# Part II: For the Teacher

## Curriculum Areas

**Problem 1 -** Number Sense, Pattern/Algebra

**Problem 2 -** Geometry

**Problem 3 -** Number Sense

**Problem 4 -** Pattern/Algebra

**Problem 5 -** Probability

**Problem 6 -** Geometry

## Hints and Suggestions:

### Problem 1

**Hint 1 -** What is the difference between each house number and the previous house number?

*Suggestion:* Once the students have noticed the differences are even, suggest they make a table showing the house numbers and successive differences.

### Problem 2

**Hint 1 -** Draw extra lines on the figures which make equal-sizes pieces.

**Hint 2 -** How can you compare fractions with different denominators?

### Problem 3

**Hint 1 -** How many points will Ye Ming have if she spends $15?

**Hint 2 -** How many $15 purchases would Ye Ming need to make to receive 350 points?

### Problem 4

**Hint 1 -** Are there squares of different sizes?

**Hint 2 -** In the $3 \times 3$ grid, how many $2 \times 2$ squares are there?

**Hint 3 -** How many more total squares are there in the $2 \times 2$ grid than in the $1 \times 1$? In the $3 \times 3$ grid than in the $2 \times 2$ gird?

### Problem 5

**Hint 1 -** b) What is the total number of ways the letters could be distributed?

**Hint 2 -** d) Suppose that Butch was “the one”. How many distributions show letter 2 going to Butch?

### Problem 6

c) *Suggestions:*

1. Have students cut out the pentominoes and try to form boxes with each one.

2. Do a web search on pentominoes for many other interesting activities.

## Solutions

### Problem 1

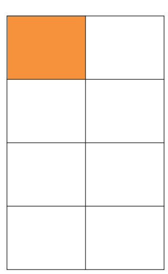
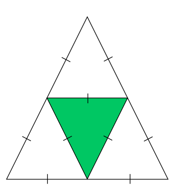
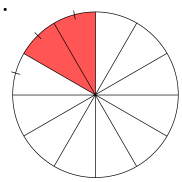
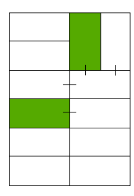
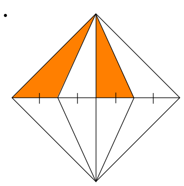
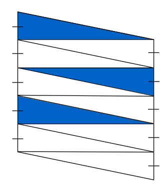
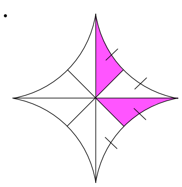
a) As revealed in the table at right, for the given five house numbers 1, 3, 7, 13, 21, the successive differences are 2, 4, 6, 8, revealing a pattern which shows the even numbers as the differences. Continuing the table reveals that there are no house numbers between 60 and 70, nor between 80 and 90.

b) There are at most 7 houses in the block with two-digit numbers, since the 11$^{\text{th}}$ house would have a three-digit number, 111, and the first three numbers 1, 3, 7, have only one digit.

c) Since adding an even number to an odd number always gives an odd number, alas there will never be an even-numbered house for Amanda.

| House | House Number | Difference |
|---|---|---|
| 1$^{\text{st}}$ | 1 | |
| 2$^{\text{nd}}$ | 3 | 2 |
| 3$^{\text{rd}}$ | 7 | 4 |
| 4$^{\text{th}}$ | 13 | 6 |
| 5$^{\text{th}}$ | 21 | 8 |
| 6$^{\text{th}}$ | 31 | 10 |
| 7$^{\text{th}}$ | 43 | 12 |
| 8$^{\text{th}}$ | 57 | 14 |
| 9$^{\text{th}}$ | 73 | 16 |
| 10$^{\text{th}}$ | 91 | 18 |
| 11$^{\text{th}}$ | 111 | 20 |

### Problem 2

| SHAPE | SHADED AREA |
|---|---|
| 1.  | $\frac{1}{8}$ |
| 2.  | $\frac{1}{4}$ |
| 3.  | $\frac{2}{12}$ or $\frac{1}{6}$ |
| 4.  | $\frac{2}{12}$ or $\frac{1}{6}$ |
| 5.  | $\frac{2}{8}$ or $\frac{1}{4}$ |
| 6.  | $\frac{3}{10}$ |
| 7.  | $\frac{2}{8}$ or $\frac{1}{4}$ |

Thus figures 3 and 4 have the same fraction $\left(\frac{1}{6}\right)$ of area shaded, and figures 2, 5, and 7 have the same fraction $\left(\frac{1}{4}\right)$ of area shaded.

**Problem 3**

Ye Ming gets 1 point for spending \$5, $1 + 2 = 3$ points for spending \$10, and $3 + 4 = 7$ points for spending \$15, Since 350 points $= 50 \times 7$ points, to get the jeans, she would have to spend $50 \times \$15 = \$750$. Most 'tweens' would judge that to be not worth it!

**Problem 4**

a) The simplest grid is a single $1 \times 1$ grid.

b) The next simplest grid is a $2 \times 2$ grid with 1 large $(2 \times 2)$ square + 4 smallest $(1 \times 1)$ squares = 5 total squares.

c) The next simplest grid is a $3 \times 3$ grid with 1 $(3 \times 3$ square) + 4 $(2 \times 2$ squares) + 9 $(1 \times 1$ squares) = 14 squares. Note that this equals the total number in the $2 \times 2$ grid (5), plus $3^2$.

Similarly, the $4 \times 4$ grid has 1 $(4 \times 4$ square) + 4 $(3 \times 3$ squares) + 9 $(2 \times 2$ squares) + 16 $(1 \times 1$ squares) = 14 + $4^2$ = 30 squares in total.

The pattern is: at each stage, add the number of 'smallest' $(1 \times 1)$ squares to the previous total to obtain the new total number of squares.

Note that the number of "smallest" squares in each case is just the square of the grid size

| Dimensions of the Grid | Number of "Smallest" Squares | Total Number of Squares |
|---|---|---|
| $1 \times 1$ | 1 | 1 |
| $2 \times 2$ | 4 | 5 |
| $3 \times 3$ | 9 | 14 |
| $4 \times 4$ | 16 | 30 |
| $5 \times 5$ | 25 | 55 |
| $6 \times 6$ | 36 | 91 |

**Problem 5**

a)

| Letter | 1 | 2 | 3 |
|---|---|---|---|
| Recipients | A | B | C |
| | A | C | B |
| | B | A | C |
| | B | C | A |
| | C | A | B |
| | C | B | A |

Table of Possible Outcomes

b) There are 6 possible ways the letters could have been distributed. Only the first one (letter 1 to Alberto, letter 2 to Butch, and letter 3 to Coolroy) is correct. Thus the probability is $\frac{1}{6}$

c) There is only 1 correct distribution, so there are 5 with at least one letter going to the wrong person. So the probability is $\frac{5}{6}$.

d) Suppose she chose Butch as 'the one', so letter 2 was supposed to go to Butch. There are 2 ways for this to happen in the table, so the probability is $\frac{2}{6}$, or $\frac{1}{3}$

e) Advice for Bev: Look before you lick (the envelopes)!

*Extension:*

1. If Bev had 4 admirers instead of 3, the table would be extended as shown.

| Letter | Recipients | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A | A | A | A | A | A | B | B | B | B | B | B | C | C | C | C | C | C | D | D | D | D | D | D |
| 2 | B | B | C | C | D | D | C | C | D | D | A | A | D | D | A | A | B | B | A | A | B | B | C | C |
| 3 | C | D | D | B | B | C | D | A | A | C | C | D | A | B | B | D | D | A | B | C | C | A | A | B |
| 4 | D | C | B | D | C | B | A | D | C | A | D | C | B | A | D | B | A | D | C | B | A | C | B | A |

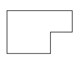
b) There are 24 possible distributions, but only 1 correct one, so the probability is $\frac{1}{24}$.

c) There are 23 ways with at least one letter going to the wrong person. So the probability is $\frac{23}{24}$.

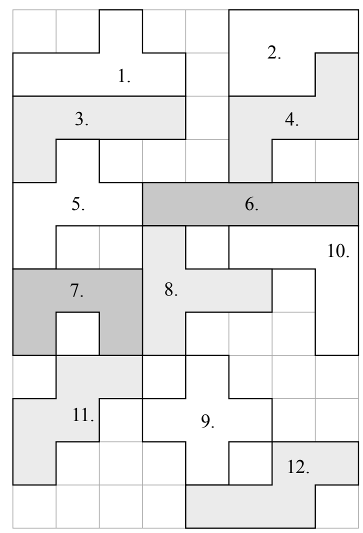
d) There are, for example, 6 ways letter 2 could go to Butch. Thus the probability is $\frac{6}{24}$ or $\frac{1}{4}$.

**Problem 6 (Pentominoes and Boxes)**

a) All 12 pentominoes have the same area, 5 square units.

b) Of the 12 pentominoes, 11 have the same perimeter, namely 12 units of length. The exception is the pentomino labelled #2 in the diagram,  which has a perimeter of 10 units of length.

c) The pentominoes labelled 1, 3, 4, 5, 8, 9, 11 and 12 can be made into open boxes. Cut them out and make the boxes to verify this.




*Extension:*

1. The diagram below (from the web) provides one solution. Try to find others!