## Problem

OLYMPIC BEACH VOLLEYBALL RULES:

1. A game must be won by at least 2 points.
2. A match has 3 games. To win the match, a team must win 2 of the 3 games.
3. The first two games are played to at least 21 points.
4. Game 3 in the match is played to at least 15 points.


Note: If the game is close (e.g., 21-20) then the winner will have to score more than 21 points (e.g., the final score could be $22-20$, or $23-21$, or $30-28$, etc). Similarly in Game 3, scores may be above 15 .

For this problem, you may assume all games were won at either 15 or 21 points.
a) What would be the total difference in points between the two teams in the closest possible match outcome?
b) Assuming a match goes to 3 games, what is the maximum number of points the losing team could score?
c) If one team scores 7 points for every 2 points scored in each game by the other team, how many points will be scored by each team in the match?
d) What is the minimum number of points a team must score to win a match?


|  | Team A | Team B |
| :--- | :--- | :--- |
| Game 1 |  |  |
| Game 2 |  |  |
| Game 3 |  |  |

## Hints

Hint 1 - How many games would be played in the closest possible match?
Hint 2 - What would be the final score in the first game of the closest possible match? And in the second game?

Hint 3 - How many games will be played if one team scores 5 points for every 2 points of the other team?

Hint 4 - Would the winning team win all three games if they score the minimum number of points in total?

Suggestion: Students may find it helpful to make a chart for each match, with two columns, one for each of the two teams, and three rows, one for each game.

|  | Team A | Team B |
| :--- | :--- | :--- |
| Game 1 |  |  |
| Game 2 |  |  |
| Game 3 |  |  |

## Solution

a) Think of the two teams as Team A and Team B. The closest possible match won by Team A would occur with Team A winning one of the first two games with a score of 21 to 19, Team B winning the other with a score of 21 to 19 , and then Team A winning the final game 15 to 13 . Thus the total point difference for the match would be 2 points.
b) From a), the closest possible match would give the losers, Team B, a total of $19+21+13=53$ points.
c) The match will be won in 2 games, in which case the winning team would have $21+21=42$ points. If the losers only score 2 points in each game for every 7 points scored by the winners, then they would have scored 6 points in each game, giving a total of 12 points.
d) The minimum number of points Team A could score to win a match would be if they win the first game (21 points), score no points in the second game, and then win the third game (15 points). Thus is would take at least 36 points in total.

