Part II: For the Teacher

Curriculum Areas

Problem 1 - Measurement

Problem 2 - Measurement

Problem 3 - Geometry

Problem 4 - Pattern/Algebra

Problem 5 - Number Sense

Problem 6 - Number Sense

Hints and Suggestions:

Problem 1

Hint 1 - How many minutes are there in one hour?

Hint 2 - How many seconds are there in one hour?

Problem 2

Hint 1 - How many megawatts per person were used by Canadians in 2006? By Americans?

Problem 3

Hint 1 - How far apart should adjacent vertices be to make a square?

Hint 2 - Do the other vertices need to be directly above A and B?

Hint 3 - Where could the right angle of the triangle be placed?

Extension:

Hint 1 - Would a compass be helpful?

Problem 4

Suggestion: Have students make a table showing values of n and $6n - 1$

n	$6n$ - 1
1	5
2	11
⋮	⋮

Problem 5

Hint 1 - What is the name for a number which has the form $n \times n$ for a whole number n? What does this tell you about the volume number $m \times m \times m$?

Suggestion:

1. You may wish to have students make a table, as shown, as a way to show their results.

m	$V{=}m{\times}m{\times}m$	Could be Area?
1	$1\times1\times1$	Yes: 1×1
2	$\vdots$	$\vdots$
3		

Problem 6

Hint 1 - If 25 is a 1-step palindrome, what can you say about 52?

Suggestion:

1. Once the groups have completed their hundred chart, have them check with other groups to verify their results.

Solutions

Problem 1

a) Since he skates at 35.78 km per hour, it would take Andrew Cogliano $7.8 \div 35.78 \approx 0.218$ hours to skate the cleared length of the Rideau Canal. Since there are 60 minutes in an hour, this is about $0.218 \times 60 \approx 13.08$ minutes.

b) Since he can run 100 m in 9.69 seconds, Usain Bolt can run 1 km in 96.9 seconds. Thus his speed is $1 \div 96.9 \approx 0.01032$ km per second. Since there are 3600 seconds in 1 hour, this is about $0.01032 \times 3600 \approx 37.15$ km per hour, which is slower than Jeremy Wotherspoon can skate. So Jeremy will take less time.

[Students may also calculate Usain Bolts' time directly, as 7.8 km $\times$ 96.9 seconds per km $= 755.82$ seconds $\equiv 755.82 \div 60 \approx 12.56$ minutes.]

Problem 2

a) Since one turbine produces 2 megawatts, to produce 70 000 megawatts would require $70000 \div 2 = 35000$ turbines.

b) For wind power to generate $\frac{1}{3}$ of the 2006 consumption of 16 378.62 million megawatts, it would need to generate $16378.62 \div 3 \approx 5459.54$ million megawatts, or 5 459 540 000 megawatts. Thus, at 2 megawatts per turbine, this production level would require

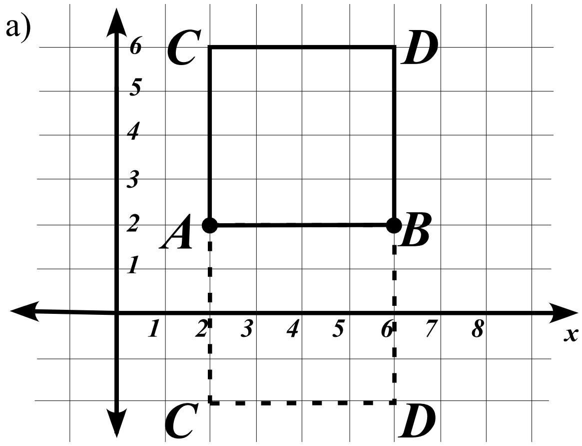
$$5\ 459\ 540\ 000 \div 2 = 2\ 729\ 770\ 000$$

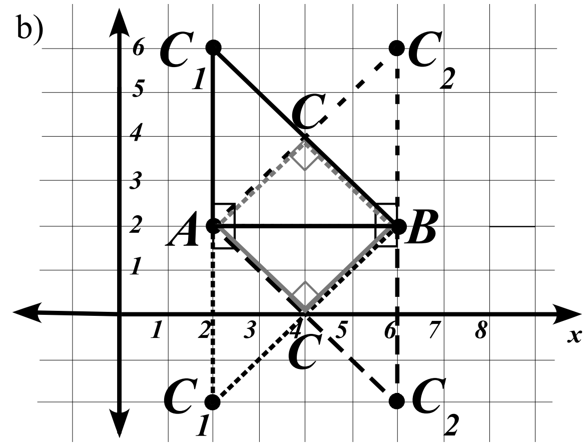
turbines, i.e., about 2.7 billion turbines, or about 1 turbine for every 3 people on earth!

c) Canada's consumption of 529.95 million megawatts for 30 million people is about $529.92 \div 30 \approx 17.7$ megawatts per person. US consumption of 3 816.85 million megawatts for about 300 million people is about $3\ 816.85 \div 300 \approx 12.7$ megawatts per person. Thus Canadian consumption per person is 5 mega watts greater than in the US, i.e. is about $17.7 \div 12.7 \approx 1.4$ times that of the US. This is likely due to the much greater need for heat during the fall/winter/spring months, being that Canada's population lives much farther north than most of the US population.

Problem 3

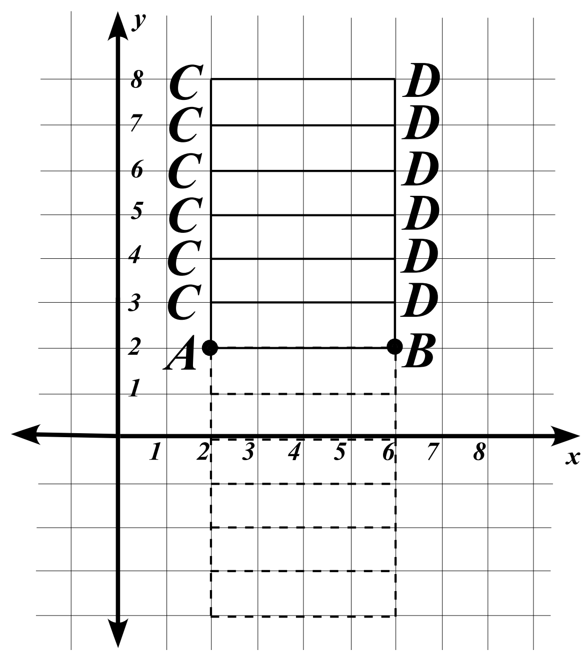
a), b) (See graph below.) Students may or may not realize that negative y-values could be used. A few students may recognize in part b) that C could be at (4, 4) or (4, 0).





c) Any pair of points $C(2, y)$ and $D(6, y)$ will work, for $y > 2$ or $y < 2$.

Students may suggest going beyong the range of 8 for y. They may also suggest the negative y possibilities.

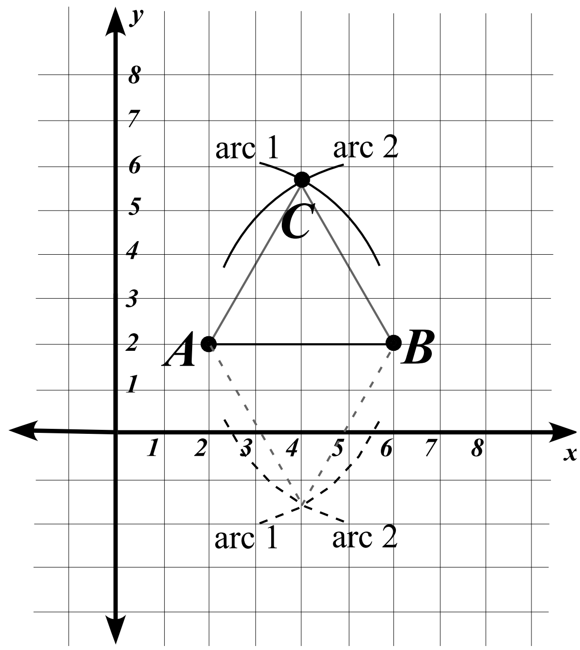


Note: The roles of C and D may be reversed in parts a) and c).

Extension:

1. Using a compass, set its span to be the distance AB. Then draw arc 1 with A as the pivot point, and arc 2 with B as the pivot point. The intersection C of arcs 1 and 2 must be the same distance from both A and B. Thus ABC is an equilateral triangle.

   This construction could be repeated below AB.



Problem 4

a) The algebraic expression is $6 \times n - 1$.

b) Substituting $n = 1, 2, 3, ...$ into this expression reveals that the smallest value of n such that $6n - 1$ is a composite number is $n = 6$, which gives $6 \times 6 - 1 = 35$.

c) Continuing the table, we see that the next value of n which gives a composite number is $n = 11$, which gives $6 \times 11 - 1 = 65$

n	$6n - 1$
1	5
2	11
3	17
4	23
5	29
6	35
7	41
8	47
9	53
10	59
11	65

Extension:

1. Careful observation of the table suggests that every fifth value of $6n - 1$ is a multiple of 5, i.e., $n = 1, 6, 11$ give $6n - 1 = 5, 35, 65$ respectively. This suggests $n = 16$ will also do so. To confirm this, note that $n = 16$ give $6 \times 16 - 1 = 95$. However, $n = 13$ give $6 \times 13 - 1 = 77$, which is a composite number as well; hence the 'next' number is $n = 13$.

Problem 5

a) For the number value of volume $V = m \times m \times m$ to equal that of the area $A = n \times n$ means $m \times m \times m = n \times n$ for some greater whole number n, i.e., $m \times m \times m$ must be the square of some number. The table reveals that the values of m between 1 and 10 that work are $m = 1, 4$, and 9.

m	$V = m \times m \times m$	? Area of Square	m	$V = m \times m \times m$	? Area of Square
1	**1**	Yes: **1** $\times$ **1**	11	1331	No
2	8	No	12	1728	No
3	27	No	13	2197	No
4	**64**	Yes: **8** $\times$ **8**	14	2744	No
5	125	No	15	3375	No
6	216	No	**16**	**4096**	Yes: **64** $\times$ **64**
7	343	No	17	4913	No
8	512	No	18	5832	No
9	**729**	Yes: **27** $\times$ **27**	19	6859	No
10	1000	No	20	8000	No

b) It appears that m must be a perfect square. (The extended table shows $m = 16$ is the next solution.)

Extension:

1. A perfect square, like 4, can be expressed as the product of two identical factors, like 2×2. Therefore when calculating the volume $4 \times 4 \times 4$, we are calculating $(2 \times 2) \times (2 \times 2) \times (2 \times 2)$. Because the factor 2 appears 6 times, it allows us to express the product as $(2 \times 2 \times 2) \times (2 \times 2 \times 2)$, which can be the area of a square. Only perfect squares, like 4, will factor in this manner.

   More formally, to have $m \times m \times m = n \times n$ requires m to be a square number because then $m = k \times k$ for some whole number k. Then $m \times m \times m = (k \times k) \times (k \times k) \times (k \times k) = (k \times k \times k) \times (k \times k \times k) = n \times n$ for $n = k \times k \times k$. No other numbers will factor in this manner.

Problem 6

a) The results are shown in the chart below.

	Zero-Step	One-Step	Two-Step	Three-Step	Four-Step
NUMBER		10, 12, 13, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 38, 40, 41, 42, 43, 45, 47, 50, 51, 52, 53, 54, 56, 60, 61, 62, 63, 65, 70	19, 28, 37, 39, 46, 48, 49, 57, 58, 64, 67	59, 68	69
		71, 72, 74, 80, 81, 83, 90, 92, 100	73, 75, 76, 82, 84, 85, 91, 93, 94	86, 95	96
PALINDROME	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22,33,44,55, 66	11, 33, 44, 55, 66, 77, 88, 99, 22, 33, 55, 66, 77, 88, 99, 121, 33, 44, 55, 77, 88, 99, 121, 44 55, 66, 77, 99, 121, 55 66, 77 88, 99, 121, 66, 77, 88, 99, 121, 77	121,121,121,363,121,363,484, 363,484,121,484	1111,1111	4884
	77, 88, 99	88,99,121,88,99,121,99,121,101	121, 363, 484, 121, 363, 484, 121, 363, 484	1111,1111	4884

b) The completed hundred chart is shown below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Observed patterns:

- The portion of the chart from 11 to 99, excluding multiples of 10, displays complete symmetry about the diagonal zero-step palindromes 11, 22, ..., 99.
- Most of the numbers are one-step palindromes.
- There are no two-step palindromes above the diagonal formed by 19, 28, 37, ..., 91.
- The three- and four-step palindromes occur for numbers greater than 58.

Some further thought may lead students to see why the above types of palindromes occur where they do in the chart.

- One-step palindromes occur if the sum, S, of the digits of the number is less than 10, or equal to 11, and the palindrome is SS if $S < 10$ (e.g., if the sum is 7 then the palindrome is 77), or 121 if $S = 11$.
- Two-step palindromes occur if $S = 10, 12$, or 13; three-step palindrome occur if $S = 14$, and four-step if $S = 15$.

Extension:

1. a) The numbers 77, 88, 99 are zero-step palindromes. The numbers 8, 9, 17, 18, 27, 29, 38, 47, predict that 80, 90, 71, 81, 72, 92, 83, 74 will be one-step palindromes, as is the number 100. The numbers 19, 28, 37, 39, 48, 49, 57, 58, 67 predict that 91, 82, 73, 93, 84, 94, 75, 85, 76 will be two-step palidromes. The numbers 59, 68 predict 95, 86 will be three step, and 69 predicts 96 will be four-step.

   b) The remaining six numbers are pairs 78 and 87, 79 and 97, and 89 and 98.