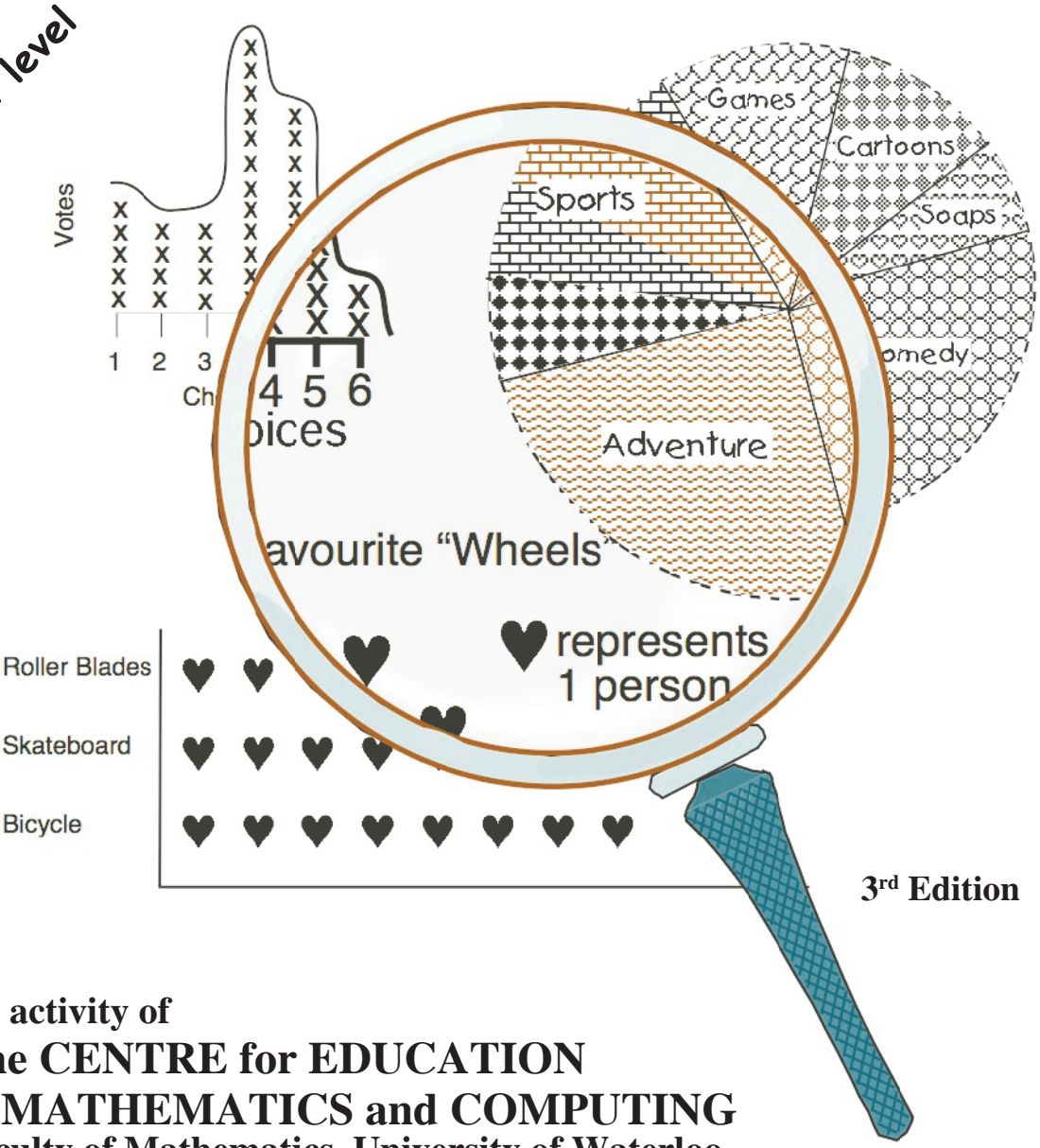
Invitations to Mathematics

Investigations in Data Management

"Our Classroom Community"

Suggested for students at the Grade 4 level



3rd Edition

An activity of
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics, among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the extensions and related activities included with individual activities/projects, provide ample scope for all students' interests and ability levels. Related "Family Math" activities to involve the students' parents/care givers are also suggested.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

Investigations in Data Management is comprised of activities to enhance the students' abilities to collect, represent, and interpret data, frequently in a problem-solving mode. Since today's media make constant use of data presented in various forms (often in order to sway opinion), it is especially important to help students learn to read and interpret these graphs, charts, or tables. Data management is equally important in the depth and variety of its connections to other subjects such as environmental studies.

Information on all the available units in the ***Invitations to Mathematics*** series can be found at the end of this booklet.

Acknowledgements

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Table of Contents

Overview

Common Beliefs

The activities in this booklet have been developed within the context of certain beliefs and values about mathematics generally, and data management specifically. Some of these beliefs are described below.

Importance of Statistics

In an information-rich society such as ours, statistics are an increasingly important aspect of daily life. "Educators and mathematicians now stress the importance of incorporating data analysis in the elementary mathematics curriculum to prepare students for living and working in a world filled with information based on data."

Corwin and Russell

Connections to Other Curriculum Subjects and Mathematics Strands

Activities which involve data management can provide a meaningful link to other content areas such as environmental studies and science. These activities help students to develop critical-thinking and problem-solving skills, and can reinforce communication skills as students discuss and write about their conclusions. Within mathematics, these activities provide opportunities for students to represent, interpret, and discuss information, as well as estimate, measure and develop number sense.

Connections to the Real World

Through collecting and analyzing real data, students encounter the uncertainty and intrigue of real mathematics. "We are living in a world of information. Stop and think a moment about the number of facts, figures, and other data that confront us each day. What do we do with all this information? We ignore some of it, we organize some of it to fit what we already know, or we summarize it by using shorter descriptions or other numbers. How is this information presented to us? It may be presented in written descriptions, in graphs or tables or in summary numbers such as averages. How do we learn to make sense of all this information? This is where we, the K-6 teachers and teacher educators, enter the picture; we need to help our students - from the time they first enter school - to make sense of data."

National Council of Teachers of Mathematics (NCTM)

> Good tasks are ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity and invite them to speculate and to pursue their hunches.
>
> NCTM

Importance of Language

A central activity in data analysis is dialogue and discussion. In a classroom setting, a significant amount of time should be devoted to reflection, discussion, and writing about the meaning of the data.

Importance of Real Data

In data analysis, students use numbers to describe, compare, predict, and make decisions. Because real data are used, there are no predetermined "answers". "Not only do you not know the answer [to the investigation] in advance, but, without seeing the data, you may not even know what the most interesting questions are going to be!"

Corwin and Russell

Essential Content

The activities in this unit deal primarily with data management, specifically, collecting, displaying, and analysing data. In addition, there are Extensions in Mathematics, Cross-Curricular Activities and Family Activities. These may be used prior to or during the activity as well as following the activity. They are intended to suggest topics for extending the activity, assisting integration with other subjects, and involving the family in the learning process.

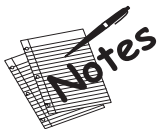
During this unit, the student will:

- collect information;
- develop methods of data collection;
- suggest possible meanings of an incomplete bar graph;
- identify a variety of graphs;
- identify components of a bar graph;
- interpret a bar graph, a circle graph, and a picto-graph;
- create a bar graph, a circle graph, and a picto-graph;
- justify opinions with coherent arguments;
- use correct mathematical language;
- collaborate with other members of a group.

Overview

CURRICULUM CONNECTIONS

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
Activity 1 **Interpreting Incomplete Bar Graphs**	• identifying parts of a bar graph • interpreting bar graphs • justifying predictions about bar graphs, using mathematical language	• use conventional symbols, titles, and labels when displaying data • recognize the purposes of different parts of a graph • read and interpret data presented on ... graphs
Activity 2 **Predicting a Bar Graph**	• identifying parts of a bar graph • interpreting bar graphs • creating bar graphs	• use conventional symbols, titles, and labels when displaying data • recognize the purposes of different parts of a graph • construct labelled graphs
Activity 3 **Collecting and Displaying Data**	• devising a questionnaire • predicting the shape of a bar graph • collecting and displaying data • creating a bar graph	• before gathering data, predict the possible results of a survey • construct labelled graphs • explain how data were collected and describe the results of a survey
Activity 4 **Types of Graphs**	• interpreting bar, circle, and picto-graphs • comparing different types of graphs • constructing circle and picto-graphs	• display data on picto-graph or bar graph • use conventional symbols, titles, and labels when displaying data • read and interpret data presented on ... graphs (e.g., circle graphs)
Activity 5 **Presenting Classroom Graphs**	• illustrating bar, circle, and picto-graphs • describing aspects of graphs using mathematical language • assessing others' graphs and presentations	• interpret displays of data and present the information using mathematical terms • explain how the data were collected and describe the results of a survey



ASSESSMENT

Assessment is a process of gathering evidence about a student's knowledge, skills, and values, and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Attention should be given to a broad range of assessment practices such as:

- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned, but mathematical competence involves such characteristics as the ability to communicate, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. Because of the nature of the activities it is suggested that a variety of types of assessment be used. Suggestions include:

(i) observing students as they work to see if they are applying various concepts; to see if they are working cooperatively; to observe their committment to the tasks;

(ii) assessing the completed project to see if instructions have been followed; to see if concepts have been applied correctly; to see if the language of mathematics has been used correctly;

(iii) assessing the students' descriptions of their completed work to see if mathematical language is used correctly; to see if students understand the concepts used;

(iv) providing opportunities for student self-assessment: have students write explanations of their understanding, opinion, or feelings about an activity. One technique is to have them write under the headings What I Did, What I Learned, and How I Felt About It. Students could be asked to write a review of one day's activities or of the whole unit's work.

(v) selecting an exemplary piece of work to be included in a portfolio for assessment purposes or for sharing with parents.

See Suggested Assessment Strategies, page 41, for further discussion and sample rubrics.

Overview

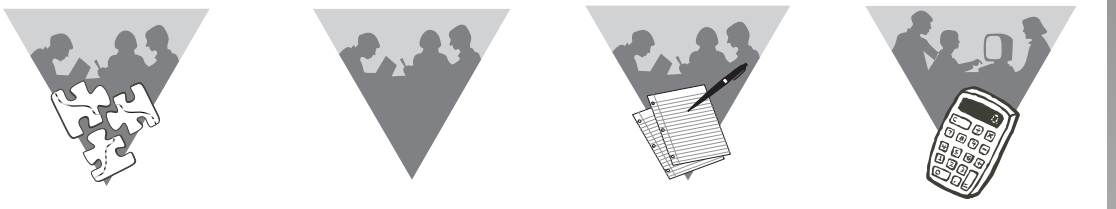
PREREQUISITES

Students need not have had extensive experience with data collection, display, or analysis, but the use of a tally when collecting or organizing data will be helpful.

LOGOS

The following logos, which are located in the margins, identify segments related to, respectively:

Problem Solving **Communication** **Assessment** **Use of Technology**



ACTIVITY	MATERIALS
Activity 1 Interpreting Incomplete Bar Graphs	• Copies of BLMs 1 and 2 • Copy of BLM 1 on acetate for use with the overhead projector • Copies of BLM 3 (optional) • Chart paper or blackboard • Small Post-Its™ or linking cubes (optional) • Copies of BLMs 10 and 11 (optional)
Activity 2 Predicting a Bar Graph	• Copies of BLM 4
Activity 3 Collecting and Displaying Data	• Copies of BLM 5 • Copies of BLM 6 (optional) • Measurement devices (metre tapes, scales) as needed to assist with data collection for topics chosen by students during the lesson
Activity 4 Types of Graphs	• Copies of BLM 7 • Copies of BLM 8 • Copies of BLMs 10 and 11 (optional) • Copies of BLM 12 (optional)
Activity 5 Presenting Classroom Graphs	• Copies of BLM 9 • Acetate of copy BLM 9, or drawing on chart paper

Letter to Parents

SCHOOL LETTERHEAD

DATE

Dear Parent(s)/Guardian(s):

For the next week students in our classroom will be participating in a unit titled "Our Classroom Community". The classroom activities will focus on data collection and organization, graph creation and interpretation, and predicting trends based on the data collected.

You can assist your child in understanding the relevant concepts by working together to look for situations where graphs might be used, and by helping in the collection of data for the purpose of creating graphs.

Various family activities have been planned for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with graphs in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher''s Signature

A Note to the Teacher:

If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.

Activity 1: Interpreting Incomplete Bar

Focus of Activity

- Creating and interpreting bar graphs

What to Assess

- Knowledge of the components of a bar graph
- Understanding of the purpose of each component of a bar graph
- Interpretation of a bar graph
- Prediction of the possible meaning of an incomplete bar graph
- Ability to justify opinions with coherent arguments

Preparation

- Make copies of BLMs 1 and 2.
- Make an acetate copy of BLM 1.
- Make copies of BLM 3 (optional).
- Make copies of BLMs 10 and 11 (optional).
- Provide small Post-Its™ or linking cubes (optional).
- Before beginning the unit, ask students to collect two pieces of data:
  (i) the number of persons currently living in student's home, and
  (ii) the number of doors in the student's home.

The expression "persons in the home" has been used, rather than "family", because the latter may have various interpretations. You may wish to discuss this with students first and come to some consensus as to the meaning of "family".

Students may ask what constitutes a door (e.g. room door, cabinet door, oven door, `fridge door, ...). If this question is raised at this time, try to reach a consensus, and discuss the importance of deciding in advance on the exact interpretation of the terms used in a survey question. If the question does not occur at this point, defer this discussion until the first day of the unit.

- Make copies of BLM 1 and BLM 2 for each student or pair or small group of students.
- Make a copy of BLM 1 for the overhead.
- Make copies of BLM 3 for all students (optional).

You may also wish to examine BLM 10 and BLM 11 which illustrate several types of graphs and identify their components. These pages are intended to help you in your planning. You may choose to duplicate these for students at this time or later in the week.

Activity:

INTRODUCTION

Using the data collected on the number of persons in each home, model a bar graph for the whole class on chart paper or with the overhead projector.

One way to "collect" the data is to present each student with a small Post-It™.

Comments in italics are explanatory, and need not be conveyed to the students.

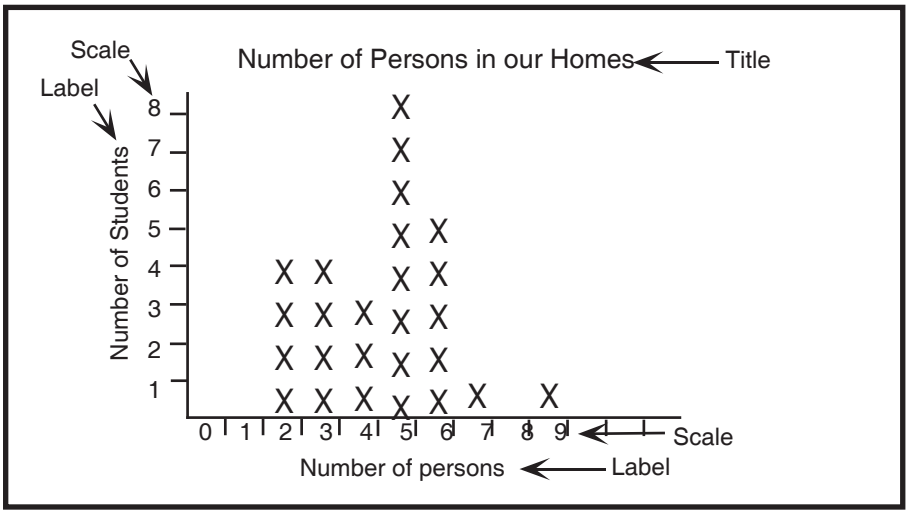
Activity 1: Interpreting Incomplete Bar

Then each child can place the Post-It™ in the correct column of the graph on chart paper. Alternatively, give each student a linking cube and have them build towers to illustrate the bars on the graph.

Make a list of components of a bar graph:

- title for the graph
- labels for the vertical and horizontal axes
- scale for each axis

Discuss the purpose of each component (i.e., ask children what they think is the reason for each component), and its position on the graph.



Ask questions that involve interpretation of the data on the graph:

- What is the most common number of people in our homes? (or "in our families"?)
- Why are there no students in our class with just one person in his/her household?
- How many students of our class are recorded in the graph? (Students should realize that they need only count the total number of Xs or squares or whatever unit you may have used to model the graph.)

Present the students with some questions such as those above as well as those that follow and ask them to decide which of the questions can be answered from the graph, and why.

For example, questions that cannot be answered are:

- How many in our class are girls?
- How many in our class have grandparents living with them?
- What size home do most of us live in?

Next, request data on the number of doors in each home. Even if the meaning of 'door' was discussed prior to data collection, you may find that new meanings emerged during the counting. If the definition of 'door' was not resolved, use this opportunity to discuss the importance of clarity in posing a survey question. Ask students to re-count the doors for next day's class. (See "Family Activities" below.)

Activity 1: Interpreting Incomplete Bar

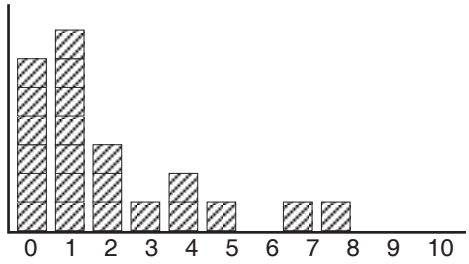
Interpreting Incomplete Graphs

Using the acetate copy of BLM 1 on the overhead projector, discuss with the class what information is missing (i.e. labels on the axes, titles). Ask students how important this is, and why.

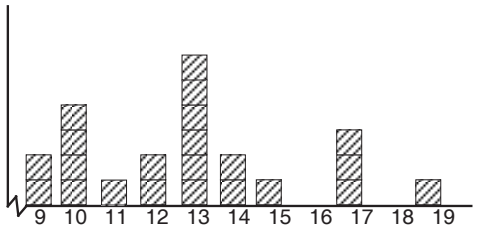
Distribute copies of BLM 1 and BLM 2 to the students in their working pairs or groups. Explain to the students that they are to match a description from BLM 2 to one of the incomplete graphs on BLM 1, and give their reasons. Stress that although these graphs are the results of grade four surveys, they should not think of it as their own grade four class, but hypothetical ones.

Excerpt from BLM 1

Graph 2



Graph 6



Possible student responses to related questions on BLM 2:

1 (a) Graph 2 could show the number of pets each student has because most students would have one or zero pets.

3. Graph 6 could not be the ages of grandmothers because grade fours are 9 or 10 and grandmothers could not be the same age.

Some students may ask if the same description can be matched to more than one graph, or if all descriptions and graphs must be matched before the task is over. This decision should be the students'. If they have valid reasons for any match, they should include it. Point out to the students that they will be expected to justify their matches to their classmates who might disagree with them.

Notes

Problem Solving

Communication

Notes

Communication

Allow sufficient time for most students to complete the task (about 15 minutes), leaving time to discuss the results. Answers may vary, but students should be expected to support their own opinions with valid reasons even in the face of disagreement. The class may not be willing to come to complete agreement about all the matches suggested, but should be willing to accept reasonable divergent views (e.g. "Yes, Graphs 2 and 5 could both represent the number of pets because children on a farm might have 9 pets")

Wrap-Up

If time permits, refer again to BLM 1 and ask students what other sets of data could be represented by each graph. Give them a few minutes to record their ideas (in a journal or on BLM 3), and then discuss briefly. If there is not enough class time, this could be assigned as a Family Activity. (See below.)

If students need help getting started, ask such questions as "Why couldn't Graph 1 represent the arm lengths of a grade four class? Why couldn't graph 2?" or "Why do you not think that Graph 4 shows the number of pets for each family? What could this graph represent? Numbers of what? Could it be a measurement of some kind?" [Students might consider it could be the number of doors per household, especially if these data have been discussed already.]

Cross-Curricular Activities

1. Construct a graph of a hypothetical grade four class during pioneer times (e.g., heights of students/ages of grandmothers/distance from home to school/number of people living in the home/number of pets. Have students suggest other topics.). What will be different from graphs of grade four students to-day? What will be the same?

   Since most pioneer schools were one-room schools, there would be very few grade four students in any school. However, the ages of the students should be reasonably close to today's data. Will the ages of mothers or grandmothers be similar to today's? Why or why not?

   You may wish to assign this to students in groups as a research problem. It is a good idea to check with the school or local librarian to see what appropriate resources are available. Students could also discuss the problem with parents and/or grandparents.

2. Explore the uses of graphs in other subjects, for example, as temperature or precipitation graphs in Social Studies.

Family Activities

1. Have students take home a copy of BLM 1 to discuss with other family members. They are to make a list of possible collections of data that might be represented by each graph. Stress that it is not important to have a topic for every graph, but that they should have very good reasons for choosing the topics they did.

Activity 1: Interpreting Incomplete Bar

2. If the definition of 'door' in the "How many doors?" question necessitates a re-count, ask students to collect this data. If there is still a question as to what a door is, tell them to decide for themselves, with the help of family members, and to be sure they have valid reasons for their choices.

Other Resources

For further details, see annotated Other Resources list on page 48, numbered as below.

1. "Making Sense of Data: Addenda Series, Grades K - 6", by Mary Lindquist et al.

11. " Our Heritage: Learning Data - Management Skills Meaningfully," by Yvonne M. Pothier and Christine M. Nickerson

Activity 2: Predicting a Bar

Focus of Activity

- Prediction of the shape and range of a bar graph given a description of data, but not actual data

What to Assess

- Knowledge of the components of a bar graph
- Indication of the awareness of the value of each component of a bar graph
- Interpretation of a bar graph
- Creation of a bar graph, given the necessary data
- Ability to justify their own opinions with coherent reasons
- Ability to distinguish between likely and unlikely

Preparation

- Make a copy of BLM 4 for each group.

Activity:

INTRODUCTION

Using the data on number of doors, construct a bar graph according to instructions from the students. ("What should I do first in making this graph? What comes next? ... Is anything missing?") There is no particular order in which the components of a bar graph should be included. Students should, however, be aware that some items should precede other items. For example, the scales should be written on the axes before the data can be entered on the graph.

PREDICTING GRAPHS

Refer to BLM 1 and ask ,"If we were going to make graphs similar to these for our classroom community, what data might we want to record?" Ask students what they would be interested in knowing about their classroom community. Encourage students to suggest sets of data different from those on BLM 2. Different types of data should be listed. For example,

Measurement: height in centimetres; length of forearm; palm width;
Preferences: favourite super hero; favourite subject; favorite ice cream flavour
Numerical: number of schools one has attended; number of books read during the past year

Record all possibilities on chart paper.

Communication

Select a specific type of data (e.g. palm width of students in the class). Ask students what the graph of these data might look like, and have each student sketch a possible graph. Discuss the need for estimation in deciding the range of the data. Remind them they can use the incomplete graphs on BLM 1 as models of what their graphs should look like. Allow a few minutes.

Have students form pairs. Each student is to explain to his/her classmate why the graph should take a certain form. The pairs should then decide which reasons and ideas they wish to present to the class, and be prepared to justify their opinions.

Activity 2: Predicting a Bar

Then ask selected student pairs to present their ideas to the class. Listen for correct use of mathematical language, and for logical reasoning. You may choose to discuss these aspects of a response with the class. ("That was well put, Alyssa. Class, what was there about Alyssa's argument that made it a good one?")

Discussion of Likelihood

Questions of likelihood of given events may help students with the shape and form of a predicted graph. Refer to BLM 1, Graph 1, and ask such questions as "How likely is it that a grade four student from the sample has a grandmother over 70? Is it very likely or very unlikely?"

Students should be pressed to say no more than , "Yes, it is likely/unlikely" or "It is very likely/unlikely" or "It is more/less likely," with no numerical probability being discussed. Such analysis will be easier for some graphs than for others. If all the data are clustered in a small range, it could be very difficult to judge.

Refer to the graph showing the number of people in the home constructed for your class. Ask each student to write a statement beginning with "It is very likely, ..." or "It is very unlikely..." about the graph. Have students present their statements with reasons.

Wrap-Up

Students in groups select one of the topics listed as interesting data about the classroom community. Each group should have a different topic. Different types of topics should be selected as well (i.e. measurement, numerical, preferences). If time permits, have students sketch a possible graph for the data by estimating and predicting what the data from the class will look like, using BLM 4. If there is not sufficient time, assign this as a Family Activity since it will be needed for Activity 3.

Extensions in Mathematics

1. Extend the discussion of likelihood with such questions as:
   a) Is a student from this class more likely to have a grandmother over 60 than to have a grandmother under 40?
   b) Which is more likely: that a student from this class has more than 2 pets or that a student has fewer than 2 pets?
   c) Which is more likely: that a student in this class has 1 or 2 pets or that a student has an arm length of over 40 cm?

   Select numbers from the actual data collected for the class so that one of the groups being compared is greater than the other. That is, for (a) above, the graph would have more grandmothers over 60 than under 40.

Cross-Curricular Activities

1. Have students predict and sketch a temperature graph for one week. During that week, collect the actual data and illustrate on the same graph. Students should compare the actual data with their predicted graphs, and make conjectures as to why some predictions are worse than others.

Assessment

Communication

For more on this idea, see "Investigations in Probability" for Grade 4 in the **Invitations to Mathematics** series.

Problem Solving

Family Activities

1. Students should discuss their predicted graph (See "Wrap-up" above) with family members so that they become very confident in both their predictions and their reasons.

2. Start a collection of graphs of different types from newspapers and magazines. This could be the beginning of a bulletin board display, or students can sort and classify the graphs by subject area (e.g., science, social studies) or by type of graph (e.g., circle graph, bar graph).

Other Resources

For further details, see annotated Other Resources list on page 48, numbered as below.

1. "Making Sense of Data: Addenda Series, Grades K - 6", by Mary Lindquist

5. "Ideas", by Helene Silverman

Activity 3: Collecting and Displaying Data

Focus of Activity

- Collecting of data and displaying it on a bar graph

What to Assess

- Prediction of the shape and range of a bar graph given a topic
- Ability to describe the nature of the data clearly and develop a data collection sheet/questionnaire
- Creation of a bar graph and correct use of the components of a bar graph
- Ability to explain reasoning, orally or in writing
- Collaboration with other members of the group

Preparation

- Make a copy of BLM 5 for each group.
- Make a copy of BLM 6 for each pair/group (optional).
- Provide measurement devices (metre tapes, scales) as needed.

Activity:

Communication

INTRODUCTION

Students should present their predicted graphs from Activity 2 (BLM 4) to the class and justify their predictions. If students wish to adjust their predicted graphs in light of questions from the class, they should be encouraged to do so.

DATA COLLECTION SHEETS

Discuss with students the importance of a good Data Collection Sheet/ Questionnaire. Stress that the nature of the data collected will depend on the questions/instructions given to the people from whom the data is collected.

Distribute copies of BLM 5 (Data Collection Sheet/Questionnaire). Give students a few minutes to discuss the questions in their groups. Allow a few minutes for them to write responses before a class discussion.

Excerpt from BLM 5

Title: Bicycle Colours

Check the colour of your bicycle:

_____ red	_____ white
_____ blue	_____ black
_____ green	_____ yellow

Some grade four students were planning to collect data to make graphs. Their Data Collections Sheet is shown above.

1. Do you think the Data Collection Sheet for bicycle colours will give them the information they want? Explain your reasons.

Activity 3: Collecting and Displaying

Possible student response to Question 1:

> Some bicycles have more than one colour. This Collection Sheet won't show that. What if somebody has a purple bike? What if somebody has no bike? This Data Collection Sheet is not a good one

Discussion

Try to elicit the facts that the first data collection sheet does not take into account students who may not have bicycles, or those whose bicycles are colours not listed. An "Other" category would count both those groups, but those without bicycles would be mixed in with those whose bicycle colours were not listed. Perhaps the sheet could begin with a question: "Do you have a bicycle?" Students may suggest other ways of improving the collection sheet.

The Questionnaire on smiles will give the data wanted, but in an unorganized fashion. Students may also suggest that most people don't know the width of their smiles. Reasonable suggestions for improvement should be accepted.

Problem Solving

Data Collection

Each group should prepare a Data Collection Sheet/Questionnaire for the class, designed to collect the data for which the group has already drawn a predicted graph (from Activity 2).

To keep the data collection process organized, you may choose to have the Data Collection Sheets passed from group to group, rather than have the students move about the classroom.

As each group receives back their completed collection sheet, they can begin constructing their graph, using the components of a bar graph as discussed in earlier activities.

Assessment

Writing About Their Work

Have students write about their predicted graphs and their actual graphs, indicating similarities and differences, and explaining why any differences occurred.

An oral discussion before the students start writing will help them focus on the issues and review mathematical language which should be included in their written responses. Some terms to include are: bar graph, data, predict, collect, range, label, record, result.

Communication

Presenting Their Work

If time permits, you may wish to have each group present its graph to the class, explain their choice of data collection technique, and compare their results with their predicted graphs.

Activity 3: Collecting and Displaying Data

Extensions in Mathematics

1. Have each group collect similar data from another grade after predicting the changes that might occur in the data, and sketching another predicted graph.

Problem Solving

2. Introduce double bar graphs (BLM 6). Have students construct a double bar graph to illustrate
   (a) their predicted and actual class data for their first data collection task.
   (b) the actual data from their own and another grade.

Cross-Curricular Activities

1. Discuss the possibility of obtaining similar data from other sources - for example, other countries with which students are familiar; other generations (e.g., pioneers, parents).

2. Collect and display data from the community about
   - people (age, occupation, country of origin)
   - occupations
   - transportation available
   - type of buildings

Communication

3. Create a personal biography to describe your "individual data" (e.g., preferences, measurements). Use your data, words, and/or pictures to create a book, poster, collage, song, poem or video about you.

4. Create a class, school, or community biography. Use collected data, words, and/or pictures about your class, school, or community to create a book, poster, collage, song, poem or video that portrays some important aspect of the subject group.

5. Based on collected data at the school or community level, identify an issue of concern (e.g., wasted paper, a needed stop sign) and write a letter to the appropriate person(s) voicing concerns and detailing suggestions for solving the problem.

Family Activities

1. Create a family biography using data collected in the home. Use words and/or pictures to create a book, poster, collage, song, poem or video about the family.

Other Resources

For further details see annotated Other Resources list on page 48, numbered as below.

4. "Ideas" by Calvin and Rosemary Irons

5. "Ideas" by Helene Silverman

Activity 4: Types of Graphs

Focus of Activity

- Reading, comparing, and constructing bar, circle, and picto-graphs

What to Assess

- Ability to interpret bar, circle, and picto-graphs
- Knowledge of components for each type of graph
- Construction of picto-graph or circle graph
- Collaboration with others

Preparation

- Make a copy of BLM 7 for each pair of students.
- Make a copy of BLM 8 for each pair of students.
- Make copies of BLMs 10 and 11 for each student (optional).
- Make copies of BLM 12 as needed (optional).

You may wish to illustrate the circle graph using students (see Extensions, next page) before, rather than after, presenting the circle graph on paper.

Activity:

INTRODUCTION

Refer to completed graphs from previous activities and ask students what all graphs have in common. If the term "Bar Graph" has not been used previously, introduce it now. Tell students that there are other types of graphs as well, and they will be exploring two more types in today's activity.

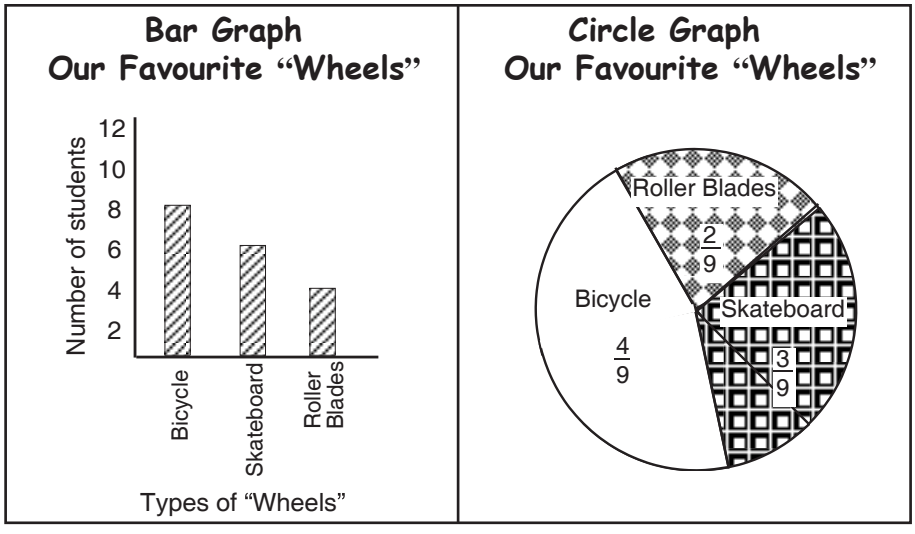
COMPARING GRAPHS

Distribute copies of BLM 7, and have children examine the three graphs and read the instructions and questions.

Excerpt from BLM 7:

(a) How are the graphs alike?

(b) How are they different?



Activity 4: Types of Graphs

You may wish at this stage to discuss, with the students, a few similarities and differences among the three graphs. This will give students a starting point when they begin their written descriptions.

If the class has been making a collection of different types of graphs from magazines and newspapers (See Activity 2, Family Activities) these could be available, as well, for students to compare.

Communication

Students should then spend several minutes writing, in pairs, their own descriptions of the three graphs. Have each pair share their descriptions with another pair. Circulate during the discussion to assess children's use of mathematical language, willingness to share reading time with others, how carefully students listen to others, how well they support their conclusions, and how accurate their descriptions of the graphs are.

Possible student responses to Questions on BLM 7:

Both graphs show the favourite wheels of Ms. Marshman's class. You can tell that more people like bicycles than skateboards or roller blades

The Graphs are different because the circle graph shows fractions that like each type of wheels but the bar graph tells how many kids like each one.

Assessment

Discussion

Summarize with the whole group the similarities and differences among the three types of graphs. These could be recorded on chart paper and posted for future reference.

Distribute copies of BLM 8. Examine the questions in the box. Stress with students that they need not answer these questions, but they will need to refer to them in their responses for the problems below the box. Give students a few minutes to respond. Discuss the results. Students should realize , for example, that the picto-graph makes it easy to count the total number in the sample; the circle graph makes it easy to compare parts of the sample to each other and to the whole.

Graphing

Using the data collected for Activity 3, students should, in the same pairs, illustrate the data using either a picto-graph or a circle graph. Provide copies of the circles on BLM 12 to help students construct circle graphs. The circle templates are divided into sixths, eighths, twelfths, and sixteenths respectively. The BLM may be distributed whole to each pair or you may wish to cut the templates apart so that pairs can select only the template they need.

Problem Solving

Alternatively, if time permits, students could (in the same or other pairs) select another topic, predict the picto- or circle graph, collect the data and sketch the graph.

Activity 4: Types of Graphs

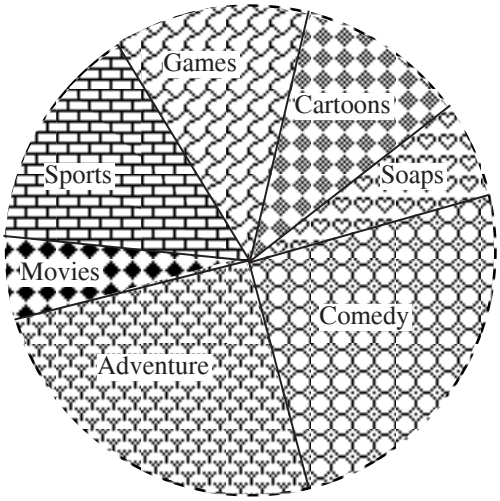
It is not intended that circle graphs be constructed exactly. Angles need not be measured with protractors. Students instead should think about each category as being a certain fraction of the whole, and estimate the size of the angle of each segment. If fraction pieces (of a circle) are available, they could be used to help students.

Sample student graph:

FAVOURITE TYPE OF TV PROGRAM

Cartoons: 4
Soaps: 2
Comedy: 8
News: 0
Adventure: 8
Movies: 2
Sports: 4
Games: 4

TOTAL: 32



Students can use the fourth template on BLM 12 to construct this graph.

Communication

Writing about your work

When the graph is complete, students should write 2 or 3 questions that can be answered from their graph. Answers can be written on the back of the paper. Have them also identify related questions that cannot be answered from the graph. You may wish to discuss this with students first to avoid totally inappropriate questions (e.g., questions completely off topic such as "How many hours does each person watch TV?").

A type of statement students frequently make, using a graph such as the above is, "Most students like comedy." The correct statement is, "More students like comedy than any of the other choices." Students should be helped to see that 8 students (the ones preferring comedy) is not "most" students.

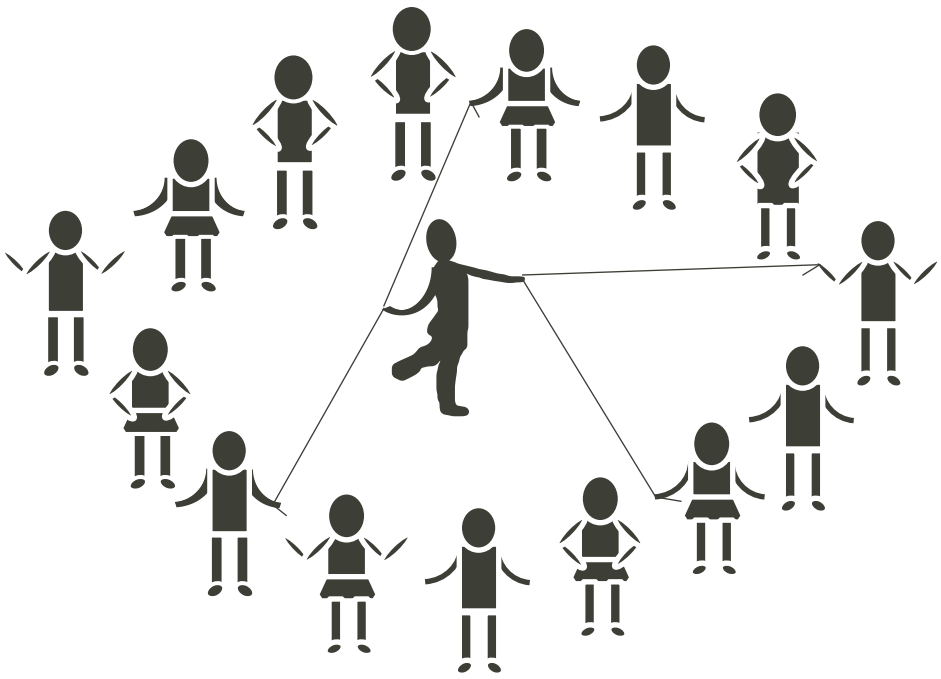
Wrap-Up

Tell students that they will be presenting their graphs and questions to the class tomorrow and if they have a few minutes now, they could start planning their presentations. Tell them they will have a few minutes next day but the presentation must be no longer than 3 or 4 minutes.

Activity 4: Types of Graphs

Extensions in Mathematics

1. To illustrate a circle graph, have students stand in a circle. Then have them sort themselves by some criterion (e.g., shoe colour). Use strings to divide the circle into sectors. The teacher, in the centre, holds the mid-points of the strings.



Discuss the resulting graph with students using fraction terms (e.g., "Almost half the class is wearing sneakers."). Ask why it is important that students space themselves evenly around the circle. Have students in one section gather closely together and see how the graph appears different.

2. Distribute copies of BLMs 10 and 11. Students could keep these in their books for future reference. Discuss the new types of graphs illustrated and the strengths and weaknesses of each. Suggest when each type of graph might be appropriate.

Cross-Curricular Activities

1. Have students collect and display data from a scientific investigation. For example:

   - How far will a toy car run from the base of a ramp after rolling down the ramp? How does the distance change as the slope of the ramp changes?

   - What kind of, and how many, birds can be seen at a school bird feeder at each hour of the day?

- How fast does a candle burn? (*A birthday candle can be marked in centimetres with a permanent marker, and then timed as each segment burns. The data can be displayed on a line graph.)*
- Which paper towel gets the wettest?

Family Activities

1. Construct, with family members, a pictograph illustrating some data collected at home (e.g., types of shoes: with laces, with straps, with velcro fasteners)

Other Resources

For further details, see annotated Other Resources list on page 48, numbered as below.

1. "Making Sense of Data: Addenda Series, Grades K - 6", Mary Lindquist

9. "Ideas", by Sharon L. Young

Activity 5: Presenting Classroom Graphs

Focus of Activity

- Presentation and discussion of graphs

What to Assess

- Clarity of explanations
- Correct use of mathematical language
- Accuracy of graphs
- Appropriateness of the alternative type of graph (i.e., circle or picto-graph)

Preparation

- Make sufficient copies of BLM 9 to use as determined. (See suggestions below).
- Make a copy of BLM 9 on acetate for the overhead projector or on chart paper or the blackboard.

Activity:

INTRODUCTION

Remind the students that they will be presenting their graphs and questions to the class. Spend a few minutes in discussion with the class making a list of things that should be a part of the presentation (e.g., readable graphs; questions that can be answered from the graph; reasons why they chose either a circle or picto-graph for the second graph). Discuss with students how to assess an oral presentation, and tell them they will be asked to do so.

Present BLM 9 (on overhead projector or chart paper) as a type of rating scale. Some of the topics listed will have emerged from the discussion above. Discuss with students the value of each, and add others (as 7 and 8) that they feel are important.

Communication

Examine each topic carefully and ask students what they think would be necessary for a graph to be given a rating of '4' on the BLM 9 scale. How would this differ from a '3' graph? What would a '2' be like? Repeat with other ratings so students have a good idea of how they will be assessed. Record some of the suggestions on the acetate or chart paper so that they are in front of the students during the presentations. (See Solutions & Notes for suggestions.)

There are several ways to utilize the rating scale. A few suggestions are given below.

Suggestion 1:

Distribute copies to each student to assess presentations of all groups except his/her own. Students assess during presentations.

Suggestion 2:

Distribute copies to each group. Assessment is done as a group following the presentations.

Activity 5: Presenting Classroom Graphs

Suggestion 3:

Distribute to individuals or groups but assign only one or two of the topics for each person/group to assess. For example, group A might assess only the quality of the graphs used in the presentations.

Suggestion 4:

Distribute sufficient copies so that each group can assess its own presentation, as well as the others.

Assessment

However the assessment is done, students should know that they will be expected to present and justify their opinions.

Allow students a few minutes to prepare their presentations, now that they know how they will be assessed.

Presentations

As each group makes their presentation, observe their use of mathematical language, sharing of the task with others in the group, accuracy of graphs and collected data.

After each presentation, spend a few minutes on peer assessment. Students should present and justify their opinions. This gives you an opportunity to assess the assessors.

Communication

Answers to the questions above could be given orally, or students could work in groups to write their opinions before presenting them to the class.

As students complete their presentations, post their work on a bulletin board.

Wrap-Up

Ask "What does all the data you collected tell us about our classroom community?" "What do you know about our class that you didn't know last week?" Written responses to these questions could be added to the bulletin board display.

Continue with: "What other things about our class might be interesting to discover?" "Which of these could be shown on a graph? What kind of graph?" These questions could be answered either in written form in journals or orally.

The collection and displaying of data could be continued for a few weeks by making a "graph of the week". After the class has decided on a topic for the graph, post a Data Collection Sheet/Questionnaire devised by a group of students on the bulletin board. Every student is to add his/her information to the sheet during the week. The graphs can then be constructed by small groups of students and discussed at the end of the week. Encourage students to write questions that can be answered from the graph. Encourage the use of different kinds of graphs, if applicable.

Activity 5: Presenting Classroom Graphs

Family Activities

1. Students could create a series of graphs to describe their homes/families. They could write about their reasons for choosing particular sets of data and what they think each graph tells others about their homes/families.

Other Resources

For further details, see annotated Other Resources list on page 48, numbered as below.

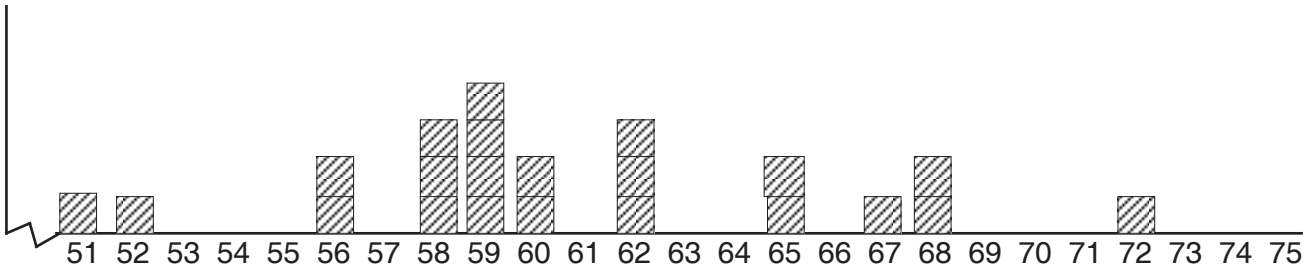
1. "Making Sense of Data: Addenda Series, Grades K-6" by Mary Lindquist.

6. "Gender, Ninja Turtles, and Pizza: Using a Classroom Database for Problem Solving"

Use of Technology

BLM 1: Incomplete Bar Graphs

1.



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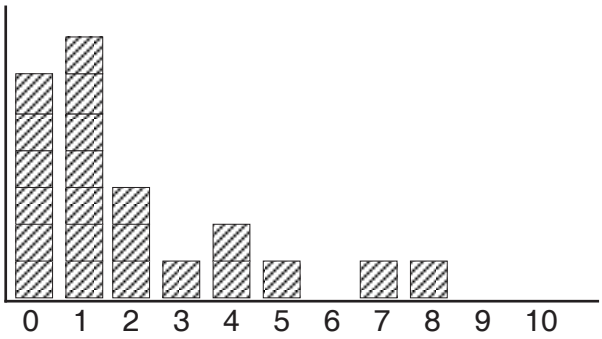

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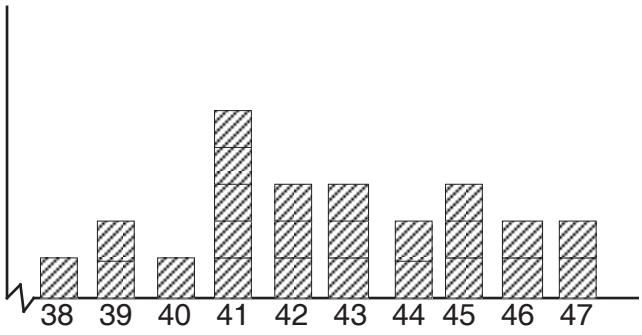

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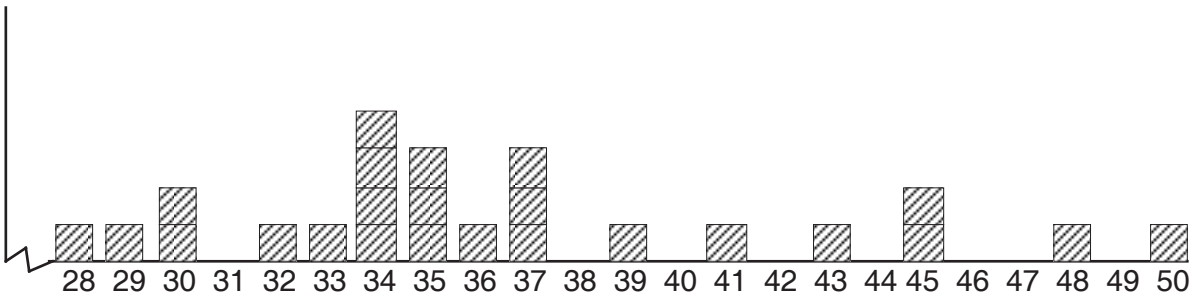
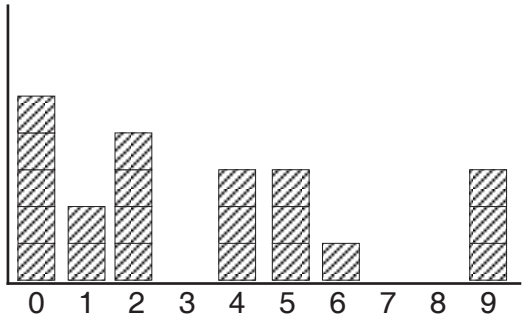
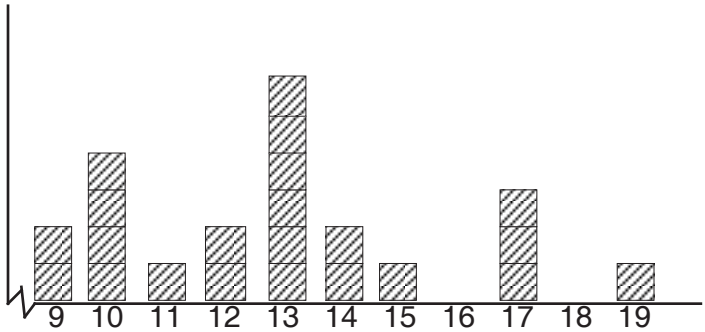


5.



6.



BLM 2: Possible Graph Descriptions

Each of the six given graphs illustrates information about a class of students in grade four.

1. Decide which graph best shows each of the following:

    (a) The number of pets each grade four student has at home.

    (b) The length, in centimetres, of the arm length (shoulder to wrist) of each of the grade four students.

    (c) The number of seconds grade four students can hold their breath.

    (d) The ages of the grade four students' grandmothers.

    (e) The number of times a grade four student can write his/her name neatly in one minute.

    (f) The number of plants in each room in the school.

2. Explain why you think the graph you picked for (d) is the one that shows grandmother's ages.

3. Explain why you think none of the other graphs could be showing grandmothers' ages.

BLM 3: Alternative Graph Descriptions

Examine the graphs on BLM 1 again.

Choose two of the graphs, and tell what else you think each graph could represent.

Give a title for each of the graphs you chose, and explain your reasoning.

Graph #____

Graph #____

BLM 4: Predicted Graphs

1. Tell what topic you chose.

2. Sketch a graph that you think could show this information.

3. Explain why you think the graph might look like your sketch.

BLM 5: Data Collection Sheet/Questionnaire

Title: Bicycle Colours

Check the colour of your bicycle:

_____ **Red**

_____ **Blue**

_____ **Green**

_____ **White**

_____ **Black**

_____ **Yellow**

Some grade four students were planning to collect data to make graphs. Their Data Collection Sheet is shown on the left.

1. Do you think the Data Collection Sheet for bicycle colours will give them the information they want? Explain your reasons.

2. If your answer to Question 1 above was "No", tell how you would improve the Data Collection Sheet.

Title: Size of Your Smile

How wide is your smile in centimeters?

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____ _____ _____

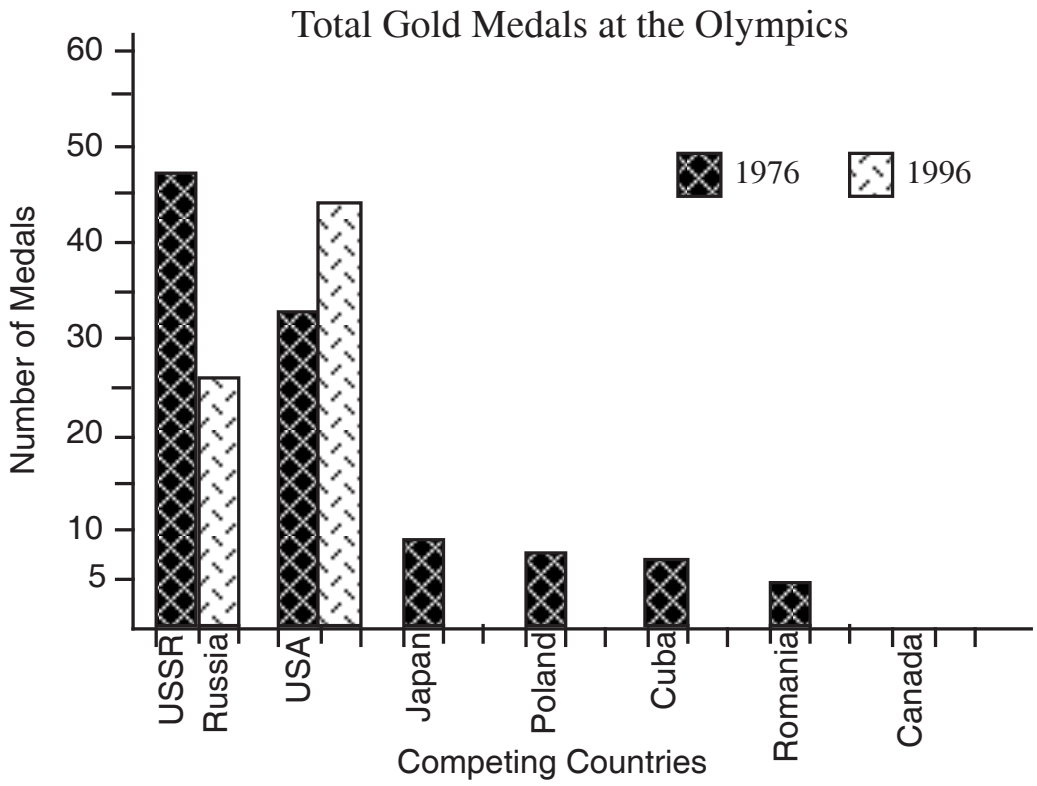
Other grade four students wanted to collect data about smile width. Their questionnaire is shown on the left.

3. Do you think the questionnaire will give the students the information they want? How could you improve the questionnaire?

BLM 6: Double Bar Graphs

Sometimes there are two sets of data that you want to compare. A Double Bar Graph is often used for this.

The graph below, when complete, will show the number of gold medals won by several countries in 1976 and twenty years later in 1996 at the Summer Olympics.



1. How is a double bar graph like a single bar graph? How is it different?

2. Why are there two different names for the first pair of bars?

3. The data for 1996 are given below. Complete the double bar graph to include these data.

Gold Medals won in 1996:

Russia	26	USA	44
Japan	3	Poland	7
Cuba	9	Romania	4
Canada	3		

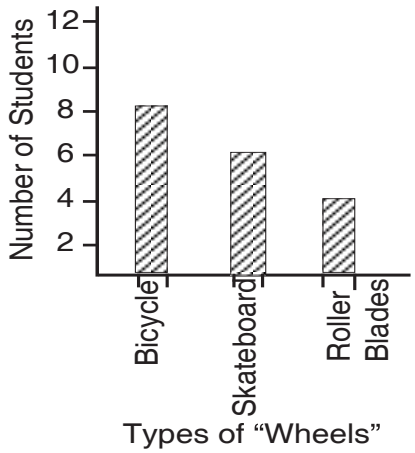
BLM 7: Different Types of Graphs

Ms. Marshman's Grade Four Class collected data on their favourite "wheels". Each student was asked, "Of the different types of "wheels" or vehicles you have, which is your favourite?"

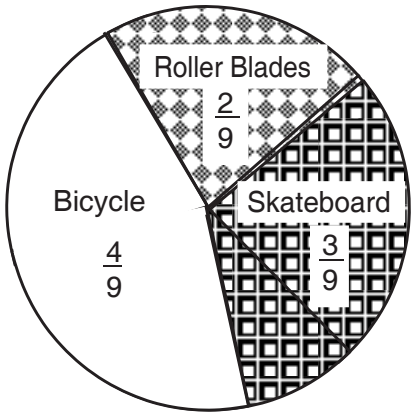
The information collected is shown on three different graphs.

How are the three graphs alike?

(a) Bar Graph

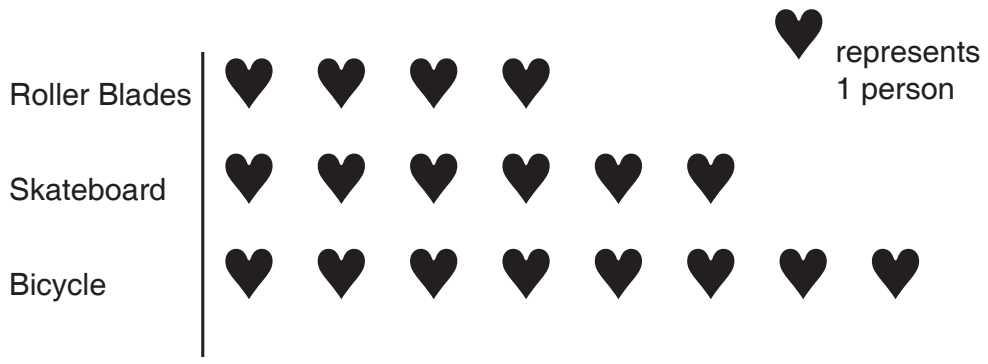


(b) Circle Graph



(c) Picto-graph

Favourite "Wheels"



How are they different?

BLM 8: Circle, Bar and Picto Graphs

Questions About the "Favourite Wheel" Graphs

A. How many students were in the class? _____

B. What was the favourite type of wheels? _____

C. Complete the following: Nearly half the class preferred _____

_____

D. Complete the following: Twice as many people preferred _____

_____ as preferred _____

E. What fraction of the class preferred skateboards? _____

1. Examine the questions in the box above. Tell whether or not any question can be answered more easily by a particular type of graph. Tell why that graph made it easier.

2. On the back of this sheet:
   (a) Write one question that can be answered most easily by the circle graph.

   (b) Write one question that can be answered most easily by the picto-graph.

BLM 9: Assessing Group Presentations

A sliding scale for assessing presentations is given below:

1. Quality of graphs: (0 - poor; 4 - very good)

0 1 2 3 4

2. Quality of questions to be answered from the graph: (0 - poor; 4 - very good)

0 1 2 3 4

3. Quality of voice during presentation: (0 - poor; 4 - very good)

0 1 2 3 4

4. Use of mathematical language: (0 - poor; 4 - very good)

0 1 2 3 4

5. Cooperation among group members: (0 - poor; 4 - very good)

0 1 2 3 4

6. Organization of presentation: (0 - poor; 4 - very good)

0 1 2 3 4

7.

0 1 2 3 4

8.

0 1 2 3 4

Other considerations: (Answer on the back of this sheet if you need more room.)

9. What was best about the presentation?

10. What could have been improved? How?

11. What did you learn from the presentation?

BLM 10: Sample Graphs - 1

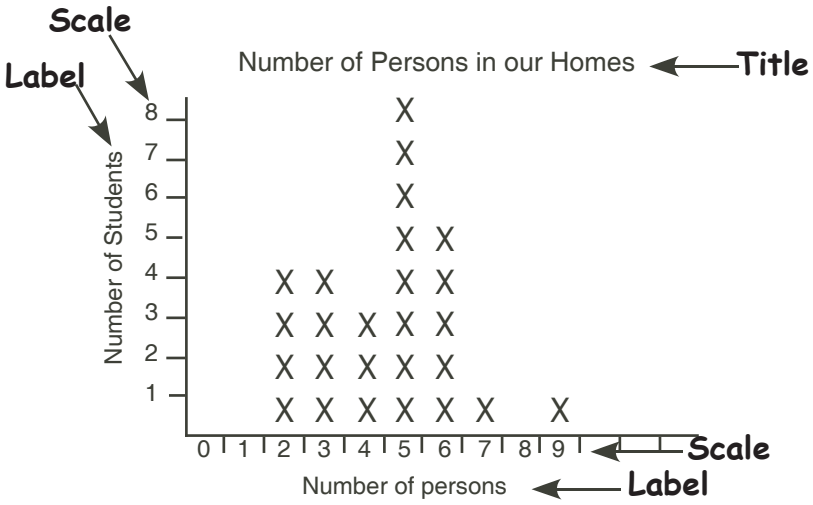
Several types of graphs, with their components identified, are given here.

BAR GRAPH

Bar graphs may be constructed with bars or with repeating elements as shown.

Each axis should have a label or title. Axes should also have scales (see first graph) or lists of categories (see second graph).

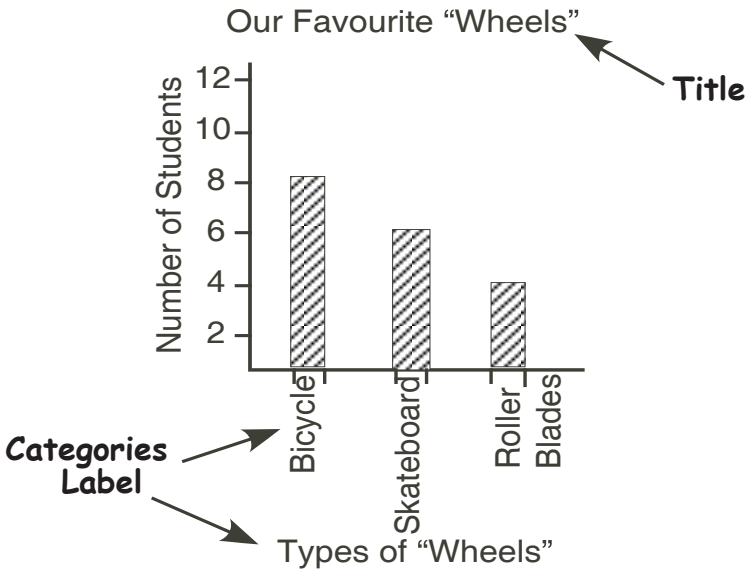
Each graph has a title.



Usually there are spaces between the bars.

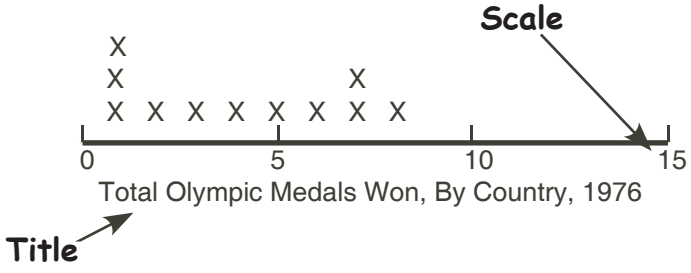
Bar graphs are used to compare items with each other.

A double bar graph (see BLM 6) is used to compare two sets of data.



LINE PLOT

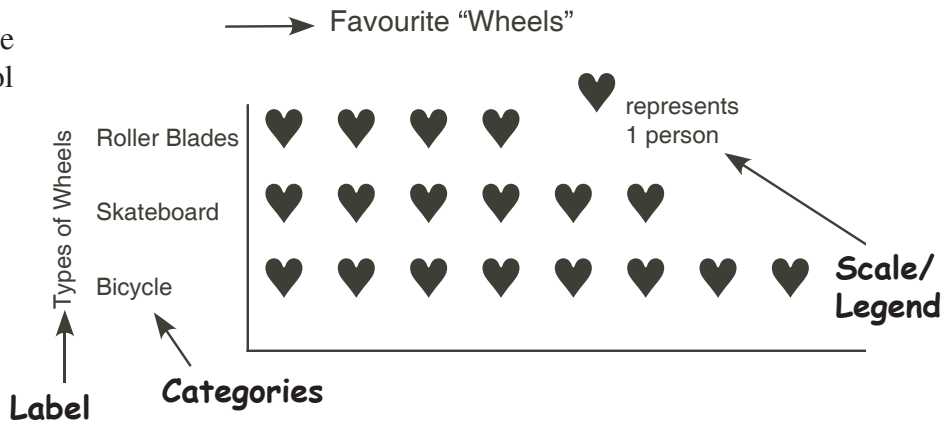
The Line Plot is a very simple type of graph and is useful when comparing bits of data.
However, the line plot does not indicate who (or, in the example shown, what country) each piece of data represents.



BLM 11: Sample Graphs - 2

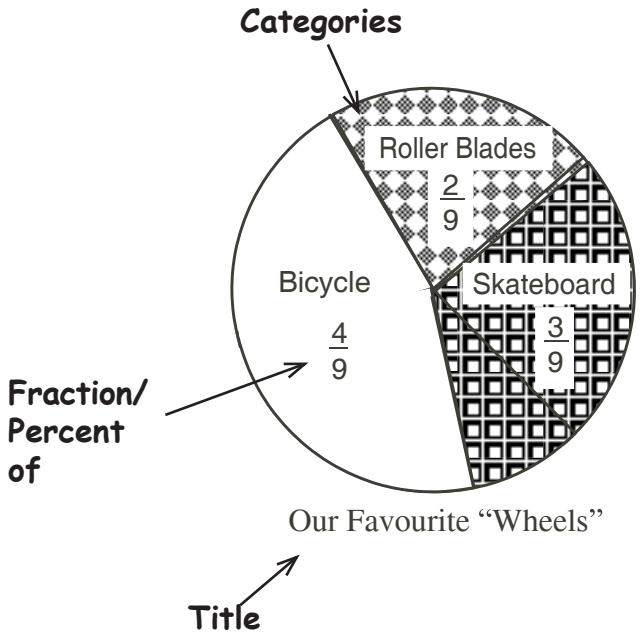
PICTO-GRAPH

A picto-graph is very like a bar graph, The major difference is the legend. One symbol may represent one person, as here, or any number of people. It is therefore useful when graphing a set of data with large numbers (e.g., populations of countries)



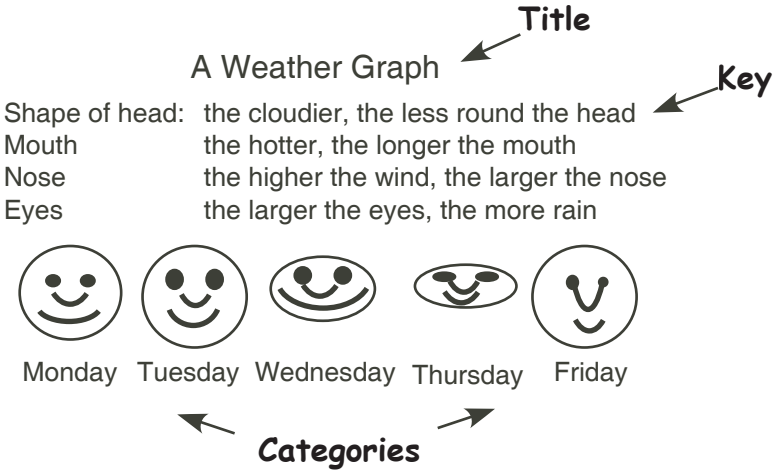
CIRCLE GRAPH

A circle graph is used to illustrate parts of a whole, in this case a whole class. Circle graphs are not appropriate for things like daily temperature for which there is no "whole".



"Glyph" GRAPH

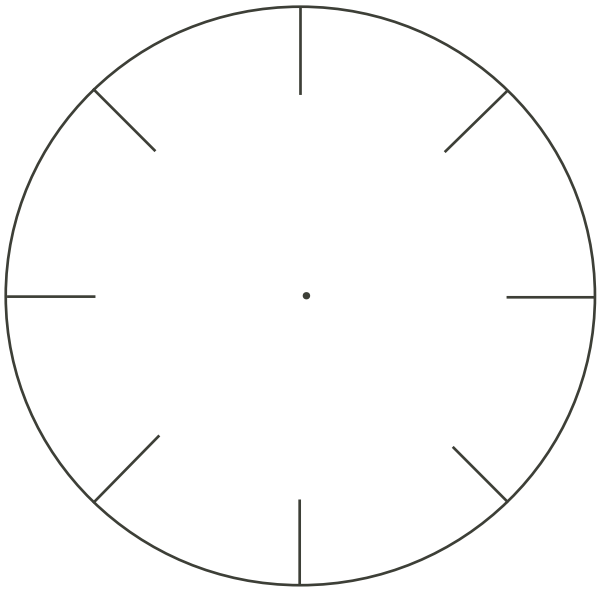
While not a traditional graph, graphs such as the one shown can be used to present several types of data, while giving students the opportunity to analyse data, which they must do in order to devise a workable "key".



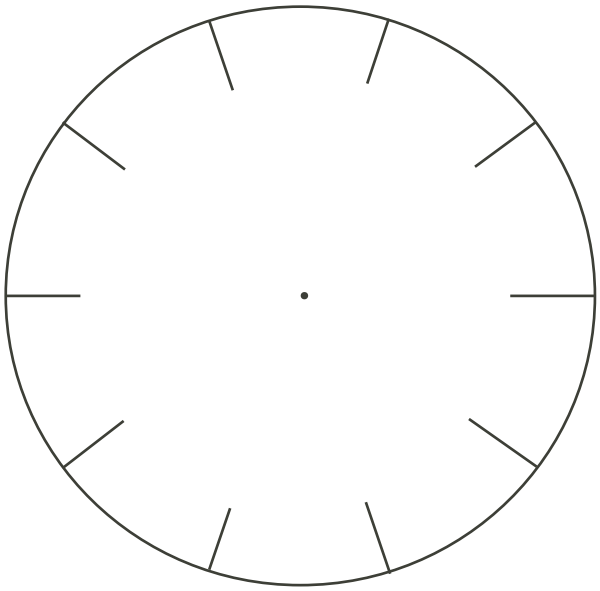
BLM 12: Circle Graph Templates

Each circle below is marked off into equal segments. Choose the template that is best for your graph.

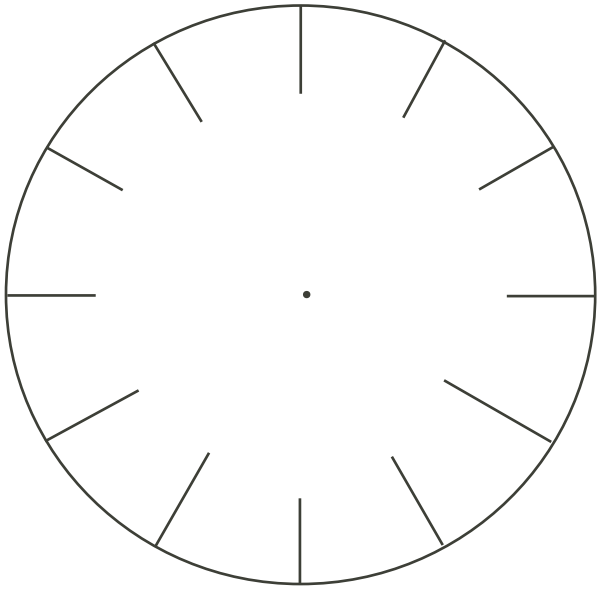
1.



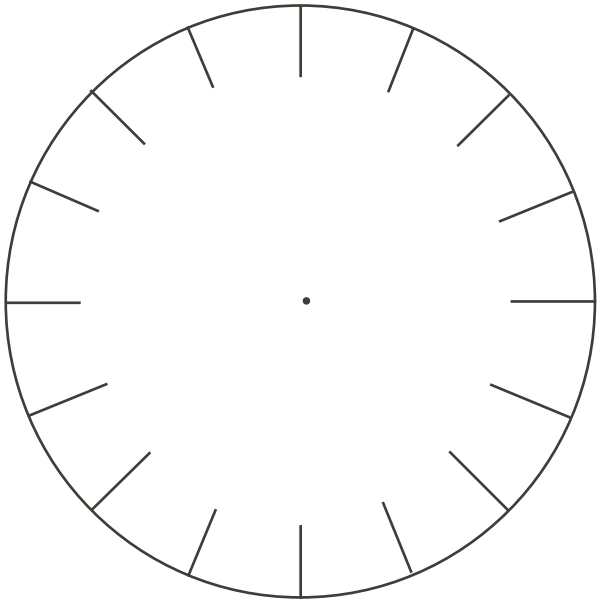
2.



3.



4.



Solutions & Notes

Activity 1: Interpreting Incomplete Bar Graphs

Probable matches of descriptions (BLM 2) with incomplete graphs (BLM 1):

Graph 1: ages of grandmothers (d)
Graph 2: number of pets (a)
Graph 3: arm length in centimetres (shoulder to wrist) (b)
Graph 4: holding breath (c)
Graph 5: number of plants (f)
Graph 6: name writing (e)

If students can justify other matches they should be encouraged to do so.

Activity 2: Predicting a Bar Graph

What to look for in the sketch of the predicted graph:

- Title on the graph
- Label on each axis
- Reasonable scale or list of categories on each axis
- Reasonable range of data (e.g., a graph of number of schools attended might range from 1 to 5; it is unlikely that it would need to extend to 10.)

Student's justification of the sketch should concentrate on the scales/categories and range, and should attempt to explain why these were selected as they were.

Activity 3: Collecting and Displaying Data

Possible responses for BLM 5:

1. Response should indicate that students who do not have bicycles will not be included in the survey. Students owning bicycles of colours not listed will not be included.

2. Begin with the question ,"Do you own a bicycle?" Include other colours; alternatively, list the most common colours and include the category "Other" for bicycles of other colours.
   Asking "What colour is your bicycle?" will collect the proper data, but in a disorganized fashion.

3. This question will give the proper data but in an unorganized fashion. Provide a tape measure so students can check the widths of their smiles.
   A range of expected widths could be given and students asked to check the appropriate one — e.g., 5 cm, 6 cm, 7cm. etc.
   Alternatively, have students check a range of measures:
   e.g., 5 cm to 9 cm; 10 cm to 14 cm; etc. , with measures rounded to the nearest centimetre;
   or 9.5 cm to 14.4 cm; 14.5 cm to 19.4 cm; etc., with measures rounded to the nearest half centimetre.

Solutions & Notes

BLM 6: The actual data already on the graph are:

USSR	47	USA	34
Japan	9	Poland	8
Cuba	6	Romania	4
Canada	0		

These countries, excepting Canada, were among the top countries in 1976 by number of gold medals won.

Ask students which countries *appear* to have done better in 1996 than in 1976. Ask what other information is needed before a true comparison is possible. In 1976 a total of 216 medals were awarded; in 1996, 668 were awarded. Students should be able to see that the number of medals in 1996 was about 3 times the number of medals in 1976. Therefore, if a country was to do better in 1996, the number of medals should be at least 3 times as great as for 1976.

The total number of all medals won by each of these countries is given below:

Country	Total Medals, 1976	Total Medals, 1996
USSR/Russia	125	63
USA	34	44
Japan	25	14
Poland	25	17
Cuba	13	25
Romania	27	20
Canada	11	22

These data could be the basis for another double bar graph.

Students could be asked to suggest reasons why so many more medals were awarded in 1996 than in 1976. What events have been added? Have any been deleted? Why?

Further research topics: What events were included in the first “modern” Olympics in 1896? What events were included in the Greek Games held around 700 B.C.?

Activity 4: Types of Graphs

Similarities and differences identified by the students for the three graphs may include the following:

	Bar Graph	Circle Graph	Picto-graph
Has a title	X	X	X
Has two axes	X		X
Has a title for each axes	X		
Has scale/categories on both axes	X		
Shows each piece of data separately			X[1]
Has no axes		X	
Show fractions of the whole		X	
Show how many people there were althogether	X		X
It is easy to see the category with the most people	X	X	X
Has a little logo to tell how many people		X	

[1]*Since the symbol represents one person, this is true for this picto-graph. Students may be familiar with picto-graphs in which the symbol represents more than one piece of data. For such a graph each piece of data is not shown separately.*

Students may suggest other characteristics, such as "I think it is easier to make a bar graph than a circle graph." or "Picto-graphs are easiest to read because it's easy to count how many people are involved." Students should be expected to justify their analyses by references to the appropriate graphs.

Responses to questions on BLM 7 may include:

1. There were 18 students; this is easiest in the picto-graph because all you have to do is count the hearts.
2. The favourite was the bicycle; all three graphs show this equally well.
3. Nearly half the class preferred bicycles; the circle graph is probably the easiest for this.
4. Twice as many people preferred bicycles as preferred roller blades; this is probably seen most easily on the circle graph, since the fractions of the class are given (2/9 and 4/9). However, it is easy to compare the lengths of the "bars" in both the other two graphs to answer this.
5. One-third or three-ninths of the class preferred skateboards. The circle graph indicates this directly. The other two graphs need a count and then a comparison of figures.

A question that might be more easily answered from the circle graph could be:

"Approximately what fraction of the class preferred skateboards?"

A question that might be more easily answered from the picto-graph is:

"How many of the class preferred skateboards?"

Solutions & Notes

Some students may argue that the picto-graph does not , for example, show how many preferred skateboards, only how many of those who had skateboards preferred them. This is quite correct. The question would be better worded "How many of those surveyed preferred skateboards if they had them?" Discuss the need for accurate language or the need to have a common understanding of the language that is used. In the example above, as long as everyone agrees to word the question "How many preferred...?" to mean "How many of those who have skateboards preferred ...?" then this language is permissible.

Activity 5: Assessing Group Representations

Possible interpretations of questions on BLM 9

1. Graphs should be neat and easy to read.

2. The questions can (or cannot) be answered from the graph.

   The questions are clear.
   The questions make us think.

3. The speakers should speak clearly.

   It should be easy to hear the speakers.
   The speakers should not just read notes.

4. Mathematical words like 'graph' are used correctly.

   The speaker uses many math words.

5. We could follow the presentations from beginning to end.

   The speakers told how they made the graph before telling us how they collected the data.

Suggested Assessment Strategies

Suggested strategies (*Ministry of Education and Training, 1995*) are as follows:

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

Suggested Assessment Strategies

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

Resources for Assessment

1. The Ontario Curriculum Grades 1-8: Mathematics, Ministry of Education and Training, 1997.

2. "Linking Assessment and Instruction in Mathematics: Junior Grades" by OAME/OMCA, Crompton et al, 1996.
   The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.

3. "Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions", by Jean Karr Stenmark (Ed.), NCTM, 1991.
   This book contains a variety of assessment techniques and gives samples of student work at different levels. Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca

4. "Assessment", Arithmetic Teacher Focus Issue, February 1992, NCTM.
   This copy of NCTM's journal for elementary school addresses several issues dealing with assessment. It also includes suggested techniques and student activities.

5. "How to Evaluate Progress in Problem Solving", by Randall Charles et al., NCTM, 1987.
   Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

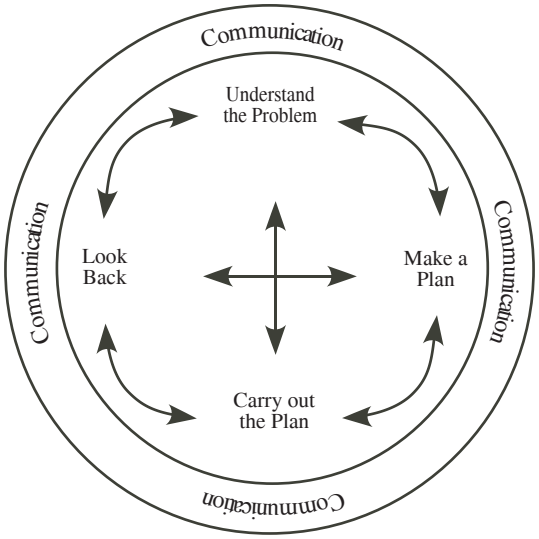
A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

"US and the 3 R''s""

There are five criteria by which each response is judged:
Understanding of the problem,
Strategies chosen and used,
Reasoning during the process of solving the problem,
Reflection or looking back at both the solution and the solving, and
Relevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA "Linking Assessment and Instruction in Mathematics", page 4) should be kept in mind at all times.



There are four levels of response considered:

Limited identifies students who are in need of much assistance;

Acceptable identifies students who are beginning to understand what is meant by 'problem solving', and who are learning to think about their own thinking but frequently need reminders or hints during the process.

Capable students may occasionally need assistance, but show more confidence and can work well alone or in a group.

Proficient students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

Suggested Assessment Strategies

LEVEL OF RESPONSE →

CRITERIA FOR ASSESSMENT ↓

	Limited	Acceptable	Capable	Proficient
UNDERSTANDING	• requires teacher assistance to interpret the problem • fails to recognize all essential elements of the task	• shows partial understanding of the problem but may need assistance in clarifying	• shows a complete understanding of the problem	• shows a complete understanding of the problem
	• needs assistance to choose an appropriate strategy	• identifies an appropriate strategy	• identifies an appropriate strategy	• identifies more than one appropriate strategy
STRATEGIES	• applies strategies randomly or incorrectly • does not show clear understanding of a strategy[1] • shows no evidence of attempting other strategies	• attempts an appropriate strategy, but may not complete it correctly[2] • tries alternate strategies with prompting	• uses strategies effectively • may attempt an inappropriate strategy, but eventually discards it and tries another without prompting	• chooses and uses strategies effectively[3] • recognizes an inappropriate strategy quickly and attempts others without prompting
REASONING	• makes major mathematical errors • uses faulty reasoning and draws incorrect conclusions • may not complete a solution	• may present a solution that is partially incorrect	• produces a correct and complete solution, possibly with minor errors	• produces a correct and complete solution, and may offer alternative methods of solution
	• describes[4] reasoning in a disorganized fashion, even with assistance • has difficulty justifying[5] reasoning even with assisstance	• partially describes[4] a solution and/or reasoning or explains fully with assistance • justification[5] of solution may be inaccurate, incomplete or incorrect	• is able to describe[4] clearly the steps in reasoning; may need assistance with mathematical language • can justify[5] reasoning if asked; may need assistance with language	• explains reasoning in clear and coherent mathematical language • justifies[5] reasoning using appropriate mathematical language
REFLECTION	• shows no evidence of reflection or checking of work • can judge the reasonableness of a solution only with assistance	• shows little evidence of reflection or checking of work • is able to decide whether or not a result is reasonable when prompted to do so	• shows some evidence of reflection and checking of work • indicates whether the result is reasonable, but not necessarily why	• shows ample evidence of reflection and thorough checking of work • tells whether or not a result is reasonable, and why
RELEVANCE	• unable to identify similar[6] problems	• unable to identify similar[6] problems	• identifies similar[6] problems with prompting	• identifies similar[6] problems, and may even do so before solving the problem
	• unlikely to identify extensions[7] or applications of the mathematical ideas in the given problem, even with assistance	• recognizes extensions[7] or applications with prompting	• can suggest at least one extension[7], variation, or application of the given problem if asked	• suggests extensions[7], variation, or applications of the given problem independently

Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.

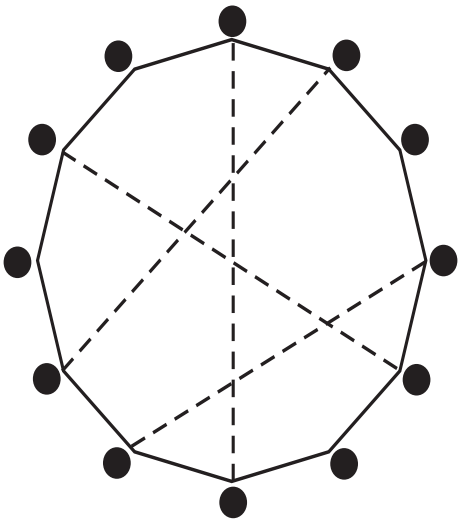
2. For example, diagrams or tables may be produced but not used in the solution.

3. For example, diagrams, if used, will be accurate models of the problem.

4. To *describe* a solution is to tell *what* was done.

5. To *justify* a solution is to tell *why* certain things were done.

6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.
   For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



Problem 1: There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

Problem 2: Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

Problem 3: How many diagonals does a 12-sided polygon have?

   Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

   The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a "what if...?" problem, such as "What if the question were reversed?", "What if we had other data?", "What if we were to show the data on a different type of graph?".

Suggested Assessment Strategies

Suggested Adapted Rubric for Activity 1

The rubric below has been adapted for Activity 1: Interpreting Incomplete Bar Graphs. Four components of the problem are considered: the matching of data descriptions (BLM 2) with given graphs (BLM 1); reasons given for these matches; reflective thinking both during the problem solving and after; presentation of the solution.

Limited	Acceptable	Capable	Proficient
• half (or more) of the matches of the data descriptions to the given graphs are unrealistic • may not identify a data description for each graph	• shows limited understanding of the relationship between the data descriptions and the given graphs • may have one or two questionable matches of data to graph	• shows some understanding of the relationship between the data description and the given graphs • all the matches of data to graph are reasonable	• shows full understanding of the relationship between the data descriptions and the given graphs • all the matches of data to graph are reasonable, and some alternative valid matches may be given
• shows a lack of flexibility in thinking: once a match of data for graph is made, it remains in place to the detriment of finding other matches	• identifies a match for each graph, and is resistant to the idea of changing any matches (i.e., to correct their initial choice)	• identifies a match for each graph; may change matches during the problem-solving process	• shows great flexibility in matching data sets to graphs and in rethinking conclusions when appropriate
• reasons are missing or very brief; they also may be superficial or inappropriate	• reasons show some logical thought, based on magnitudes of data sets, but may be incomplete	• reasons are well thought out and show consideration of the magnitudes of the data sets	• reasons are clear and complete and use appropriate language; they are based on, or include reference to, the magnitudes of the data sets
• cannot respond in any meaningful way to classmates' questions or challenges about their reasoning	• responds to classmates' questions or challenges with brief, sometimes repetitious statements	• responds to most classmates' questions or challenges with clear reasons	• responds confidently to classmates' questions or challenges using appropriate language

Other Resources

1. "Making Sense of Data: Addenda Series, Grades K-6", by Mary Lindquist et al., NCTM, 1992.
   Activities for each grade are presented with lesson outlines and selected black line masters. Students collect, represent and analyze data.

2. "Dealing with Data and Chance: Addenda Series, Grades 5-8", by Judith S. Zawojewski et al., National Council of Teachers of Mathematics, 1991.
   Data gathering and reasoning are two of the themes of this book. Ways people use data and chance in their everyday lives are also discussed.

3. "Developing Graph Comprehension: Elementary and Middle School Activities", by Frances R. Curcio, NCTM, 1989.
   The book contains 25 activities for classroom use dealing with interpreting and constructing various types of graphs.

4. "Ideas", by Calvin and Rosemary Irons, *Arithmetic Teacher*, NCTM, October 1991, pages 26-33.
   The activities presented involve mostly pictographs. Student pages are available for copying. One activity focuses on "Favourite Type of Music."

5. "Ideas", by Helene Silverman, *Arithmetic Teacher*, NCTM, December 1989, pages 27-32, and April 1990, pages 27-32.
   These two articles involve generating and collecting data, as well as making charts and graphs. Student pages are available for copying.
   The Dec. '89 article uses U.S. geography for data; you could just as easily use Canadian geography.
   The April '90 article involves making various measurements, including the jump length of the "Incredible Cotton-ball Frog".

6. "Gender, Ninja Turtles, and Pizza: Using a Classroom Database for Problem Solving", by Judith Day Siegel, *Teaching Children Mathematics,* NCTM, December 1996, pages 192-199.
   Students collect a wide range of classroom data as a database in Clarisworks™, and select various sets of data to illustrate with graphs.

7. "Ideas", by Joan Westley, *Arithmetic Teacher*, NCTM, February 1991, pages 30-36.
   Students use balloons to collect data for a variety of graphs from sources of free balloons to lung capacity. Students pages are available for copying, and a letter for parents/caregivers is included.

8. "Ideas", by Sharon L. Young, *Arithmetic Teacher*, NCTM, April 1991, Pages 26-33.
   Students discuss different ways to collect data, and then use one of the techniques, comparing their data with national data provided. Black Line Masters and a Letter to Parents are included.

9. " Ideas" by Sharon L. Young, Arithmetic Teacher, NCTM, December 1990, pages 23 - 33. Students collect data on bicycles, such as, colour, type of wheels, bicycle racing records, etc., and graph the data. Sample activity sheets are included.

Other Resources

10. " Consumer Investigations: What Is The 'Best Chip'". by Dixie Methany, *Teaching Children Mathematics*, NCTM, March 2001, Pages 418 - 420
    Students design surveys and collect data to determine favourites, to compare crispiness and to identify calories per serving for several varities of chips (potatoe, salsa, corn, etc.). They write reports " for a consumer magazine" that include charts and graphs.

11. " Our Heritage: Learning Data - Management Skills Meaningfully," by Yvonne M. Pothier and Christine M. Nickerson, *Teaching Children Mathematics*, NCTM, October 1997, pages 82 -88
    Students collect data about themselves and their families, such as 'my choice of occupation', 'toys I own', 'allowances my grandparents received', 'sports my grandparents played', and so on. Sample questionnaires are included.

12. " Graphing in the Information Age: Using Data from the World Wide Web," by Juli K. Dixon and Christy J. Falba, *Mathematics Teaching in the Middle School*, NCTM, March - April 1997, pages 298 - 304.
    Ideas for activities include interpretation and creation of bar graphs, line graphs, and circle graphs. Examples of the type of data that can be found on the web page given (e.g., average earnings versus education, NHL stats, frequency of winning lotto numbers).

13. " Adventures in Statistics", by Thomas R. Scavo and Byron Petraroja, *Teaching Childred Mathematics*, NCTM, March 1998, pages 394 - 400.
    Students collect and graph data on the length and width of classrooms and the number of students, then investigate the area per student in each classroom. Sample recording sheets are included.