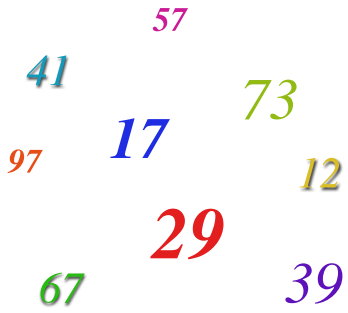
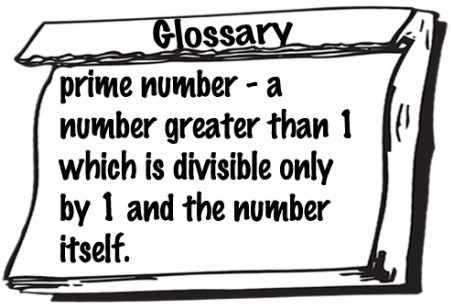
Emmy Noether - Circle 1 for 2010-2011

Part I: Problems

Problem 1

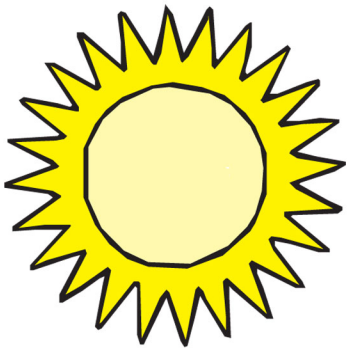


The number 17 is a prime number. When you reverse its digits, you get 71, which is also a prime number. How many two-digit prime numbers are still prime when you reverse their digits?



Problem 2

The circumference of the Earth, (i.e., the distance around the equator) is 40 075 kilometres (km). The distance from the Earth to the Moon is 384 403 km. The distance from the Earth to the Sun is about 149 600 000 km.

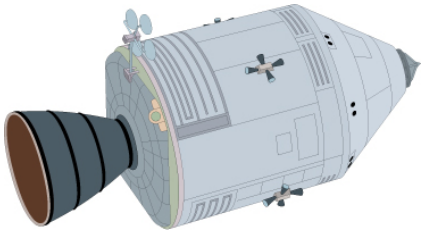


a) Estimate how many times you would have to travel around the equator in order to cover the same distance as from the Earth to the Moon. Then calculate the actual number of times, using the given data. Was your estimate high or low?

b) Repeat part a) for the distance from the Earth to the Sun.

c) A Boeing 747 flies at an average speed of 893 km per hour. If such an aircraft could fly to the Moon, how many hours would it take? How many days? How many years?

d) Repeat part c) for the distance from the Earth to the Sun.



e) Apollo 13 took about 4 days to reach its closest approach to the Moon. What was its average speed (in km per hour) for this part of its journey?

Problem 3

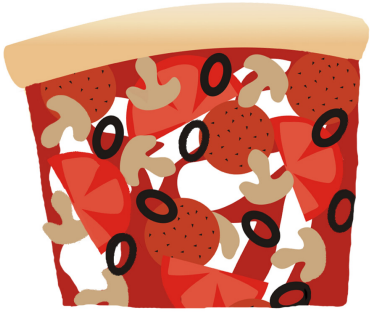
At "Wanna Piece 'a Pizza", they sell only square pizzas. A 15 cm by 15 cm pizza (an individual slice) feeds one person.

a) If each person eats just one individual slice, how many people would a 'medium' 30 cm by 30 cm pizza feed?

b) How many times bigger is the 'large' 45 cm by 45 cm pizza than the individual slice?

c) How many people could be fed with the 'Super Duper' 60 cm by 60 cm pizza?

d) Suppose an individual slice costs \$2.00, a medium \$5.00, a large \$10.00, and a super \$15.00. If you wish to feed 23 people at least one slice each, what pizzas should you buy in order to minimize the cost?



Problem 4

a) All the license plates in the hamlet of Rickville have the form 6__8 QWL, where the blank is a single digit. How many cars can be licensed in Rickville with the pattern 6__8 QWL?

b) All the license plates in Becville have the form 648 __WL, where the blank is a letter from the alphabet. How many cars can be licensed in Becville with the pattern 648 __WL?

c) Suppose, instead, the license plates in Becville can have the form 6_ _ QW_. Now how many cars can be licensed in Becville?

***Extensions* :**

1. Below are pictures of typical license plates in Quebec and New Brunswick. If the digits can be any of 0,1,...,9 and the letters can be any of A,B,C,...Y,Z, how many cars can be licensed using either of these patterns of numbers and letters?

2. How would your answer to Extension 1. change for Quebec if the digit 0 is not permitted as the first of the three digits?

3. On the right is a picture of a typical Ontario license plate. How does the number of cars that could be licensed in Ontario compare to the number that could be licensed in Quebec or New Brunswick?

Problem 5

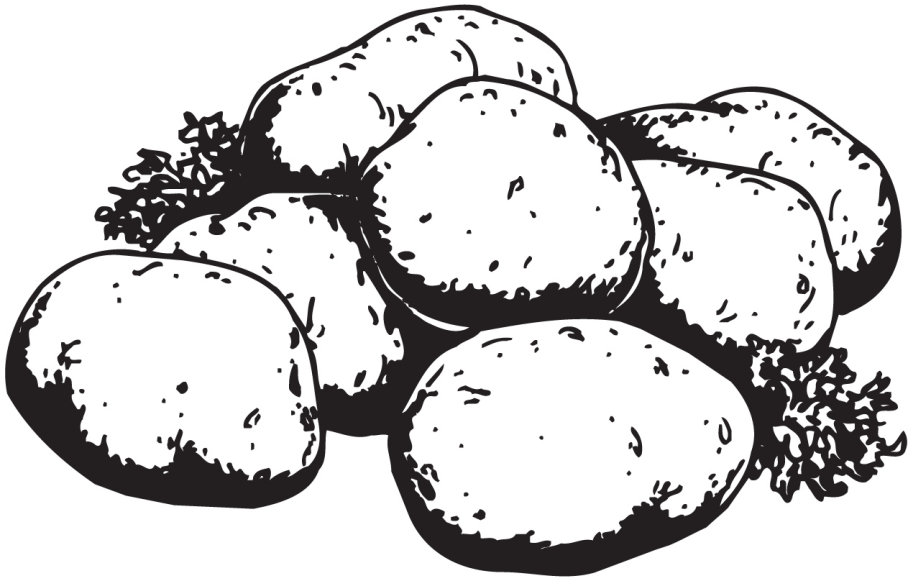
Sarah buys a 1 kilogram (kg) bag of potatoes at the market. There are five potatoes in the bag, with individual weights of 190 grams (g), 195 g, 200 g, 205 g, 210 g. She reaches into the bag and selects 4 potatoes without looking (i.e., at random) to cook for dinner.

a) Use the table to list the weights of the four potatoes in each of the five possible sets of potatoes Sarah could select. Then calculate the total weight and the average weight of each set.

b) What is the probability that the average (mean) weight of the four potatoes she selects is exactly 200 g?

c) What is the probability that the average (mean) weight of the four potatoes she selects is at least 200 g?

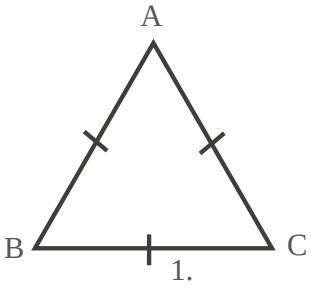
Set	Weights of potatoes Sarah could select				Total Weight	Average Weight
	Potato 1	Potato 2	Potato 3	Potato 4		
1						
2						
3						
4						
5						

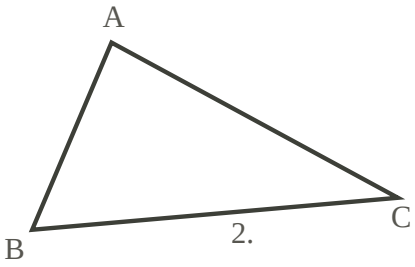


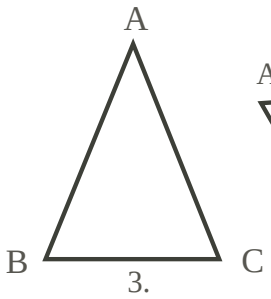
Problem 6: Polygons: How Many Degrees per Vertex? (For pairs or groups of students)

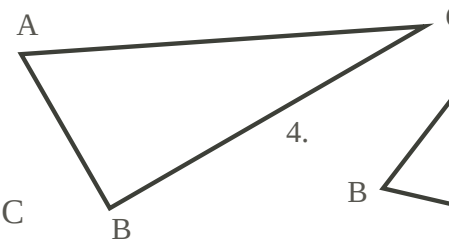
a) Below are several triangles. For each triangle, measure the angles at each vertex A, B, C, find their sum, and enter your results in the table at right. What conclusion do your observations seem to show about the sum of the angles in a triangle?

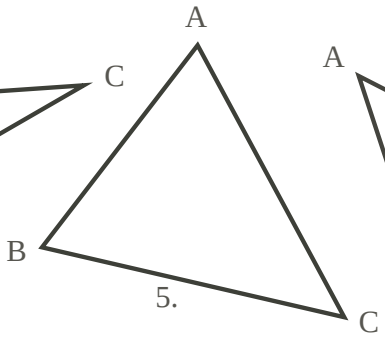
Triangle	Angles			Sum of Angles
	A	B	C	
1	60°	60°	60°	
2				
3				
4				
5				
6				

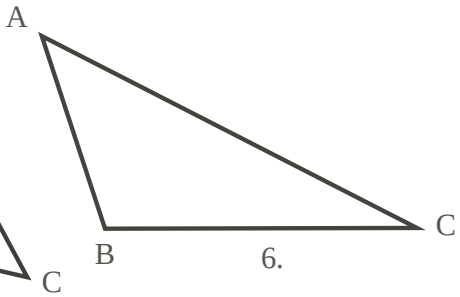




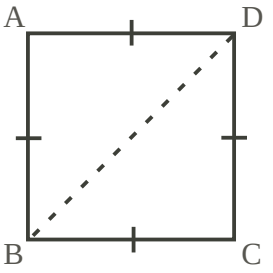




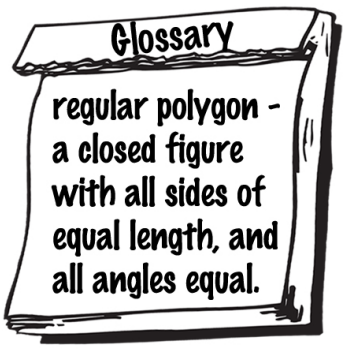


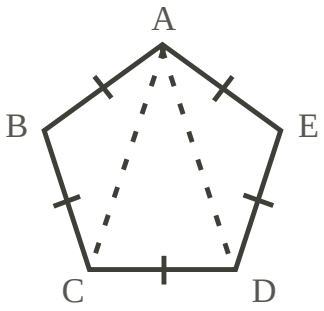


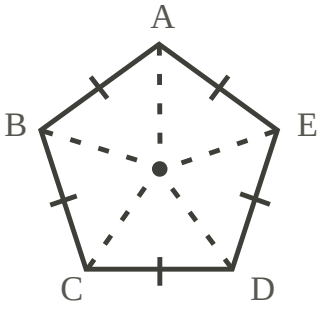
b) For the square at right, use your result from part a) to show that the sum of the angles at the vertices A, B, C, D equals 360°.



c) (i) The diagrams below suggest two ways you could find the sum of the angles in a regular pentagon. Show that each method leads to the same result. (Recall that a complete rotation, (i.e., a 'round' angle whose two sides coincide), measures 360°, so the sum of the angles at the centre of the diagram on the right is 360°.)

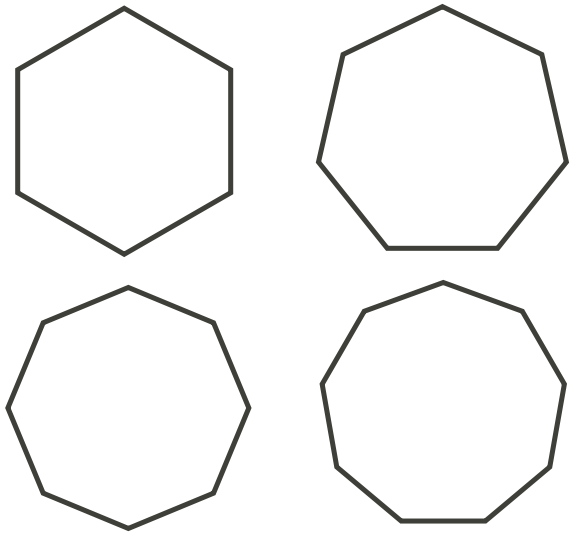






(ii) What is the number of degrees in the angle at each vertex? How do you know?

d) (i) Below are several more regular polygons. Find the sum of the angles at all vertices, and the size of the angle at each vertex. Write your results in the chart at right for 6, 7, 8, and 9 sides.



No. of Sides	Sum of all angles at vertices	Size of angle at each vertex
3	180°	60°
4	360°	90°
5		
6		
7		
8		
9		
10		
11		
12		

(ii) Look for a pattern in your chart. Then use it to predict the results for a 10-sided, 11-sided, and 12-sided regular polygon.

***Extension* :**

1. Suppose the polygons in a) to e) above were not regular, i.e., they could have unequal sides. Which of your two columns of the results would remain unchanged? Experiment with the three non-regular polygons below.