Problem

Sally wanted an allowance. Her Mother offers two options:

1. $10 per week for a year, or

2. $204.80 in January, $102.40 in February, and so on, halving her allowance each month for the rest of the year.

   a) Which option would you choose? 1. ☐ or 2. ☐

   b) Determine which option gives the greatest total allowance.

   c) Could you answer part b) without summing the allowances for the entire year?

Extension :

1. Suppose, in option 2, Mother tells Sally her March allowance will be $75, instead of telling her January's and February's. Would your answer to b) change?

Hints

Hint 1 - a) If you chose Option 1, how could you determine the total allowance for the year?

Hint 2 - b) If you chose Option 2, what would be your allowance in March? In May?

Hint 3 - c) Given that May's allowance is $12.80, how does the size of Sally's allowance compare to this for the remaining seven months of the year?

Suggestion: Have students assume there are 52 weeks in a year.

Extension :

Hint 1 - What would Sally's allowance be for February in this case? In January?

Solution

b) Option 1 would give Sally a total allowance over the year of $10 \times 52 = \$520$. To find the total for Option 2, we halve each month's allowance to get the next month's, and sum them. Starting with January and February, and continuing, we have:

$204.80 + \$102.40 + \$51.20 + \$25.60 + \$12.80 + \$6.40 + \$3.20 + \$1.60 + \$0.80 + \$0.40 + \$0.20 + \$0.10 = \$409.50$.

Thus Option 1 gives the greatest total allowance.

c) Noting that the first four months' allowance sum to $384.00, and that the remaining eight months must be less than $13.00 each, we see that the total for Option 2 must be less than $384.00 + (\$13.00 \times 8) = \$488.00$. So it is sufficient to just sum the first four months, and then estimate the remainder of the year, in order to see that Option 1 is better.

Extension :

1. In this case, Sally's February allowance would be twice her March allowance, or $150, and her January allowance would be twice $150, or $300. Hence, by the end of March, she would already have $300 + \$150 + \$75 = \$525$, which is greater than the whole year's allowance for Option 1.