Problem

A 'Lorna' number has 3 digits, and the tens (middle) digit equals the hundreds (left) digit minus the units or ones (right) digit. For example, 752 is a 'Lorna' number, since $5 = 7 - 2$ (which could also be written $7 = 5 + 2$).

a) If the hundreds digit is a 3, what are the possible 'Lorna' numbers?

b) If the hundreds digit is a 5, what are the possible 'Lorna' numbers?

c) What is the least (smallest) possible 'Lorna' number? What is the greatest 'Lorna' number?

d) How many 'Lorna' numbers are there in total?

Extension **:**

1. A 'Dennis' number also has 3 digits, but the tens digit is the units digit minus the hundreds digit. Is there the same number of 'Dennis' numbers as 'Lorna' numbers? Explain your answer.

Hints

Suggestion: Before beginning the problem, discuss with the class whether numbers with hundreds digit 0 are to be included (e.g., 077). The solutions below have assumed they are NOT allowed.

Part a)

Hint 1 - What pairs of digits have a sum of 3?

Part b)

Hint 1 - What pairs of digits have a sum of 5?

Part c)

Hint 1 - What will be the hundreds digit of the least 'Lorna' number? Of the greatest?

Part d)

Hint 1 - How many 'Lorna' numbers have 1 as the hundreds digit? How many have 2 as the hundreds digit? How many have 3? Can you see a pattern?

Suggestion: For a more challenging version of this problem, instead of starting with the definition of a 'Lorna' number, pose this initial question:

These are 'Lorna' numbers: 202, 312 440, 523, 514, 752.
These are NOT 'Lorna' numbers: 222, 311, 443, 521, 732, 908.
Write the definition of a 'Lorna' number.

Have students procede with parts a), b), c), d) as given.

***Extension* :**

Suggestion: Ask students the same questions as suggested in the Hints for 'Lorna' numbers. Do they have the same answers for 'Dennis' numbers?

Solution

a) Possible Lorna numbers with hundreds digit equal to 3 are: 303, 312, 321, and 330.

b) Possible Lorna numbers with hundreds digit equal to 5 are: 505, 514, 523, 532, 541 and 550.

c) The least possible Lorna number is 101; the greatest is 990.

d) Using a chart to record all possible 'Lorna' numbers reveals a pattern:

Hundreds Digit	Possible 'Lorna' Numbers	Number of 'L' Numbers
1	101, 110	2
2	202, 220, 211	3
3	303, 312, 321, 330	4
4	404, 413, 422, 431, 440	5
...	...	...
9	909, 918, 927, 936, 945, 954, 963, 972, 981, 990	10

Thus the total number of 'Lorna' numbers is 2+3+4+5+6+7+8+9+10=54.
Clearly, for each hundreds digit H there are H+1 'Lorna' numbers H T U with T = H - U, or H = T + U, giving possible values T = 0,1,2,...,H while U = H, H-1, H-2,...,0. For example, for H = 7, the 8 'Lorna' numbers are 707, 716, 725, 734, 743, 752, 761, 770.

***Extension* :**

A chart recording all possible 'Dennis' numbers also reveals a pattern:

Units Digit	Possible 'Dennis' Numbers	Number of 'Dennis' Numbers
1	101	1
2	112, 202	2
3	123, 213, 303	3
4	134, 224, 314, 404	4
...	...	...
9	189, 279, 369, 459, 549, 639, 729, 819, 909	9

Clearly, for each units digit U there are exactly U 'Dennis' numbers H T U, with T = U - H, or U = H + T, giving possible values H = 1,2,3,...,U while T = U-1, U-2,...,1,0. (Note that we have not permitted H = 0, i.e., 022 is not allowed, even though T=2, U=2, H=0 gives T=U-U.) For example, if U = 7, the 7 'Dennis' numbers are 167, 257, 347, 437, 527, 617, 707.
Thus the total number of 'Dennis' numbers is 1+2+3+4+5+6+7+8+9=45, so it is NOT the same as the number of 'Lorna' numbers.