Problem

In Wonky-Town, the house numbers on one block follow an unusual pattern. The first house is numbered 1, the next house 3, the third house 7, the fourth 13, the fifth 21, and so on.

a) If the house numbers continue to follow this pattern, is there a house number between 60 and 70? Between 80 and 90?

b) If the house numbers are less than 100, what is the greatest number of houses there could be?

c) Amanda is convinced that an even house number brings good luck. If the house numbers can have any number of digits, and still follow the given pattern, will there ever be an even-numbered house for Amanda? Explain your reasoning.

Hints

Hint 1 - What is the difference between each house number and the previous house number?

Suggestion: Once the students have noticed the differences are even, suggest they make a table showing the house numbers and successive differences.

Solution

House	House Number	Difference
1^{st}	1	
2^{nd}	3	2
3^{rd}	7	4
4^{th}	13	6
5^{th}	21	8
6^{th}	31	10
7^{th}	43	12
8^{th}	57	14
9^{th}	73	16
10^{th}	91	18
11^{th}	111	20

a) As revealed in the table at right, for the given five house numbers 1, 3, 7, 13, 21, the successive differences are 2, 4, 6, 8, revealing a pattern which shows the even numbers as the differences. Continuing the table reveals that there are no house numbers between 60 and 70, nor between 80 and 90.

b) There are at most 7 houses in the block with two-digit numbers, since the 11^{th} house would have a three-digit number, 111, and the first three numbers 1, 3, 7, have only one digit.

c) Since adding an even number to an odd number always gives an odd number, alas there will never be an even-numbered house for Amanda.