Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Measurement and Number Sense

Problem 3 - Measurement and Problem Solving

Problem 4 - Number Sense and Pattern/Algebra

Problem 5 - Probability and Data Management

Problem 6 - Geometry and Pattern/Algebra

Hints and Suggestions:

Problem 1

Hint 1 - If a prime has an even first digit, what kind of number will you get when you reverse the digits?

Hint 2 - If a prime has 5 as its first digit, will you get a prime number when you reverse the digits?

Suggestion: You may want to have the students use a 100s chart.

Problem 2

Hint 1 - a),b) If a string 50 centimetres (cm) long will wrap around a ball, and your bedroom is 300 cm wide, how many such strings (i.e., circumferences of the ball) could you place end-to-end across your room? What arithmetic is needed to answer this question?

Hint 2 - c),d) How many hours are there in a day? Days in a year?

Problem 3

Hint 1 - a),b),c) Make a sketch.

Hint 2 - d) Would you need two Super Duper pizzas? Would you need three large pizzas?

Hint 3 - d) Does the qustion require that you buy a total of exactly 23 slices of pizza?

Problem 4

Hint 1 - c) What is the least number with the form 6_ _, where the blanks are digits? What is the greatest such number?

Hint 2 - c) For each choice of the pair of digits following the 6, how many choices are there for the last letter in the license plate?

Suggestion: Once students realize there are 100 possibilities $(600, 601, 602, ..., 699)$, point out that they could think of this as a product, (10 choices for the first digit) $\times$ (10 choices for the second digit). Then ask what product is needed to find the total number of license plates.

Extension:

Hint 1 - 1. What is the least number with three digits? The greatest?

Hint 2 - 1. How many letters are there? How many pairs A_, where the blank is a letter? How many pairs B_? How many pairs _ _, where both blanks are letters? How many triples _ _ _, where all three blanks can be filled with any letter?

Hint 3 - 1. For each number with three digits, how many triples of letters are possible to make a full license plate?

Hint 4 - 2. How many choices are there now for the first digit? Which choice will now give the least number with three digits? The greatest?

Hint 5 - 3. For every New Brunswick license plate, there are how many Ontario plates?

Problem 5

Suggestions:

1. Do the first part of part a) with the class, indicating that they will need to delete one potato to make each of the five sets of four potatoes. Then have students assist in filling in the weights of each potato in each set in the table, so that the whole class will be working with the same table. Finally, have them proceed with completing the table by finding totals and averages for each set.

2. Before going on to parts b) and c), remind the class that probability $= \dfrac{\text{number of desired outcomes}}{\text{total number of outcomes}}$.

Problem 6

Suggestions: This activity works best if students work in small groups with some direction from the teacher. Here are some suggestions.

1. For part a), divide students into six small groups, and have each group do the measurements for one triangle. Then collect the data for the whole class to verify that every triangle's angles sum to 180°. If you have already covered this topic in geometry, you may wish to do just one triangle to remind them of this fact.

2. For part b), ask "If you draw a diagonal on the square, what sort of figures appear?"

3. For part c), have half the groups do one method, and half the other, and report to the class.

4. For part d)(i), assign one figure to each group of students, and then have each group report its results to complete the table up to 9 sides. Discuss the pattern and the predictions for the 10-, 11-, and 12-sided polygons.

Solutions

Problem 1

The two digit primes are: 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97. Those with an even tens digit 2, 4, 6, or 8 will be even numbers when the digits are reversed, eliminating 23, 29, 41, 43, 47, 61, 67, 83. Those with tens digit 5 will be divisible by 5 when the digits are reversed, eliminating 53 and 59. This leaves 9 primes which are still prime when the digits are reversed: 11, 13, 17, 31, 37, 71, 73, 79, and 97.

Problem 2

a) If the circumference of the Earth, 40 075 km, is estimated as 4×10^4 km, and the distance to the moon, 384 403 km is estimated as 384×10^3 km, then the number of times you would have to travel around the equator in order to cover the distance from the earth to the moon would be roughly $384\,000 \div 40\,000 = 9.6$, i.e., about ten trips around the equator. (An alternative estimate for the distance to the moon is $384\,403$ km $\approx 4 \times 10^5$ km, which gives the estimate of $400\,000 \div 40\,000 = 10$ trips directly.)
The actual number of times is $384\,403 \div 40\,075 = 9.592089832$, which is pretty close to our first estimate of 9.6 times.

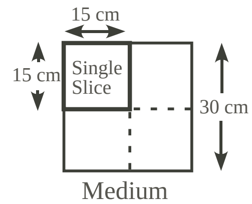
b) Estimating the distance 149 600 000 km from the Earth to the sun as 15×10^7 km, the number of trips around the equator that would be roughly equivalent to the trip to the sun is $150\,000\,000 \div 40\,000 = 3\,750$.
The actual number of trips is $149\,600\,000 \div 40\,075 \approx 3\,733.0006 \approx 3\,733$ times around the Earth.

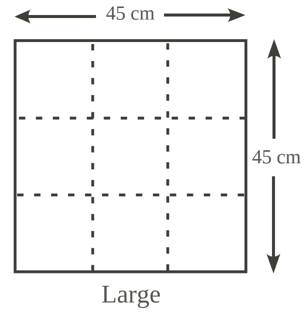
c) Since a Boeing 747 flies at 893 km/hr, the time for the trip from the Earth to the moon would be distance $\div$ speed $= 384\,403 \div 873 \approx 430.462486$ hr.
Since there are 24 hours in one day, the time in days would be $430.462486 \div 24 \approx 17.93593692$ days $\approx$ 18 days.
Since there are 365 days in one year, the time in years would be about $18 \div 365 \approx \dfrac{1}{20}$ of a year.

d) The time for a Boeing 747 trip to the sun would be $149\,600\,000 \div 893 \approx 167\,525.196 \approx 167\,525$ hours which is about 6 980.2 days, or 19.124 years.

e) Since Apollo 13 took about 4 days, or 96 hours to reach the moon, its average speed was the total distance travelled in km divided by the time in hours, or $384\,403 \div 96 \approx 4\,004.1979 \approx 4\,004.2$ km/hr.

Problem 3





a) Since a 'medium' pizza is 30 cm by 30 cm, it will consist of four individual slices, as shown in the diagram. Thus it will feed four people.

b) A 'large' pizza is 45 cm by 45 cm, so it is 3 slices by 3 slices, or 9 slices of pizza. Thus it is 9 times larger than the individual slice.

c) The 'Super Duper' 60 cm by 60 cm pizza is 4 slices by 4 slices, and thus will feed 16 people.

d) You could feed 23 people at least one slice in many ways. Here are a few possibilities:

1. 23 individual slices at \$2 each for \$46;
2. 6 medium pizzas at \$5 for \$30 (i.e., 24 slices, with one extra slice);
3. 5 medium pizzas at \$5 plus 3 individual pieces at \$2 for \$31;
4. 3 large pizzas at \$10 for \$30 (i.e., 27 slices, with four extra slices);
5. 2 large at \$10, 1 medium at \$5, and 1 individual at \$2 for \$27;
6. 1 Super Duper at \$15, 1 medium at \$5, plus 3 individual at \$2 for \$26;
7. 1 Super Duper at \$15, 2 medium at \$5 for \$25 (exactly 23 slices);
8. 1 Super Duper at \$15, 1 large at \$10 for \$25 (i.e., 25 slices, so two extra).

Clearly the last choice is the minimum cost for the greatest value since it gives two extra slices. But either of the last two choices is correct. It is worth exploring with the students how we know there is no cheaper way. The KEY IDEA is that the larger the pizza, the lower the cost per piece. Individual slices cost \$2 each; the pieces in a medium pizza cost $\$5 \div 4 = \$\ 1.25$ each; in a large, they cost $\$10 \div 9 \approx \$\ 1.11$ each; and in a Super Duper, they cost $\$15 \div 16 \approx 94$ cents each. Thus the least expensive solution has to involve the larger pizzas, as in choice 7 or 8 above.

Problem 4

a) Since there are 10 possible digits that could fill the blank between the 6 and the 8, ten cars could be licensed in Rickville.

b) Since there are 26 possible letters that could fill the blank before the W, 26 cars could be licensed in Becville.

c) The two blanks after the 6 could be filled by any of the 100 pairs of digits 00, 01, 02, 03,...,97, 98, or 99. For each of these pairs, the last letter could be any one of the 26 letters A, B, ..., Z. So now $100 \times 26 = 2600$ cars could be licensed in Becville.

Extensions:

1. Both licenses have a triple of digits and a triple of letters. If all ten digits 0, 1,...,9 are permitted, and we know that a pair of digits has 100 possibilities, then a triple of digits could be formed by combining any pair of digits with any of the ten digits, so there would be $100 \times 10 = 1000$ possible triples of digits. We can similarly argue that a pair of letters would have 26 pairs of form A_, 26 of form B_, 26 of form C_,..., 26 of form Z_. Thus there are $26 \times 26 = 676$ pairs of letters, each of which could be followed by any one of the 26 letters to give a triple of letters. Thus there are $676 \times 26 = 17\ 576$ triples of letters. Since each of these could be used in combination with any one of the 1000 triples of digits, a total of $17\ 576 \times 1000 = 17\ 576\ 000$ cars could be licensed in either Quebec or New Brunswick.

2. If the Quebec plate could not start with a 0, then there would only be 9 choices for the first digit, and hence only $100 \times 9 = 900$ possible triples of digits. Thus the total number of cars that could be licensed would be reduced to $17\ 576 \times 900 = 15\ 818\ 400$ cars.

3. Since the fourth letter could be any of the 26 letters, for every plate from Quebec or New Brunswick, there would be 26 Ontario plates. Thus there are 26 times as many possibilities for Ontario plates as for Quebec or New Brunswick.

Problem 5

a) The five possible sets of potatoes are shown in the table below, along with the total weight and average weight of each set. (The order of the potatoes in each set may vary.)

b) Since only 1 of the 5 sets has an average weight of 200 gm, the probability that the four potatoes Sarah selected have an average weight of 200 gm is 1 in 5, or $\frac{1}{5}$.

c) Since 3 sets have an average weight greater than or equal to 200 gm, the probability that the set Sarah selected is one of these is 3 in 5, or $\frac{3}{5}$.

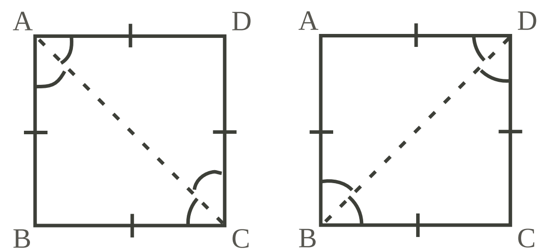
Set	Weights of potatoes Sarah could select				Total Weight	Average Weight
	Potato 1	Potato 2	Potato 3	Potato 4		
1	190	195	200	205	790	197.5
2	195	200	205	210	810	208.5
3	200	205	210	190	805	201.25
4	205	210	190	195	800	200
5	210	190	195	200	795	198.75

Problem 6

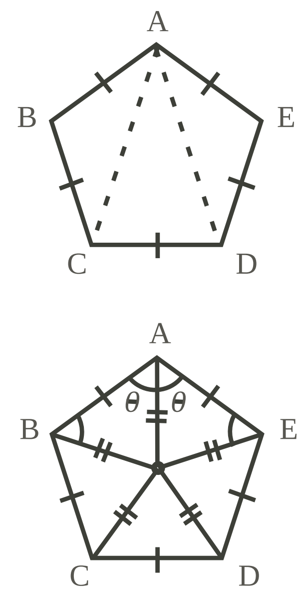
Triangle	Angles A	Angles B	Angles C	Sum of Angles
1	60°	60°	60°	180°
2	83°	63°	34°	180°
3	42°	69°	69°	180°
4	64°	90°	26°	180°
5	65°	65°	50°	180°
6	45°	108°	27°	180°

a) The sum of the angles in each triangle will be 180°, give or take errors in measuring the angles. At right is a table which should roughly coincide with the students' measurements.

b) Divide the square into two congruent triangles, using either diagonal. Then each triangle has angles that sum to 180°, as in part a), giving a total of 360°. *Note:* Since the square is symmetric about both diagonals, and by definition has one right angle, the triangles are congruent. Thus the angles at the vertices are all equal, each being $360° \div 4 = 90°$.

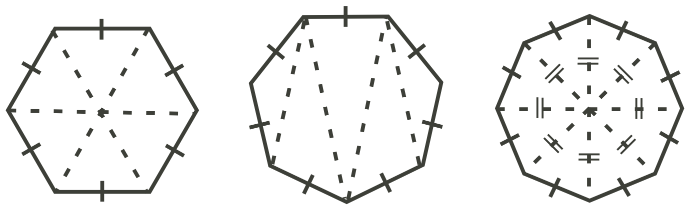


c)(i) In the upper diagram, the pentagon is divided into three triangles, each having angles that sum to 180°. Since $\angle C = \angle BCA + \angle ACD$, $\angle D = \angle CDA + \angle ADE$, and $\angle A = \angle BAC + \angle CAD + \angle DAE$, if we add in $\angle B$ and $\angle E$, we have all the angles in the three triangles. Thus $\angle C + \angle D + \angle A + \angle B + \angle E = 3 \times 180° = 540°$, i.e., the sum of the angles in the pentagon is 540°.
On the other hand, if we use the lower diagram, the pentagon has been divided into 5 identical triangles, each with angles summing to 180°, giving a total of $5 \times 180° = 900°$. But the angles at the centre of the pentagon form a complete revolution, i.e., a 'round' angle which measures 360°. Thus the five vertices must have angles that sum to $900° - 360° = 540°$, as before.



(ii) Since all five triangles are congruent and isosceles, each vertex angle equals 2θ, i.e., all five vertices have equal angles, each being $540° \div 5 = 108°$.

d)(i) For each of the regular polygons, the sum of the angles at the vertices and the size of each angle can be found by subdividing the polygon into triangles in one of the ways shown for the pentagon in part c). Three samples are shown below, and the values for each are given in the chart at right.



Hexagon Heptagon Octagon

(ii) For each increase of 1 in the number of sides of the regular polygons, there is an increase of 180° in the sum of the angles at the vertices. Thus the totals for a 10-sided, 11-sided, and 12-sided polygon are 1440°, 1620°, and 1800°, respectively. The vertices are equal angles in all cases; answers are given in the chart.

No. of Sides	Sum of all angles at vertices	Size of angle at each vertex
3	180°	60°
4	360°	90°
5	540°	108°
6	720°	120°
7	900°	$128\frac{4}{7}°$
8	1080°	135°
9	1260°	140°
10	1440°	144°
11	1620°	$147\frac{3}{11}°$
12	1800°	150°

Extension:

1. For the irregular polygons, we can always use the first of the two methods from part c) to find the total number of degrees in the vertices, as shown below. However, since the sides are no longer of equal length, the vertices will not be equal angles. Thus only the first column of the table remains unchanged.