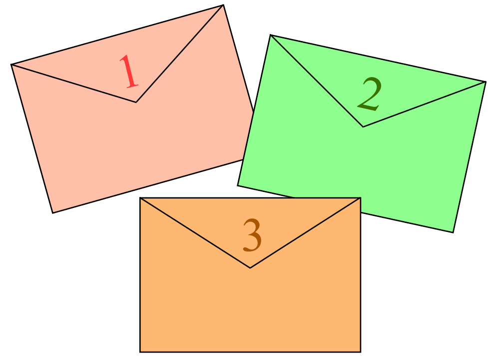
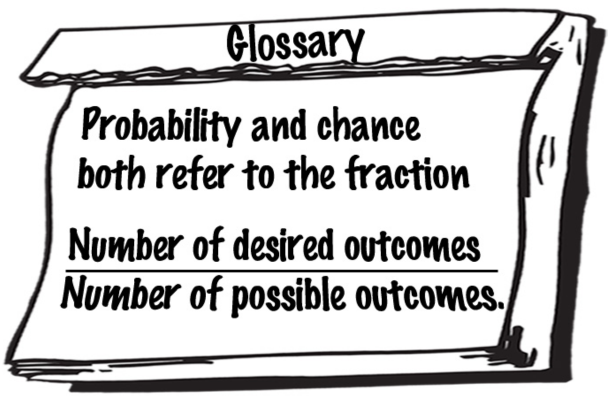
Problem

Three love-struck boys (Alberto, Butch and Coolroy) each invited Bev to the dance. Bev replied with a special letter to each boy, letting them know whether they were 'the one', or not. The first letter was supposed to go to Alberto, the second to Butch, and the third to Coolroy, but Bev forgot to write their names on the outside of the envelopes before sealing them. So now she doesn't know which is which! She decides to just take a chance and label them and mail them anyhow.



a) Complete the chart below to show the possible outcomes. For example, if letter 1 went to Alberto (A), then letter 2 could have gone to Butch (B), and letter 3 to Coolroy (C), or vice versa.



Letter	1	2	3
Recipients	A	B	C
	A	C	B
	B		
	B		
	C		
	C		

b) What is the chance that each boy got the letter he was supposed to get?

c) What is the chance that at least one boy got the wrong letter?

d) What is the probability that the boy she chose as 'the one' got the correct letter?

Extension **:**
How would your answers change if Bev had 4 admirers instead of 3?

Hints

Hint 1 - b) What is the total number of ways the letters could be distributed?

Hint 2 - d) Suppose that Butch was "the one". How many distributions show letter 2 going to Butch?

Solution

a)

Letter	1	2	3
Recipients	A	B	C
	A	C	B
	B	A	C
	B	C	A
	C	A	B
	C	B	A

Table of Possible Outcomes

b) There are 6 possible ways the letters could have been distributed. Only the first one (letter 1 to Alberto, letter 2 to Butch, and letter 3 to Coolroy) is correct. Thus the probability is $\frac{1}{6}$

c) There is only 1 correct distribution, so there are 5 with at least one letter going to the wrong person. So the probability is $\frac{5}{6}$.

d) Suppose she chose Butch as 'the one', so letter 2 was supposed to go to Butch. There are 2 ways for this to happen in the table, so the probability is $\frac{2}{6}$, or $\frac{1}{3}$

e) Advice for Bev: Look before you lick (the envelopes)!

Extension:

1. If Bev had 4 admirers instead of 3, the table would be extended as shown.

Letter	Recipients																							
1	A	A	A	A	A	A	B	B	B	B	B	B	C	C	C	C	C	C	D	D	D	D	D	D
2	B	B	C	C	D	D	C	C	D	D	A	A	D	D	A	A	B	B	A	A	B	B	C	C
3	C	D	D	B	B	C	D	A	A	C	C	D	A	B	B	D	D	A	B	C	C	A	A	B
4	D	C	B	D	C	B	A	D	C	A	D	C	B	A	D	B	A	D	C	B	A	C	B	A

b) There are 24 possible distributions, but only 1 correct one, so the probability is $\frac{1}{24}$.

c) There are 23 ways with at least one letter going to the wrong person. So the probability is $\frac{23}{24}$.

d) There are, for example, 6 ways letter 2 could go to Butch. Thus the probability is $\frac{6}{24}$ or $\frac{1}{4}$.