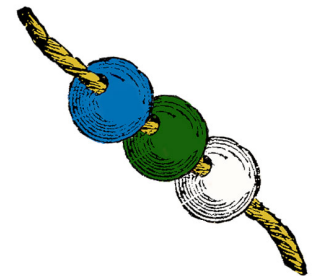


Problem

Ekaterina is making a bracelet with different coloured beads. In the centre are six special beads, two blue, two green, and two white. The beads are all different sizes: 0.5 cm, 0.6 cm, 0.7 cm, 0.8 cm, 0.9 cm, and 1 cm. Use the following clues to discover the colour of each size of bead.

Bead Size (cm) Colour	0.5	0.6	0.7	0.8	0.9	1.0
Larger Blue						
Smaller Blue						
Larger Green						
Smaller Green						
Larger White						
Smaller White						



1. The smallest bead is not blue.
2. The largest bead is blue.
3. One of the white beads is 0.2 cm longer than the other white bead.
4. The 0.7 cm bead is not white.
5. The lengths of the blue beads differ by more than 0.1 cm.

Extension :

1. If you were given only the first four clues, how many solutions would there be to this problem?

Hints

Hint 1 - Could the 0.9 cm bead be blue? Why, or why not?

Hint 1 - Could the 0.5 cm bead be white? Why, or why not?

Hint 1 - Could the 0.9 cm bead be white? Why, or why not?

Suggestions:

1. Have students use the clues to fill in the chart, using ✓ if true, X if false. Initially, there may be more than one possibility for the position of the some beads, but as they look at the consequences of each clue, they will gradually eliminate all but one for each size of bead.
2. This problem could be done in small groups, generating active discussions among students. It is especially valuable for them to explain their reasoning to one another.

Solution

The clues can be used sequentially to reason the correct sizes and colours of the beads. The steps in the reasoning are explained below, and depicted by numbered entries in the table.

1. Clues 1 and 2 reveal that the larger blue bead is the 1 cm bead; thus we can fill in a check mark for that square in the table, and put X's in all the other squares in the first row and last column, and in the first square of the second row. (These are shown as X_1 s in the table below.)
2. Clue 4 reveals that the 0.7 cm bead is not white (hence the two blue X_2 's in the third column). Clue 3 thus implies that neither the 0.5 cm nor the 0.9 cm bead could be white, since those would give a 0.2 cm difference from 0.7 cm, which cannot be white. (These are the remaining X_2 s in the table.)
3. As a consequence, the white beads must be 0.6 cm and 0.8 cm, which puts a check mark in those positions, and X's in all the remaining 0.6 and 0.8 cm squares. (These are X_3 s in the table.)
4. Clue 5 implies that the smaller blue bead cannot be 0.9 cm, leaving the only possibility as the 0.7 cm bead. Thus we place a check mark in that square, and X's in the remaining squares for 0.7 cm, and the second row of the 0.9 cm column. (These are X_4 s in the table.)
5. Finally, the only choices left for the two green beads are 0.5 cm for the smaller green and 0.9 cm for the larger green. (These are X_5 s in the table.)

Bead Size (cm) Colour	0.5	0.6	0.7	0.8	0.9	1.0
Larger Blue	X_1	X_1	X_1	X_1	X_1	✓ ₁
Smaller Blue	X_1	X_3	✓ ₄	X_3	X_4	X_1
Larger Green	X_5	X_3	X_4	X_3	✓ ₅	X_1
Smaller Green	✓ ₅	X_3	X_4	X_3	X_5	X_1
Larger White	X_2	X_3	X_2	✓ ₃	X_2	X_1
Smaller White	X_2	✓ ₃	X_2	X_3	X_2	X_1

Extension :

1. If the last clue is not given, then the smaller blue bead could be either 0.7 cm (as in the above solution), or 0.9 cm. In the latter case, the two green beads would be 0.5 cm and 0.7 cm. Thus there are two possible solutions in this case.