Part II: For the Teacher

Curriculum Areas

Problem 1 - Data Management

Problem 2 - Number Sense and Measurement

Problem 3 - Measurement and Problem Solving

Problem 4 - Problem Solving and Number Sense

Problem 5 - Logic/Problem Solving

Problem 6 - Geometry and Pattern/Algebra

Hints and Suggestions:

Problem 1

Hint 1 - b)(i) What is the total number of minutes Kevin Garnett played?

Hint 2 - b)(ii) What is the total number of rebounds he made?

Hint 3 - b)(iii) What is the total number of points he scored?

Problem 2

Hint 1 - a) If you chose Option 1, how could you determine the total allowance for the year?

Hint 2 - b) If you chose Option 2, what would be your allowance in March? In May?

Hint 3 - c) Given that May's allowance is $12.80, how does the size of Sally's allowance compare to this for the remaining seven months of the year?

Suggestion: Have students assume there are 52 weeks in a year.

Extension **:**

Hint 1 - What would Sally's allowance be for February in this case? In January?

Problem 3

Hint 1 - a) What is a quick way to see how far Curly has run after 10 seconds? How far has Hoppy run?

Hint 2 - b) In every second, how much closer does Curly get to Hoppy?

Hint 3 - c) How long will it take Hoppy to reach the forest?

Hint 4 - c) How long will it take Curly to reach the forest?

Problem 4

Hint 1 - a) If you sum 2 consecutive numbers, what kind of number is the sum?

Hint 2 - b) What are the factors of each of the two sums in the second table?

Hint 3 - c) What does your result from b) tell you to check about 58?

Hint 4 - d) Could 58 be the sum of four or more consecutive single digit numbers? Why, or why not?

Hint 5 - d) Could 58 be the sum of three or more consecutive numbers in the 20s? Why, or why not?

Problem 5

Hint 1 - Could the 0.9 cm bead be blue? Why, or why not?

Hint 1 - Could the 0.5 cm bead be white? Why, or why not?

Hint 1 - Could the 0.9 cm bead be white? Why, or why not?

Suggestions:

1. Have students use the clues to fill in the chart, using ✓ if true, X if false. Initially, there may be more than one possibility for the position of the some beads, but as they look at the consequences of each clue, they will gradually eliminate all but one for each size of bead.

2. This problem could be done in small groups, generating active discussions among students. It is especially valuable for them to explain their reasoning to one another.

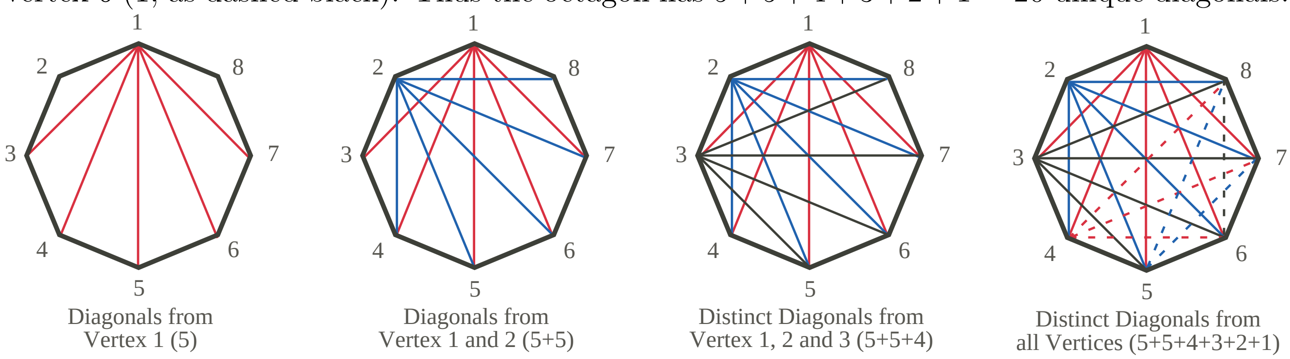
Problem 6

Suggestions: This is a discovery activity which could be done by individual students or by small groups; if groups are used, assign one of the figures a), b) or c) to each group. Suggestions which may help are:

1. Have students use a different coloured crayon or pencil for the diagonals they draw from each vertex. This will help them to avoid counting a diagonal more than once.

2. Once students/groups have completed a), b), c), work with the class to complete the table in part d) up to 8 sides.

3. Trying to see a pattern requires some organized counting. To illustrate, sketch the diagonals of the octagon in stages with the students, on a large octagon on the board or an overhead, using the diagrams below as a guide. Note that, from vertex 1, we can draw 5 diagonals (red on the diagrams), since we can only use non-adjacent vertices. From vertex 2, we can draw 5 new and different diagonals (blue on the diagrams). But from vertex 3, we can only draw 4 new diagonals (black on the diagrams). At this point, ask students how many could be drawn from vertex 4 (3 new diagonals, shown as dashed red on the last diagram), then vertex 5 (2, as dashed blue), and vertex 6 (1, as dashed black). Thus the octagon has $5+5+4+3+2+1 = 20$ unique diagonals.



4. To start them thinking about a 12-sided polygon, ask "How many diagonals could be drawn from the first vertex? The second vertex?" This should help them get started on finding the total number by adapting the method used above for the octagon.

Solutions

Problem 1

a) The completed table:

League	NBA	NFL	NHL	MLB
Average salary (millions of $)	4.9	1.3	1.8	2.5
Number of Games in a regular season	82	16	82	162
Average salary per regular season game	$59 756	$81 250	$21 951	$15 432

(Averages have been rounded to the nearest dollar.)

b)(i) Since he played a total of $71 \times 32.8 = 2328.8$ minutes during the regular season, his salary per minute is $\$24\,750\,000 \div 2328.8 \approx \$10\,627.79$, i.e., about $10 628 per minute. Since there are 60 minutes in 1 hour, his pay is about $\$10\,627.79 \times 60 \approx \$637\,667.40$ per hour.

(ii) His total rebounds were $71 \times 9.2 = 653.2$, or 653 rebounds. Thus he made $\$24\,750\,000 \div 653 \approx \$37\,901.91$, or about $37 902 per rebound.

(iii) He scored a total of $71 \times 18.8 = 1334.8$, or 1335 points. Thus he made $\$24\,750\,000 \div 1335 = \$18\,539.33$, or about $18 539 per point.

Problem 2

b) Option 1 would give Sally a total allowance over the year of $\$10 \times 52 = \$520$. To find the total for Option 2, we halve each month's allowance to get the next month's, and sum them. Starting with January and February, and continuing, we have:

$\$204.80 + \$102.40 + \$51.20 + \$25.60 + \$12.80 + \$6.40 + \$3.20 + \$1.60 + \$0.80 + \$0.40 + \$0.20 + \$0.10 = \$409.50.$

Thus Option 1 gives the greatest total allowance.

c) Noting that the first four months' allowance sum to $384.00, and that the remaining eight months must be less than $13.00 each, we see that the total for Option 2 must be less than $\$384.00 + (\$13.00 \times 8) = \$488.00$. So it is sufficient to just sum the first four months, and then estimate the remainder of the year, in order to see that Option 1 is better.

Extension :

1. In this case, Sally's February allowance would be twice her March allowance, or $150, and her January allowance would be twice $150, or $300. Hence, by the end of March, she would already have $\$300 + \$150 + \$75 = \$525$, which is greater than the whole year's allowance for Option 1.

Problem 3

a) See the table at right. After 10 seconds, the gap between Curly and Hoppy is 140 m.

b) Since Curly runs at 7 metres per second (m/s) and Hoppy runs at 5 m/s, Curly gains 2 m on Hoppy each second. Thus, to make up the 160 m gap between them will take Curly $160 \div 2 = 80$ s, i.e., Curly will catch up to the rabbit in 80 seconds. By that time, Curly will have run 560 m.

c) Since the forest is 400 metres from where Hoppy starts running, and Curly is 160 metres further away, i.e., 560 metres from the forest, we see that Curly will catch up to Hoppy JUST as Hoppy reaches the forest! So if Hoppy hides quickly, he might just make it!

Time (sec)	Total Distance Run		Gap Between
	Curly	Hoppy	
0	7 m	5 m	160 m
1	14 m	10 m	158 m
2	21 m	15 m	156 m
3	28 m	20 m	154 m
4	35 m	25 m	152 m
5	42 m	30 m	150 m
6	49 m	35 m	148 m
7	56 m	40 m	146 m
8	63 m	45 m	144 m
9	70 m	50 m	142 m
10	77 m	55 m	140 m

Problem 4

a) Each of the sums $20 + 21 = 41$, $21 + 22 = 43$, and $23 + 24 = 47$ is an odd number. This will be true of the sum of any two consecutive numbers. Thus, since 58 is even, it cannot be the sum of only two consecutive numbers.

b) Each of the sums $16 + 17 + 18 = 51$, and $23 + 24 + 25 = 72$ is divisible by 3 and is equal to 3 times the middle number ($51 = 3 \times 17$, and $72 = 3 \times 24$). This is true of any three consecutive numbers. For example, consider $20 + 21 + 22$. Before adding, we can subtract 1 from 22 and add it to 20 without changing the sum:

$$\overset{+1}{20} + 21 + \overset{-1}{22}$$

Thus we see that $20 + 21 + 22 = 21 + 21 + 21 = 3 \times 21 = 63$. Clearly we could do this with any three consecutive numbers. So any number which is the sum of three consecutive number must be divisible by 3.

c) Since 58 is not divisible by 3, it cannot be the sum of three consecutive numbers. (Students may also discover this by systematic trials: $16 + 17 + 18 = 51$, $17 + 18 + 19 = 54$, $18 + 19 + 20 = 57$, $19 + 20 + 21 = 60$, so 58 is 'missing' in the set of sums.)

d) Trying for four consecutive numbers reveals that $58 = 13 + 14 + 15 + 16$.

Extension :

1. Systematic trials of sums of five consecutive whole numbers give: $8 + 9 + 10 + 11 + 12 = 50$, $9 + 10 + 11 + 12 + 13 = 55$, $10 + 11 + 12 + 13 + 14 = 60\ldots$ again, 58 is 'missing'. In fact, the sum of the five consecutive whole numbers is always divisible by 5, as can be the seen from the diagram for the example $10 + 11 + 12 + 13 + 14$:

$$\overset{+2}{10} + \overset{+1}{11} + 12 + \overset{-1}{13} + \overset{-2}{14}$$

Thus we see that $10 + 11 + 12 + 13 + 14 = 12 + 12 + 12 + 12 + 12 = 5 \times 12$, i.e., 5 times the middle number. We could do this with any such sum, so a sum of five consecutive whole numbers must be divisible by 5, which 58 is not.

The sum of six consecutive numbers consists of the sum of two sets of three consecutive numbers, hence must be divisible by 3, as above. So 58 cannot be the sum of six consecutive numbers.

A similar approach shows that the sum of seven consecutive whole numbers equals 7 times the middle number. A similar approach shows that 58 cannot be the sum of eight, nine, or ten whole numbers. And since the smallest sum of 11 whole numbers is $1+2+3+4+5+6+7+8+9+10+11 = 66$, we've run out of possibilities. Thus 58 can only be written as the sum of four consecutive whole numbers, as above.

Problem 5

The clues can be used sequentially to reason the correct sizes and colours of the beads. The steps in the reasoning are explained below, and depicted by numbered entries in the table.

1. Clues 1 and 2 reveal that the larger blue bead is the 1 cm bead; thus we can fill in a check mark for that square in the table, and put X's in all the other squares in the first row and last column, and in the first square of the second row. (These are shown as X$_1$s in the table below.)
2. Clue 4 reveals that the 0.7 cm bead is not white (hence the two blue X2's in the third column). Clue 3 thus implies that neither the 0.5 cm nor the 0.9 cm bead could be white, since those would give a 0.2 cm difference from 0.7 cm, which cannot be white. (These are the remaining X$_2$s in the table.)
3. As a consequence, the white beads must be 0.6 cm and 0.8 cm, which puts a check mark in those positions, and X's in all the remaining 0.6 and 0.8 cm squares. (These are X$_3$s in the table.)
4. Clue 5 implies that the smaller blue bead cannot be 0.9 cm, leaving the only possibility as the 0.7 cm bead. Thus we place a check mark in that square, and X's in the remaining squares for 0.7 cm, and the second row of the 0.9 cm column. (These are X$_4$s in the table.)
5. Finally, the only choices left for the two green beads are 0.5 cm for the smaller green and 0.9 cm for the larger green. (These are X$_5$s in the table.)

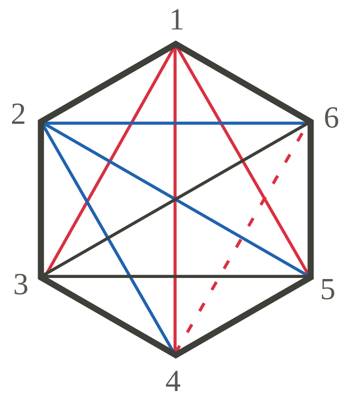
Bead Size (cm) / Colour	0.5	0.6	0.7	0.8	0.9	1.0
Larger Blue	X$_1$	X$_1$	X$_1$	X$_1$	X$_1$	✓ 1
Smaller Blue	X$_1$	X$_3$	✓ 4	X$_3$	X$_4$	X$_1$
Larger Green	X$_5$	X$_3$	X$_4$	X$_3$	✓ 5	X$_1$
Smaller Green	✓ 5	X$_3$	X$_4$	X$_3$	X$_5$	X$_1$
Larger White	X$_2$	X$_3$	X$_2$	✓ 3	X$_2$	X$_1$
Smaller White	X$_2$	✓ 3	X$_2$	X$_3$	X$_2$	X$_1$

Extension :

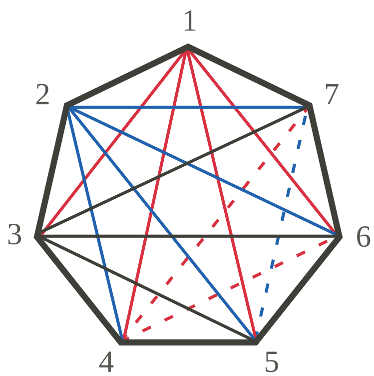
1. If the last clue is not given, then the smaller blue bead could be either 0.7 cm (as in the above solution), or 0.9 cm. In the latter case, the two green beads would be 0.5 cm and 0.7 cm. Thus there are two possible solutions in this case.

Problem 6

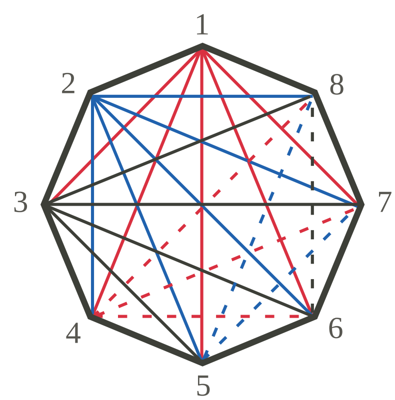
a), b), c) The hexagon has $3+3+2+1 = 9$ diagonals; the septagon has $4+4+3+2+1 = 14$ diagonals; and the octagon has $5+5+4+3+2+1 = 20$ diagonals, as illustrated on the diagrams below. (See also the detailed discussion of the octagon in the Hints and Suggestions for Problem 6 above.)



Hexagon

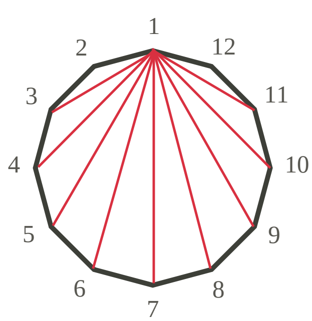


Septagon

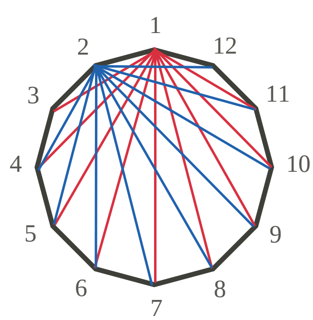


Octagon

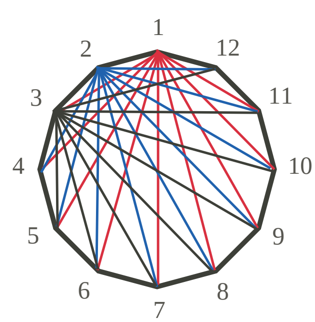
d) Following the pattern established for the octagon, we see that the dodecagon will have $12-3 = 9$ diagonals from each of the first two vertices (red and blue on diagrams below), 8 from vertex 3 (black on diagrams), 7 from vertex 4 (dashed red on last diagram), 6 from vertex 5 (dashed blue on last diagram), 5 from vertex 6 (dashed black on last diagram), etcetera, giving a total of $9+9+8+7+6+5+4+3+2+1 = 54$ distinct diagonals.



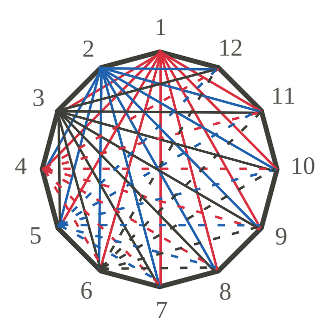
Diagonals from
Vertex 1 (9)



Diagonals from
Vertex 1 and 2 (9+9)



Distinct Diagonals from
Vertex 1, 2 and 3 (9+9+8)



Distinct Diagonals from
Vertices 1, 2, 3, 4, 5, and 6
(9+9+8+7+6+5)

Extension :

1. Since the irregular polygons have the same number of vertices, and the number of diagonals depends only on the number of vertices, not the lengths of the sides, the number of diagonals would be exactly the same for irregular polygons as for regular polygons with the same number of sides.