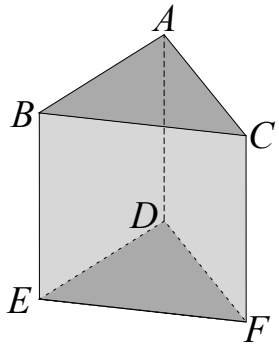
Problem

Minimal Paths (suitable for pairs or groups of students)

In the solid shown below, all the edges are 1 cm in length. A 'path' is defined as follows:



1. A path goes from one vertex to a different vertex.
2. A path follows only edges.
3. A path cannot pass through any vertex more than once.

a) Name the solid.

b) Find the path of minimum total length from vertex A to vertex F. How many vertices does this path go through (not counting the end-points A and F)?

c) Repeat part b) for the path of maximum total length from A to F.

d) Are there any other paths from A to F? If so, name them.

e) List all the possible paths from A to F, their lengths, and the number of vertices, in a table like the one shown below.

Path	Length	Vertices
ACF	2	1

***Extension* :**

Suppose the rules for a path change so that it can traverse any edge only once, but can pass through any vertex once or twice. How would your answer to part c) change?

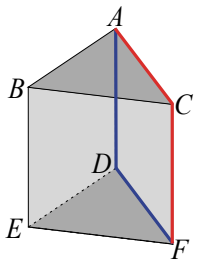
Hints

Suggestion: If possible, allow students to try finding the various possible paths on a wooden or plastic model of this geometric solid.

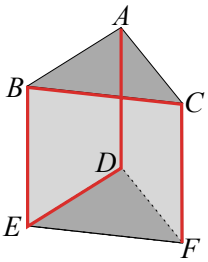
Solution

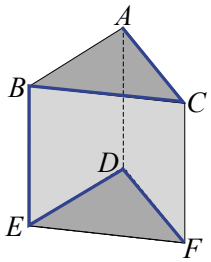
a) The solid is a trianglular prism.

b) There are two possible paths which have minimum length of 2 cm, ACF and ADF, each with 1 vertex.



c) There are two possible paths with maximum length 5 cm, $ADEBCF$ and $ACBEDF$, each with 4 vertices (not counting the end-points A and F).





d) Other paths are $ABEF, ADEF, ABEDF, ACBEF$.

e)

Path	Length	Vertices
ACF	2	1
ADF	2	1
$ABEF$	3	2
$ADEF$	3	2
$ABEDF$	4	3
$ACBEF$	4	3
$ADEBCF$	5	4
$ACBEDF$	5	4

Note that the number of vertices is always 1 less than the path length.

Extension:

The answer to part c) could now include three possible paths, each of lenth 6 cm, with 4 vertices, namely

$ADEBACF$, $ABEDACF$, and $ABCADEF$