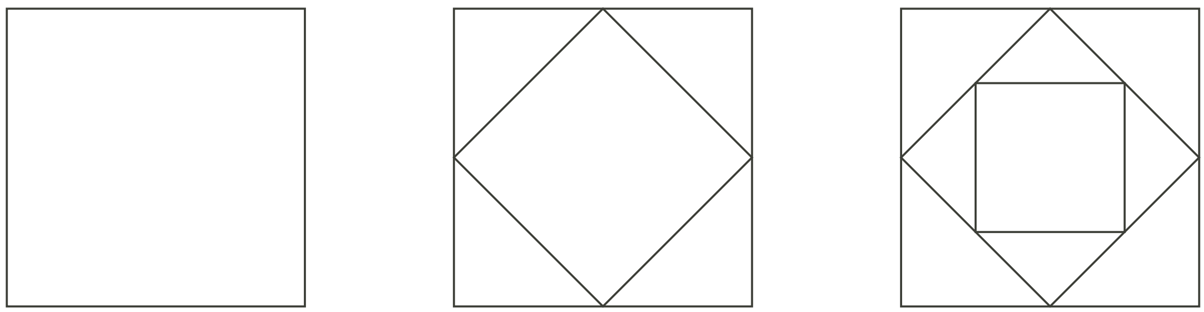
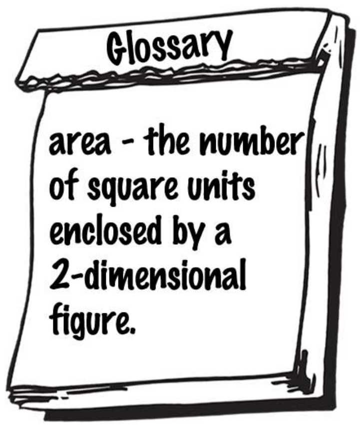
Problem

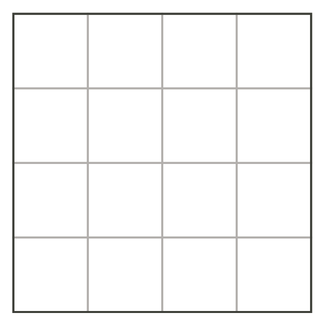
a) Count the number of non-overlapping sections in each figure. Following this pattern, how many figures would you have to draw to have 41 sections?



b) In the second figure, each of the triangles has area 1/8 of the larger (outer) square. In the third figure, there are two sizes of triangles. What fraction of the larger (outermost) square is the area of each of the smaller triangles?

c) Draw the next figure in the fourth square below. What fraction of the largest (outermost) square is the area of the smallest triangle?





d) For what figure will the fraction of the area of the smallest triangle to the whole square be 1/1024?

Hints

Problem 5 a)

Hint 1 - How many new sections in the next figure are formed within the middle square of the previous figure?

Part 5 b)

Hint 1 - How many of the smaller triangles in the third Figure would fit into one of the larger triangles?

Part 5 c)

Hint 1 - If each square on the grid is taken to have side length 1, and hence 1 square unit of area, what is the area of the whole square?

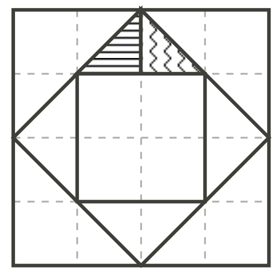
Hint 2 - What is the area of one of the smallest triangles?

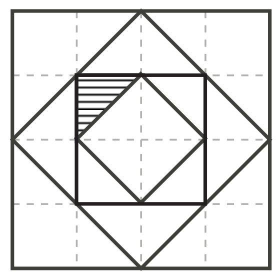
Suggestion: A table may be helpful here, with rows for the number of non-overlapping sections, and the area of the smallest triangle, for each figure.

Solution

a) Figure 1 has 1 section; figure 2 has 5 sections; figure 3 has 9 sections. In each case, the innermost square of the previous figure gets divided into a smaller square and four triangles, increasing the number of non-overlapping sections by 4. Thus the pattern of non-overlapping sections is given by the sequence 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41,..., i.e., the 11^{th} figure will have 41 non-overlapping sections.

b) If we think of each square of the light grey grid in the diagram at the right as being one square unit of area, then the outside (largest) square has area $4 \times 4 = 16$ square units. The shading shows that each of the smaller triangles in the third figure consists of 2 halves of one square unit, i.e., has area 1 square unit in total, which is $\frac{1}{16}$ of the whole (largest) square.





c) The fourth figure is shown at right. Each of the smallest triangles (one is shaded) is $\frac{1}{2}$ square unit in area, which is $\frac{1}{32}$ of the whole area.

d) Since the area of each smallest triangle is halved with the creation of each new figure, the pattern of areas of the smallest triangles as a fraction of the whole (outer) square is

$$0, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024} \ldots .$$

Thus the 9^{th} figure will have smallest triangles of area $\frac{1}{1024}$ of the whole (outer) square.

Below is a table showing all the above results.

	Figure 1	2	3	4	5	6	7	8	9	10	11
# of sections	1	5	9	13	17	21	25	29	33	37	41
smallest triangle as a fraction of the whole	0	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1024}$	$\frac{1}{2048}$	$\frac{1}{4096}$