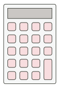
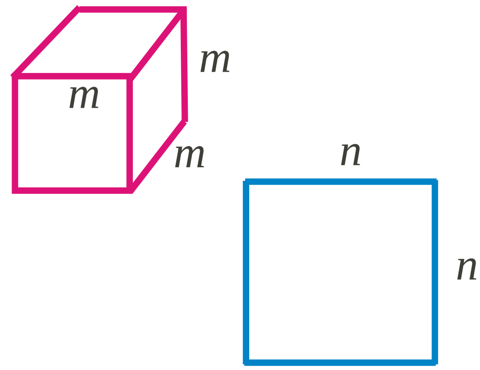
Problem



The volume V of certain cubes with side length 'm', a whole number, has the same number value as the area A of certain squares of side length 'n', also a whole number. For example, the volume of a cube of side length $m = 4$ has the same *number value* ($V = 4 \times 4 \times 4 = \mathbf{64}$) as the area of a square of side $n = 8$ ($A = 8 \times 8 = \mathbf{64}$).



a) For what other cubes with side length 'm' less than 10 is this true?

b) What is special about these numbers 'm'?

Extension :

Try to explain why these 'special' numbers are the only values of 'm' that work.

Hints

Hint 1 - What is the name for a number which has the form $n \times n$ for a whole number n? What does this tell you about the volume number $m \times m \times m$?

Suggestion:

1. You may wish to have students make a table, as shown, as a way to show their results.

m	$V = m \times m \times m$	Could be Area?
1	$1 \times 1 \times 1$	Yes: 1×1
2	$\vdots$	$\vdots$
3		

Solution

a) For the number value of volume $V = m \times m \times m$ to equal that of the area $A = n \times n$ means $m \times m \times m = n \times n$ for some greater whole number n, i.e., $m \times m \times m$ must be the square of some number. The table reveals that the values of m between 1 and 10 that work are $m = 1, 4$, and 9.

m	$V = m \times m \times m$	? Area of Square	m	$V = m \times m \times m$	? Area of Square
1	**1**	Yes: **1** $\times$ **1**	11	1331	No
2	8	No	12	1728	No
3	27	No	13	2197	No
4	**64**	Yes: **8** $\times$ **8**	14	2744	No
5	125	No	15	3375	No
6	216	No	**16**	**4096**	Yes: **64** $\times$ **64**
7	343	No	17	4913	No
8	512	No	18	5832	No
9	**729**	Yes: **27** $\times$ **27**	19	6859	No
10	1000	No	20	8000	No

b) It appears that m must be a perfect square. (The extended table shows $m = 16$ is the next solution.)

Extension:

1. A perfect square, like 4, can be expressed as the product of two identical factors, like 2×2. Therefore when calculating the volume $4 \times 4 \times 4$, we are calculating $(2 \times 2) \times (2 \times 2) \times (2 \times 2)$. Because the factor 2 appears 6 times, it allows us to express the product as $(2 \times 2 \times 2) \times (2 \times 2 \times 2)$, which can be the area of a square. Only perfect squares, like 4, will factor in this manner.

   More formally, to have $m \times m \times m = n \times n$ requires m to be a square number because then $m = k \times k$ for some whole number k. Then $m \times m \times m = (k \times k) \times (k \times k) \times (k \times k) = (k \times k \times k) \times (k \times k \times k) = n \times n$ for $n = k \times k \times k$. No other numbers will factor in this manner.