Emmy Noether - Circle 2 for 2011-2012

Part I: Problems

Problem 1

The Rideau Canal system is 202 km long, running from Ottawa, the capital of Canada, to Kingston, on Lake Ontario. During the winter, 7.8 km of the frozen canal are cleared within the city of Ottawa for skating.

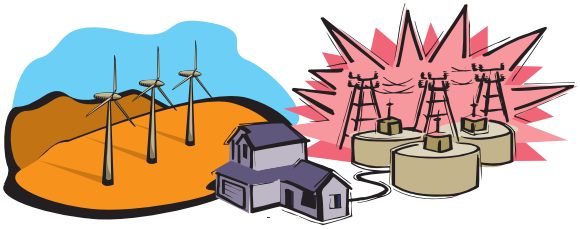
a) The fastest skater in the 2009 NHL Skills Competition was Andrew Cogliano, who skates about 35.78 km per hour. How long would it take him to skate the cleared portion of the canal? Give your answer in both hours and minutes.

b) Jeremy Wotherspoon, one of Canada's fastest speed skaters, skates about 52.89 km per hour. Usain Bolt, the fastest runner in the world in 2011, can run 100 m in 9.69 seconds. If Jeremy skates the cleared length of the canal while Usain runs along the parallel path along the bank, who will take the least amount of time?

Problem 2

A single wind turbine generates about 2 megawatts of power, enough energy for about 500 homes. (You may assume that all wind turbines in this problem are similar and have similar production capabilities.)

a) Total world electricity production by wind power in 2010 was 70 000 megawatts. How many wind turbines would have been needed to produce all of this by wind power?



b) Some people predict that by 2050, wind power will generate about $\frac{1}{3}$ of the world's electricity needs. If world consumption did not change from 2006 (which is unlikely), when it was about 16 378.62 million megawatts, how much energy would be produced by wind power? How many wind turbines would be needed to generate that much electricity?

c) In 2006, Canada's electricity consumption was 529.95 million megawatts for a population of about 30 million people, while the United State's consumption was about 3816.85 million megawatts for about 300 million people. Compare and contrast Canada's and the United States' electricity consumption.

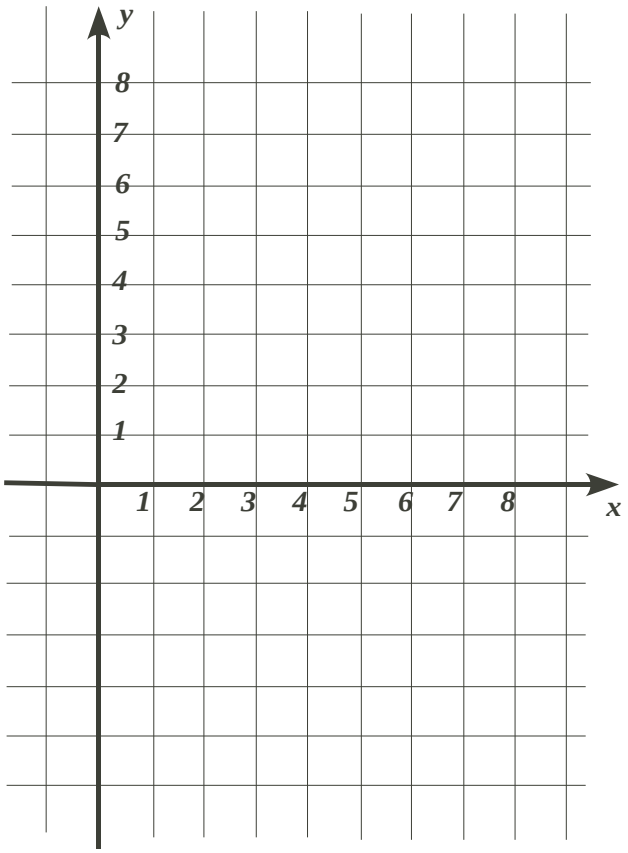
Problem 3

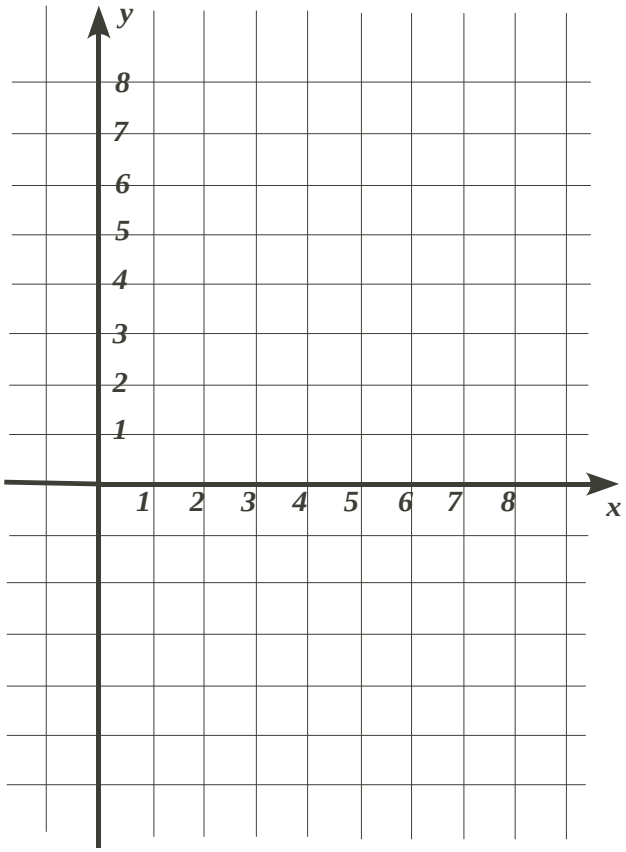
a) Plot the points $A(2, 2)$ and $B(6, 2)$ on the left graph below. If A and B are two consecutive vertices of a square, what is another pair of points, C and D, that would complete the square? Can you find more than one answer?

b) Plot the same two points A and B on the right graph below. If these points are two vertices of a right angled triangle, what would be the coordinates of the third vertex, C? Is there more than one answer?

c) If A and B are two consecutive vertices of a rectangle, how many other pairs of points C and D could be used to form a complete rectangle?

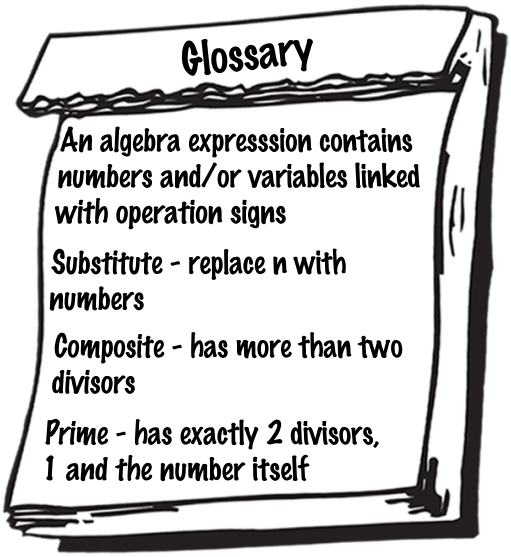
***Extension* :**
Suppose the point C in part b) is the third vertex of an equilateral triangle, rather than that of a right angled triangle. Locate the point C by construction (no need for coordinates).





Problem 4

A number n is multiplied by 6, and then 1 is subtracted.



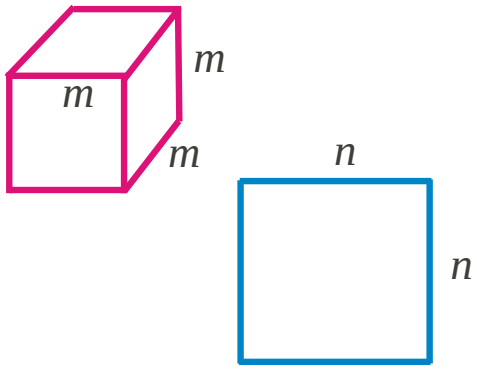
a) Write an algebraic expression for this statement.
b) If you substitute different whole numbers for n in your expression from a), what is the smallest number n which gives a composite number as the answer?
c) What is the next smallest whole number n that does not have a prime number as the answer?

n	$6n$ - 1
1	5
2	11
⋮	⋮

***Extension* :**
Predict the next number n which will give a composite number the expression in part a). Explain your reasoning.

Problem 5

The volume V of certain cubes with side length 'm', a whole number, has the same number value as the area A of certain squares of side length 'n', also a whole number. For example, the volume of a cube of side length $m = 4$ has the same *number value* ($V = 4 \times 4 \times 4 = \mathbf{64}$) as the area of a square of side $n = 8$ ($A = 8 \times 8 = \mathbf{64}$).



a) For what other cubes with side length 'm' less than 10 is this true?

b) What is special about these numbers 'm'?

***Extension* :**
Try to explain why these 'special' numbers are the only values of 'm' that work.

Problem 6: The Palindromic Two-Step (suitable for groups of four students)

A palindrome is an expression which is the same written forwards as backwards. For example, 2, 44, 101, 8118, 'race car' and 'radar' are palindromes, while 13, 24, 245, and 'auto' are not.

The number 13 is not a palindrome, but if you reverse its digits and then sum the two numbers, you get $13+31=44$, which is a palindrome. Numbers like this are called one-step palindromes.

The number 37 is also not a palindrome, but if you reverse its digits and add, you get $37+73=110$, not a palindrome. But if you repeat this process, reversing the digits of 110 and adding, you get $110+011=121$ which is a palindrome. Numbers like this are called two-step palindromes. The number 68 is a three-step palindrome:

$$\begin{array}{r} 37 \\ 73 \\ \hline 110 \\ 011 \\ \hline 121 \end{array} \qquad \begin{array}{r} 68 \\ 86 \\ \hline 154 \\ 451 \\ \hline 605 \\ 506 \\ \hline 1111 \end{array}$$

a) Working in groups of four, test all the numbers from 10 to 70 and discover how many additonal steps are required to achieve a palindrome.

Have one group member do the numbers $10-25$, one do $26-40$, one do $41-55$, and one do $56-70$.

As you work, fill in the chart below, identifying each number as a one-step, two-step, three-step, or four-step palindrome, and state the palindromic sum.

Number of Steps	Zero-Step	One-Step	Two-Step	Three-Step	Four-Step
Number	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22, 33, 44, 55, 66, 77, 88, 99	13,	37,	68,	
Final Palindrome	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22, 33, 44, 55, 66, 77, 88, 99	44,	121,	1111,	

b) Below is a hundred chart. The cells with numbers which are already palindromes are shaded grey. Colour the cells with one-step palindromes red, the two-step green, the three-step blue, and the four-step yellow, for all the other numbers up to 70. Can you see any patterns?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

***Extension* :**

a) Your answers to part *a*) allow you to predict results for some of the numbers between 71 and 100. What are these predictable numbers? Explain your reasoning. (Note that 100 is a one-step palindrome.)

b) What are the six remaining numbers that you can't predict? Fill in the remainder of the hundred chart with the appropriate colours, leaving these six unknown numbers white.