Problem

Problem 6: Plasticine Geometry (Suggested for pairs or groups of students)

For this investigation, you will need some plasticine and a piece of fishing line (or thin straight wire, such as a cheese cutter, or a straightened paper clip) for each student.
First, use plasticine to make one of each of the following three-dimensional solid figures:

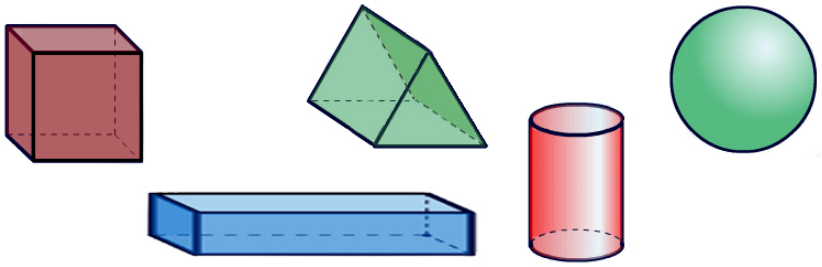
a) a cube;

b) a rectangular prism;

c) a sphere;

d) a cylinder;

e) a triangular prism.



Now, follow the instructions below:

1. Imagine slicing each solid into two pieces with a thin cutting tool. While slicing, your tool can be held at an angle, but it must move in a straight line from one face or side of the solid to another face or side. The cut face on each piece is called a *cross-section*, or a *section* of the solid.

2. Predict what two-dimensional figures the cross sections will be, **before** you cut; enter your guesses in the table below for each three-dimensional solid figure.

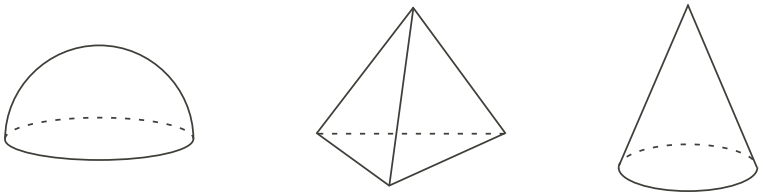
3. Then, using the fishing line like a thin knife, carefully make each of the cuts you thought about, and describe the actual resulting cross section in the appropriate row of the table.

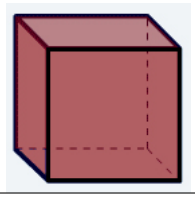
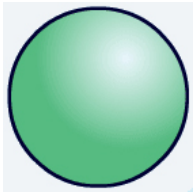
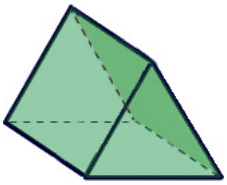
How many different cross sections can you find for each solid?

Use the table on the following page to show your answers.

***Extension* :**

1. Repeat the suggestions above for a hemisphere (half a sphere), for a triangular-based pyramid, and for a cone.



Solid	Guesses for Cross Sections	Actual Cross Sections	# of Cross Sections
1. Cube 			
2. Rectangular Prism 			
3. Sphere 			
4. Cylinder 			
5. Triangular Prism 			

Hints

Part 6 a)

Hint 1 - Through how many sides of the cube should you slice in order for the cross section to be a triangle?

Hint 2 - Are there four-sided cross sections which are NOT rectangular?

Hint 3 - Is it possible to slice through five sides of the cube with a 'flat' cut? Six sides?

Part 6 b)

Hint 1 - Are there cross-sections of the rectangular prism which are the same type of polygon as cross-sections of the cube? Are there any that are different?

Parts 6 c),d)

Hint 1 - Could a sphere (or a cylinder) have a cross section that is a polygon?

Part 6 e)

Hint 1 - How could you slice the triangular prism so that you cut through four sides of the prism?

Hint 2 - Is there a way to 'tilt' your slice so that it cuts through all five sides of the prism?

***Extension* :**

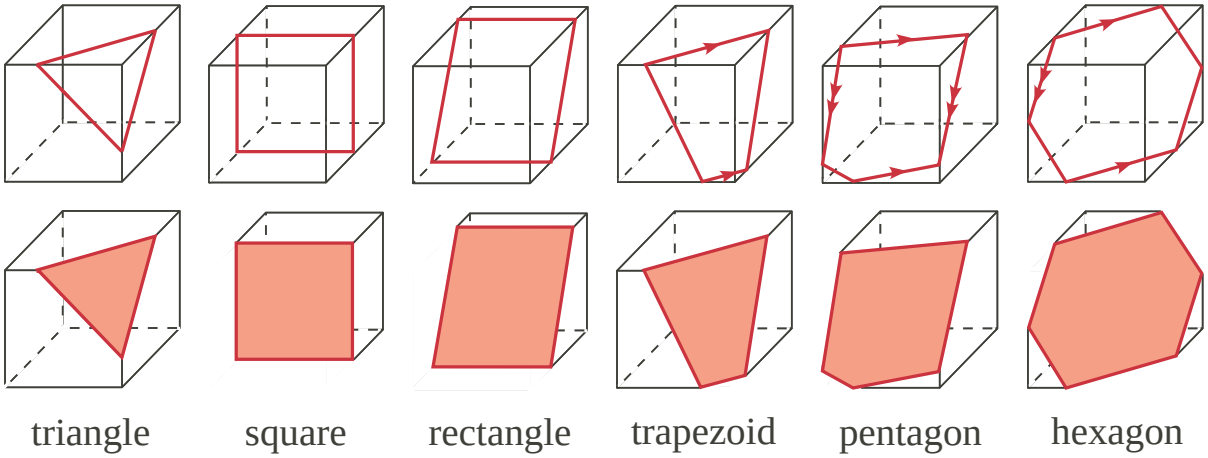
Hint 1 - Are there any cross sections of the hemisphere which are NOT circles?

Hint 2 - Can you slice the triangular pyramid in a way that cuts through all four sides?

Hint 3 - Are there cross sections of the cone which are polygons?

Solution

a) Cross sections of a cube (there are other ways to get such cross sections):



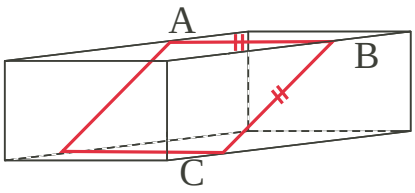
Suggestions:

1. Have students show the class their cross sections, and create a list on the blackboard. This is an excellent opportunity for reviewing polygons. You may wish to discuss with the class how moving, tilting and/or turning the slice changes the polygonal cross section.

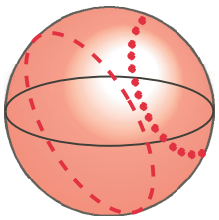
2. Possible questions for a class discussion:

   - Could you have a cross section with more than six sides?
   - How could you get an equilateral triangle as a cross section?
   - Could you have a regular hexagon as a cross section?
   - Could you have a regular pentagon as a cross section?

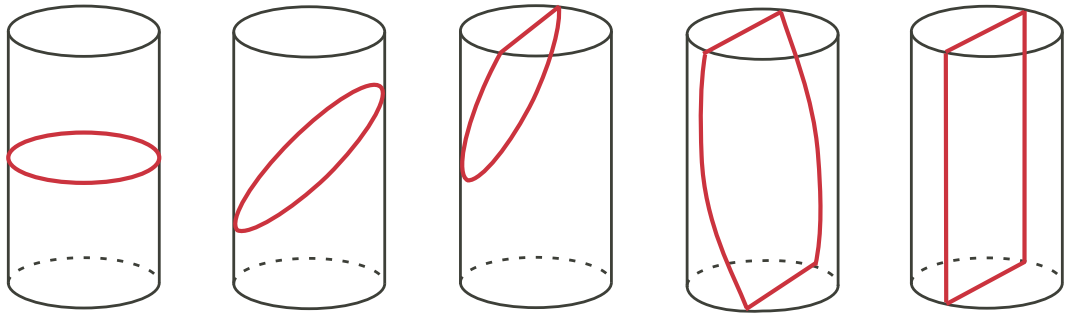
b) Cross sections of a rectangular prism are of the same types as for a cube. Note that, to obtain a square cross section, use a slice at an angle such that side BC equals side AB in length. This assumes the prism is long enough to permit this.



c) All cross sections of a sphere are circles. Shown at right are a diametral cross section **(black)**, and two non-diametral cross sections **(dotted and dashed)**.

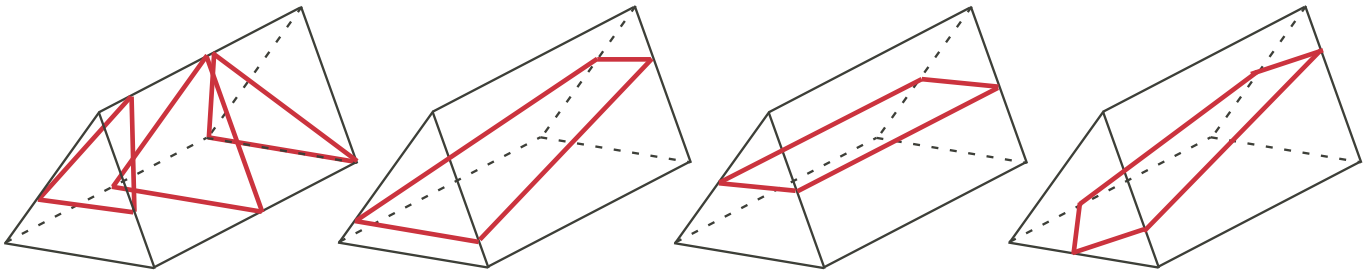


d) Cross sections of a cylinder:



Circle | oval (ellipse) | truncated oval | oval truncated at both ends | Rectangle

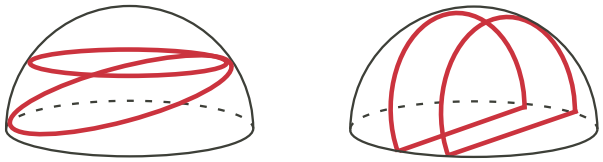
e) Cross sections of a triangular prism:



triangle | trapezoid | rectangle | pentagon

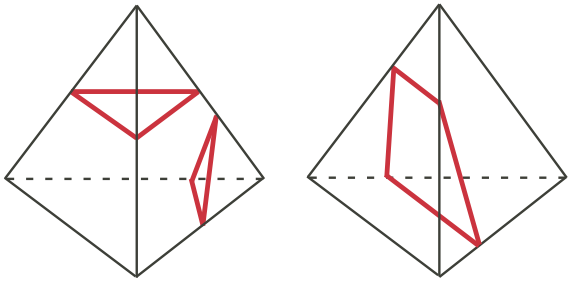
***Extension* :**

1. Cross sections of a hemisphere.



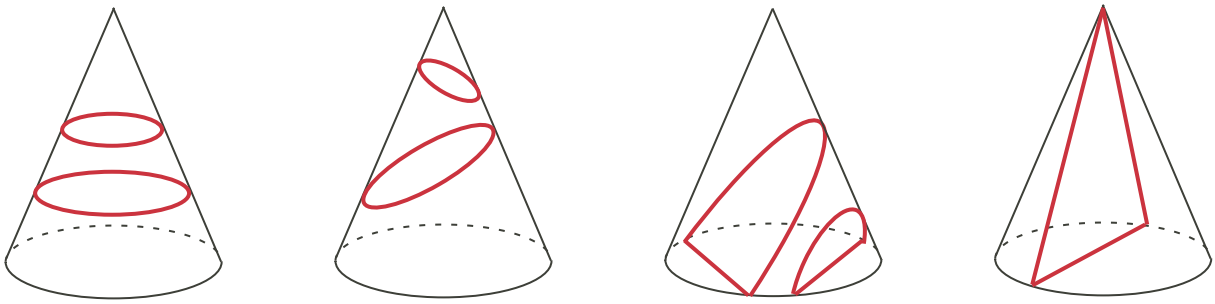
circle | truncated circle

2. Cross sections of a triangular-based pyramid:



triangle | trapezoid

3. Cross sections of a cone:



circle | oval (ellipse) | truncated ovals | triangle

Comment: An excellent web reference showing cross sections of solids is http://www.learner.org/courses/learningmath/geometry/session9/