## Problem

Bella loves to play "Rockers" guitar. Having devoted 1500 hours to this game, she has some questions about the available combinations of notes. For example, if there were just two colours (notes), say Red and Yellow (labelled R and Y for convenience), then the possible combinations (i.e., two-note riffs), would be RR, RY, YR, and YY.

When playing "Rockers" on Easy Level, there are three colours to press, Green, Red, and Yellow (label them G, R, and Y), so there are 3 one-note riffs.

a) If you play a two-note riff on Easy Level, what are all the possibilities you could play for this riff? Enter the total number of riffs in the table.

b) If you play a three-note riff on Easy Level, what are the possible riffs that begin with a G? With a R? With a Y? Enter the total number of riffs in the table. How does this compare with your answers to a)?

c) Considering your results in a) and b), try to predict how many possible combinations of notes you could play for a four-note riff, and for a five-note riff. Enter your predictions in the table. Explain your reasoning. Look for a pattern.
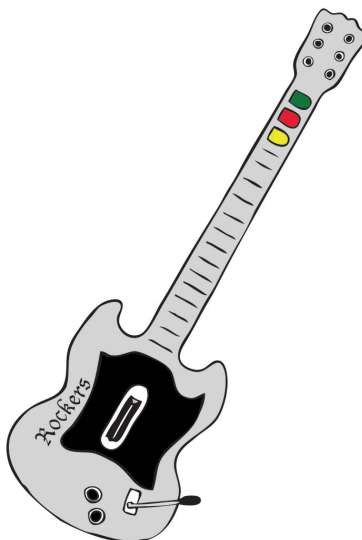
Number of notes	Number of riffs
1	3
2	
3	
4	
5	

Hints

Hint 1 - b) If you play a Green note after each two-note riff from a), how many three-note riffs will you get? Why?

Hint 2 - c) How could you use your answer from part b) to find how many four-note riffs are possible?

Solution

a) There are three colours, G, R, Y. One way to organize the counting of possible riffs is to list separately those that begin with G, then R, then Y. Thus there are three sets of three riffs, GG, GR, GY, RR, RG, RY, and YY, YG, YR, giving a total of $3 \times 3 = 9$ two-note riffs.

b) The easiest way to think of creating three-note riffs is to add a note to each of the two-note riffs. Since any one of G, R, or Y, can be added to each of the nine two-note riffs, there are three times as many, i.e., $9 \times 3 = 27$ three-note riffs.

c) Similar reasoning show that there are three times as many four-note riffs as three-note riffs, i.e., $3 \times 27 = 81$ four-note riffs, and $3 \times 81 = 243$ five-note riffs.

Number of notes	Number of riffs
1	3
2	9
3	27
4	81
5	243