Emmy Noether - Circle 2 for 2010-2011 Part I: Problems

2010

Problem 1

In 2009, the average (mean) salaries for players in various professional sports were as follows:

- NBA (National Basketball Association) players, \$4.9 million;
- NFL (National Football League) players, \$1.3 million;
- NHL (National Hockey League) players, \$1.8 million; and
- MLB (Major League Baseball) players, \$2.5 million.

Source: Wikipedia.org

a) Consider and complete the information in the following chart.

League	NBA	NFL	NHL	MLB
Average salary (millions of \$)	4.9	1.3	1.8	2.5
Number of Games in a regular season	82	16	82	162
Average salary per regular season game				

- b) Kevin Garnett of the NBA's Boston Celtics made 24.75 million dollars in 2009. In the 71 games in which he participated, he played an average of 32.8 minutes per game. He averaged 9.2 rebounds per game, and scored an average of 18.8 points per game.
 - (i) If paid solely according to the number of minutes he played in a season, how much money did he make per minute played? Per hour?
 - (ii) If paid solely according to how many rebounds he made in a season, how much money did he make per rebound?
 - (iii) If paid solely according to how many points he scored in a season, how much money did he make per point scored?

Problem 2

Sally wanted an allowance. Her Mother offers two options:

- 1. \$10 per week for a year, or
- 2. \$204.80 in January, \$102.40 in February, and so on, halving her allowance each month for the rest of the year.
 - a) Which option would you choose? 1. \(\sqrt{o} \) or 2. \(\sqrt{o} \)
 - b) Determine which option gives the greatest total allowance.
 - c) Could you answer part b) without summing the allowances for the entire year?



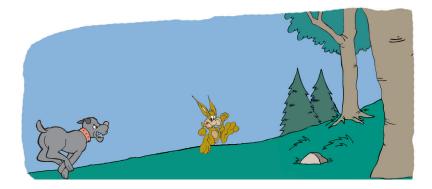
Extension:

1. Suppose, in option 2, Mother tells Sally her March allowance will be \$75, instead of telling her January's and February's. Would your answer to b) change?

Problem 3

Curly the hound is chasing Hoppy, the rabbit. Every second, Curly runs 7 metres, and Hoppy runs 5 metres. Right now, they are 160 metres apart.

- a) Fill in the table at right. How close is Curly to Hoppy after 10 seconds?
- b) How many seconds will it take for Curly to catch the rabbit? How far will Curly have run?
- c) A forest is 400 metres from where the rabbit starts running. Can the rabbit make it to the safety of the forest? Finish the story.



Time (sec)	Total Dist	ance Run Hoppy	Gap Between
0	7 m	5 m	160 m
1	14 m	10 m	158 m
2	21 m	15 m	
3			
4			
5			
6			
7			
8			
9			
10	. 11.		

Problem 4

The number 58 is the sum of some consecutive whole numbers.

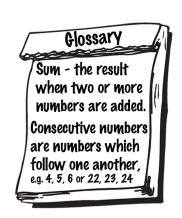
#1	#2	Sum
20	21	
21	22	
23	24	

#1	#2	#3	Sum
16	17	18	
23	24	25	

- a) Add the two numbers in each row of the given table. What do the sums have in common? Could 58 be the sum of **two** consecutive numbers? Why, or why not?
- b) Add the **three** numbers in each row of the second table. What do the sums have in common? What consecutive numbers have a sum of 63?
- c) Could 58 be the sum of three consecutive numbers? Why, or why not?
- d) What are the consecutive numbers which have a sum of 58? (Are there 4 numbers which add to 58? 5 numbers? 6 numbers?)

Extension:

1. Try to find another set of consecutive numbers which have a sum of 58.



Problem 5

Ekaterina is making a bracelet with different coloured beads. In the centre are six special beads, two blue, two green, and two white. The beads are all different sizes: 0.5 cm, 0.6 cm, 0.7 cm, 0.8 cm, 0.9 cm, and 1 cm. Use the following clues to discover the colour of each size of bead.

Bead Size (cm) Colour	0.5	0.6	0.7	0.8	0.9	1.0
Larger Blue						
Smaller Blue						
Larger Green						
Smaller Green						
Larger White						
Smaller White						



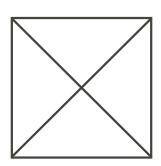
- 1. The smallest bead is not blue.
- 2. The largest bead is blue.
- 3. One of the white beads is 0.2 cm longer than the other white bead.
- 4. The 0.7 cm bead is not white.
- 5. The lengths of the blue beads differ by more than $0.1~\mathrm{cm}$.

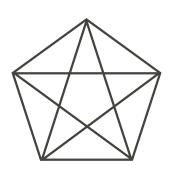
Extension:

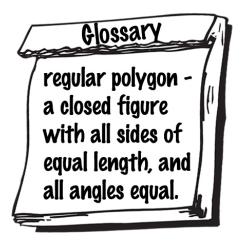
1. If you were given only the first four clues, how many solutions would there be to this problem?

Problem 6: Polygons: How Many Diagonals? (For pairs or groups of students)

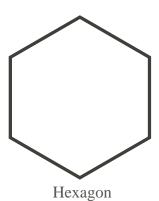
A diagonal of a regular polygon is a straight line joining two vertices which are not adjacent. Thus a square has two diagonals, and a regular pentagon has five, as shown below.

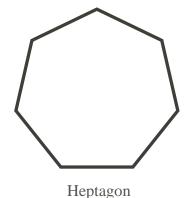






- a) How many diagonals does a regular hexagon have? (Draw them.)
- b) How many diagonals does a regular septagon have? (Draw them.)
- c) How many diagonals does a regular octagon have?







d) Complete the table at right, up to 8 sides, using the information from a), b), c). Then predict the number of diagonals for a regular dodecagon (a 12-sided polygon), using the pattern in the table. Draw the diagonals on the figure below to see whether your prediction is correct.

}

Number of sides	Number of diagonals
3	0
4	
5	
6	
7	
8	
12	

Extension:

1. Suppose the polygons do not have to be regular, i.e., they could have sides of different lengths. Would your answers to the above problems change?