoning and draws incorrect conclusions     may not complete a solution	may present a solution that is partially correct	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
ASSESSME	I N G R E	<ul> <li>describes<sup>4</sup> reasoning in a disorganized fashion, even with assistance</li> <li>has difficulty justifying<sup>5</sup> reasoning even with assisstance</li> </ul>	<ul> <li>partially describes<sup>4</sup>         a solution and/or         reasoning or explains         fully with assistance</li> <li>justification<sup>5</sup> of solution         may be inaccurate,         incomplete or incorrect</li> </ul>	• is able to describe <sup>4</sup> clearly the steps in reasoning; may need assistance with mathematical language • can justify <sup>5</sup> reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies <sup>5</sup> reasoning using appropriate mathematical language
	F L E C T	shows no evidence of reflection or checking of work     can judge the reasonableness of a solution only with assistance	<ul> <li>shows little evidence of reflection or checking of work</li> <li>is able to decide whether or not a result is reasonable when prompted to do so</li> </ul>	<ul> <li>shows some evidence of reflection and checking of work</li> <li>indicates whether the result is reasonable, but not necessarily why</li> </ul>	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
7	R N E L E	• unable to identify similar problems	• unable to identify similar problems	• identifies similar <sup>6</sup> problems with prompting	• identifies similar <sup>6</sup> problems, and may even do so before solving the problem
<b>\</b>	V A N C E	• unlikely to identify extensions <sup>7</sup> or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions <sup>7</sup> or applications with prompting	• can suggest at least one extension <sup>7</sup> , variation, or application of the given problem if asked	• suggests extensions <sup>7</sup> , variation, or applications of the given problem independently

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#### Notes on the Rubric

- 1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.
- 2. For example, diagrams or tables may be produced but not used in the solution.
- 3. For example, diagrams, if used, will be accurate models of the problem.
- 4. To *describe* a solution is to tell *what* was done.
- 5. To *justify* a solution is to tell *why* certain things were done.
- 6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.

For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



**Problem 1:** There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

**Problem 2:** Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

**Problem 3:** How many diagonals does a 12-sided polygon have?

Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a "what if...?" problem, such as "What if the question were reversed?", "What if we had other data?", "What if we were to show the data on a different type of graph?".

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#### ADAPTING THE RUBRIC

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described" by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

## Activity 1

STRATEGIES: How well do students identify characteristics of a good survey/data collection sheet?

For example,

- The "Limited" student may not understand that his/her question(s) may not elicit the desired information or may elicit extraneous and confusing material.
- The "Capable" student will recognize the need for unambiguous questions, but may not be sure exactly how to achieve this.

### Activity 2

REFLECTION: How appropriate are students' comments and questions about their own and others' graphs?

For example,

The "Limited" student may confine his/her comments to statements such as "John watched 3 hours of TV on Monday night."

The "Capable" student shows an ability to analyse the data, making such statements as "Almost all the graphs show that students spend more hours watching TV than they do playing video games."

# Activity 3

REASONING: How well can students suggest suitable sets of data for a double-bar or back-to-back stem-and-leaf plot?

For example,

The "Acceptable" student recognizes that double-bar and back-to-back stem-and-leaf plots can be used
to compare two sets of data, but may not recognize that both sets of data should use the same scales on
the axes.

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• The "Proficient" student is able to compare the two types of graph and recognize that the stem-and-leaf plot is similar to both a bar graph and a pictograph.

## **Activity 4**

UNDERSTANDING: How detailed a comparison can students make between a scatterplot and other types of graph, such as bar graphs?

For example,

- The "Acceptable" student recognizes that the data points are individual, but may interpret a vertical 'line' of dots as a bar in a bar graph.
- The "Proficienct" student recognizes that each point on a scatterplot identifies a single point of data unlike a bar graph in which, for example, all students with an "A" grade are represented within one bar.

UNDERSTANDING, STRATEGIES, and REASONING: Can students identify techniques used to distort data (as shown on BLM 14)? How consistent are they in selecting data points from BLM 6 to 'prove' a particular conclusion? How well do they recognize the effects of stretching or shrinking a scale on the axis of a graph?

## Activity 5

REFLECTION and RELEVANCE: How well do they recognize that the data used does not support the argument being made? What suggestions can they make for improvements in both data selection and presentation?

For example,

- The "Limited" student may not recognize when an invalid argument is being put forward. This student may not be able to identify techniques used in newspaper or magazine advertisements to support a particular conclusion.
- The "Proficient" student checks the validity of the data points chosen to support the argument, and identifies various methods of distorting data, and how these might be employed. This student will be able to identify graphs from newspapers or magazines using any particular method of data distortion.

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Other Resources

- 1. "Making Sense of Data: Addenda Series, Grades K-6", by Mary Lindquist et al., NCTM, 1992. Activities for each grade are presented with lesson outlines and selected black line masters. Students collect, represent and analyze data.
- 2. "Dealing with Data and Chance: Addenda Series, Grades 5-8", by Judith S. Zawojewski et al., National Council of Teachers of Mathematics, 1991. Data gathering and reasoning are two of the themes of this book. Ways people use data and chance in their everyday lives are also discussed.
- "Developing Graph Comprehension: Elementary and Middle School Activities", by Frances R. Curcio, NCTM, 1989.
   The book contains 25 activities for classroom use dealing with interpreting and constructing various types of graphs.
- 4. "A Television Programming Challenge: A Cooperative Group Activity That Uses Mathematics", by David L. Pagni, *Arithmetic Teacher*, NCTM, January 1989, pages 7-9. The article describes an interdisciplinary activity in which students gather and analyze data related to television and then offer suggestions based on their analysis.
- 5. "Gender, Ninja Turtles, and Pizza: Using a Classroom Database for Problem Solving", *Teaching Children Mathematics*, NCTM, December 1996, pages 192-199.
  Students collect a wide range of classroom data as a database in Clarisworks<sup>TM</sup>, and select various sets of data to illustrate with graphs.
- 6. "The Environmental Math Workbook", by Evannah Sakamoto, Courseware Solutions Inc., Toronto, 1992.
- 7. "Ideas", by Sharon L. Young, Arithmetic Teacher, NCTM, May 1991, pages 25 33. Students collect data on their families TV viewing patterns, favourite programs and commercials, and graph the data using bar graphs and scatterplots. Student worksheets are available for copying.
- 8. "Mathematics Teaching in the Middle School", March 1999, Focus Issue on "Data and Chance", NCTM. The entire journal is devoted to articles dealing with Data Management & Probability. One article deals with students' TV watching. Another records data about peanut butter (cost per unit, saltiness, colour) and records data using line plots and stem and leaf plots.
- 9. "Power Over Trash", by Robert N. Ronau and Karen S. Karp, *Mathematics Teaching in the Middle School*, NCTM, September 2001, pages 26 31. Students collect garbage from school grounds and classify different types (paper, plastic, metal). They record this on "strip graphs" which are then looped into circles to help develop circle graphs. Some use graphing calculators (computer programs can also be used). Data was then compared with national data from landfill sites.
- 10. "Graphing in the Information Age: Using Data from the World Wide Web," by Juli K. Dixon and Christy J. Falba, *Mathematics Teaching in the Middle School*, NCTM, March April 1997, pages 298 304. Ideas for activities include interpretation and creation of bar graphs, line graphs, and circle graphs. Examples of the type of data that can be found on the web are given (e.g., average earnings versus education, NHL stats, frequency of winning lotto numbers).

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### Other Resources

- 11. "Interdisciplinary Projects Enhance Teaching and Learning", by Jo Clay Olson, *Mathematics Teaching in the Middle School*, NCTM, January 2003, pages 260 266. Several projects that are cross-curricular are described. Language Arts and Geography, in particular, are involved as students design a country given area and perimeter, topology (e.g. 20% mountains), and other constraints. Students draw maps, build models, construct graphs, and write reports.
- 12. "Students' Interpretations of Misleading Graphs", by Suzanne R. Harper, Mathematics Teaching in the Middle School, NCTM, February 2004, pages 340 343.
  Misleading graphs of various types (pictographs, broken line graphs and bar graphs) are presented, and students are questioned about why different modes of presentation lead to different interpretations of the data.

Other Resources Page 53