Part II: For the Teacher

Curriculum Areas

Problem 1 - Pattern/Problem Solving

Problem 2 - Number Sense

Problem 3 - Measurement, Data Management

Problem 4 - Logic/Problem Solving

Problem 5 - Pattern/Algebra and Measurement

Problem 6 - Geometry

Hints and Suggestions:

Problem 1 a)

Hint 1 - If you use the colours red, black, and white, and you colour the top circle red, what colours must be used for the circles in the row below?

Hint 2 - Does choosing the colours for the top three circles determine exactly the colours for the three circles in the third row down?

Suggestion: The teacher may wish to make copies of the problem page (with the diagram) for students to use in trying this problem. Pencil (with eraser) is best!

Problem 2

Hint 1 - Is it possible to write 9 (or 99, or 998) as a sum or product of numbers other than 9?

Extension **:**

Hint 1 - If you cannot use multiplication, what other operation could substitute?

Hint 2 - By what number(s) can you multiply easily without a calculator?

Problem 3

Hint 1 - How long will it take for 135 litres of water to leak into the boat?

Hint 2 - How long would it take Bailey to reach the shore, with the boat travelling 2 km per hour?

Problem 4

Suggestion: Before students begin this problem, have them make the table as suggested in the problem.

Hint 1 - If more than one witness says the dog is fluffy, can the dog be fluffy?

Hint 2 - What does your table tell you immediately about the dog's tail?

***Extension* :**

Hint 1 - Could you solve this problem if all four people's descriptions were completely different from each other, according to the given rules?

Problem 5 a)

Hint 1 - How many new sections in the next figure are formed within the middle square of the previous figure?

Problem 5 b)

Hint 1 - How many of the smaller triangles in the third Figure would fit into one of the larger triangles?

Problem 5 c)

Hint 1 - If each square on the grid is taken to have side length 1, and hence 1 square unit of area, what is the area of the whole square?

Hint 2 - What is the area of one of the smallest triangles?

Suggestion: A table may be helpful here, with rows for the number of non-overlapping sections, and the area of the smallest triangle, for each figure.

Problem 6 a)

Hint 1 - Through how many sides of the cube should you slice in order for the cross section to be a triangle?

Hint 2 - Are there four-sided cross sections which are NOT rectangular?

Hint 3 - Is it possible to slice through five sides of the cube with a 'flat' cut? Six sides?

Problem 6 b)

Hint 1 - Are there cross-sections of the rectangular prism which are the same type of polygon as cross-sections of the cube? Are there any that are different?

Problem 6 c),d)

Hint 1 - Could a sphere (or a cylinder) have a cross section that is a polygon?

Problem 6 e)

Hint 1 - How could you slice the triangular prism so that you cut through four sides of the prism?

Hint 2 - Is there a way to 'tilt' your slice so that it cuts through all five sides of the prism?

***Extension* :**

Hint 1 - Are there any cross sections of the hemisphere which are NOT circles?

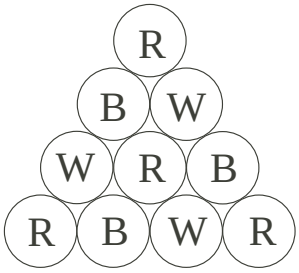
Hint 2 - Can you slice the triangular pyramid in a way that cuts through all four sides?

Hint 3 - Are there cross sections of the cone which are polygons?

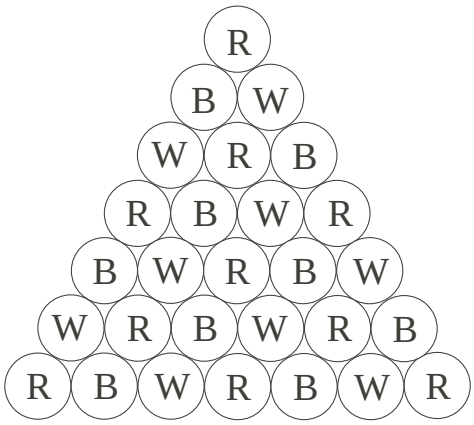
Solutions

Problem 1

a) Using R = red, B = black, W = white as the choice of colours, one solution is shown at the right. Others can be found by interchanging the colours (e.g., one way would be to replace R with B, B with W, and W with R).



b), c), d) Using the diagram at the right, the completed table is:



No. of rows	No. of Circles	No. of Colour R	No. of Colour B	No. of Colour W
2	3	1	1	1
3	6	2	2	2
4	10	4	3	3
5	15	5	5	5
6	21	7	7	7
7	28	10	9	9

Thus we see that there will be the same number of circles of each colour provided that the total number of circles is divisible by 3. The next time this will NOT occur is when there are 10 rows, with a total of 28+8+9+10=55 circles.

Problem 2

The answers will vary a lot. Here are some simple possibilities.

a) $9 \times 23 = 3 \times 3 \times 23$, or $23 \times (2 + 7)$, or $23 \times (5 + 4)$

b) $6 \times 99 = 6 \times 3 \times 3 \times 11$, or $6 \times (88 + 11)$, or $2 \times 27 \times 11$

c) $11 \times 998 = 11 \times (610 + 388)$, or $11 \times (120 + 878)$, or $11 \times 2 \times (321 + 178)$

d) $9 \times 750 = 3 \times 3 \times 3 \times 250$, or 270×25, or $6 \times 3 \times 375$

Extension **:**

Without the $\times$ sign, we must use +, -, or $\div$. Here are some possible answers.

a) $9 \times 23 = 23 + (8 \times 23) = 23 + 4 \times (2 \times 23) = 23 + 46 + 46 + 46 + 46$,
or $(10 - 1) \times 23 = 230 - 23$ (assuming $\times 10$ and $\times 1$ can be done without a calculator)

b) $6 \times 99 = 6 \times (100 - 1) = 600 - 6$

c) $11 \times 998 = 11 \times (1000 - 2) = 11000 - 22$

d) $9 \times 750 = (10 - 1) \times 750 = 7500 - 750$

Problem 3

Completing the table as suggested, with time equal to 0 when Bailey hits the log, the boat's speed at 2 kilometres per hour, water leaking in at $3 \times 60 = 180$ litres per hour, and out at $1.5 \times 60 = 90$ litres per hour, we have:

Time (in hours)	Distance (km) $2 \times$ time	Water In (litres) $= 180 \times$ time	Water Out (litres) $= 90 \times$ time	Accumulated Water in Boat
$\frac{1}{2}$	1	90	45	45
1	2	180	90	90
$1\frac{1}{2}$	3	270	135	135
2	4	360	180	180

Thus we conclude:

a) Since there are 135 litres of water in the boat when Bailey has travelled only 3 km, the boat will sink before Bailey reaches the shore.

b) The boat sinks when Bailey is 4 - 3 = 1 km from shore.

c) If Bailey is only 3 km from shore when she hits the log, then she will just make it to shore as the boat sinks .. hopefully in shallow water!

Problem 4

The completed table of attributes gives:

Child's Name	Colour	Hair Type	Collar Colour	Tail Length
Dan	white [4]	fluffy	red	long
Karen	black	short [3]	red	long
Max	brown	long silky	blue [2]	long
Emma	spotted	fluffy	red	short [1]

Reasoning in the order indicated by the numbered items in the table, we can see immediately that the correct tail length is short (Emma's answer is unique), and the correct collar colour is blue (Max's answer is unique). Thus Karen must be right about the hair type, short, since Max and Emma can only be correct about one item each, and Dan gave the same answer as Emma for hair type. So Dan must be correct about colour, since each witness is correct about one attribute.
Thus the guilty pooch is white with short hair, a blue collar, and a short tail.

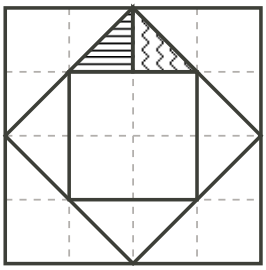
Extension **:** Answers will vary.
Suggestion: Have students post their lists of four descriptions, and discuss with the class what reasoning will lead to the answer in each case. If a given set of descriptions does not determine a unique

answer, have members of the class try to explain why.

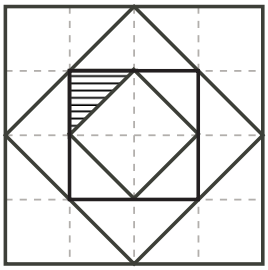
Problem 5

a) Figure 1 has 1 section; figure 2 has 5 sections; figure 3 has 9 sections. In each case, the innermost square of the previous figure gets divided into a smaller square and four triangles, increasing the number of non-overlapping sections by 4. Thus the pattern of non-overlapping sections is given by the sequence 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41,..., i.e., the 11^{th} figure will have 41 non-overlapping sections.

b) If we think of each square of the light grey grid in the diagram at the right as being one square unit of area, then the outside (largest) square has area $4 \times 4 = 16$ square units. The shading shows that each of the smaller triangles in the third figure consists of 2 halves of one square unit, i.e., has area 1 square unit in total, which is $\frac{1}{16}$ of the whole (largest) square.



c) The fourth figure is shown at right. Each of the smallest triangles (one is shaded) is $\frac{1}{2}$ square unit in area, which is $\frac{1}{32}$ of the whole area.



d) Since the area of each smallest triangle is halved with the creation of each new figure, the pattern of areas of the smallest triangles as a fraction of the whole (outer) square is

$$0, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024} \ldots .$$

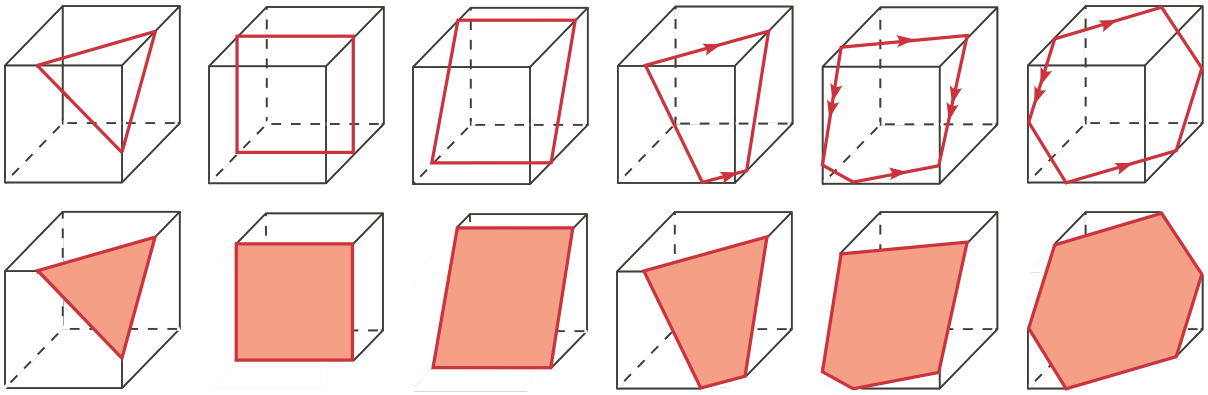
Thus the 9^{th} figure will have smallest triangles of area $\frac{1}{1024}$ of the whole (outer) square.

Below is a table showing all the above results.

	Figure 1	2	3	4	5	6	7	8	9	10	11
# of sections	1	5	9	13	17	21	25	29	33	37	41
smallest triangle as a fraction of the whole	0	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1024}$	$\frac{1}{2048}$	$\frac{1}{4096}$

Problem 6

a) Cross sections of a cube (there are other ways to get such cross sections):



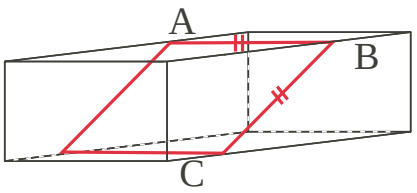
Suggestions:

1. Have students show the class their cross sections, and create a list on the blackboard. This is an excellent opportunity for reviewing polygons. You may wish to discuss with the class how moving, tilting and/or turning the slice changes the polygonal cross section.

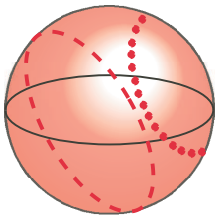
2. Possible questions for a class discussion:

   - Could you have a cross section with more than six sides?
   - How could you get an equilateral triangle as a cross section?
   - Could you have a regular hexagon as a cross section?
   - Could you have a regular pentagon as a cross section?

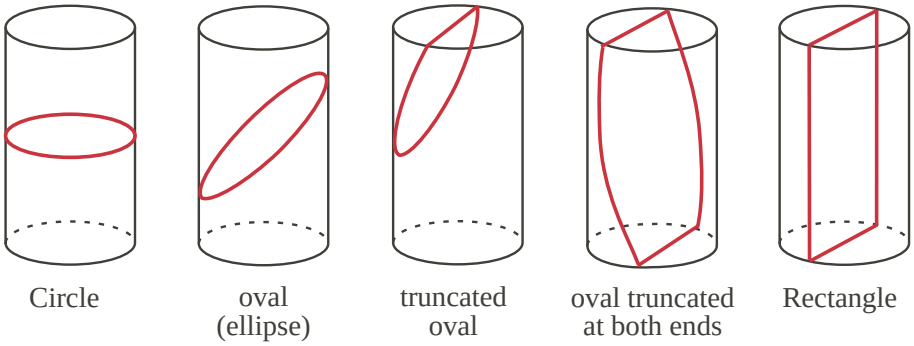
b) Cross sections of a rectangular prism are of the same types as for a cube. Note that, to obtain a square cross section, use a slice at an angle such that side BC equals side AB in length. This assumes the prism is long enough to permit this.



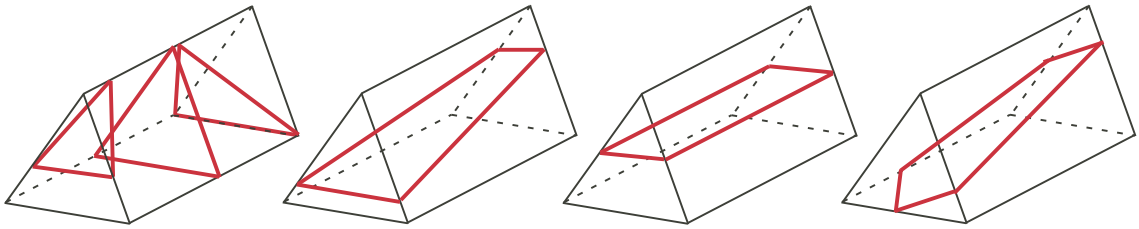
c) All cross sections of a sphere are circles. Shown at right are a diametral cross section **(black)**, and two non-diametral cross sections **(dotted and dashed)**.



d) Cross sections of a cylinder:



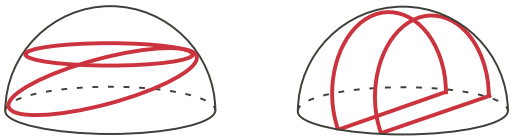
e) Cross sections of a triangular prism:



triangle trapezoid rectangle pentagon

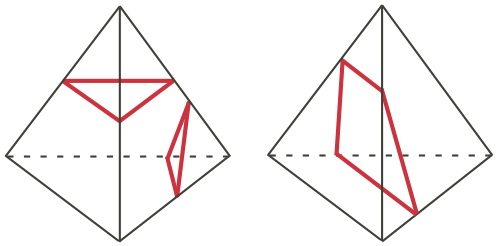
***Extension* :**

1. Cross sections of a hemisphere.



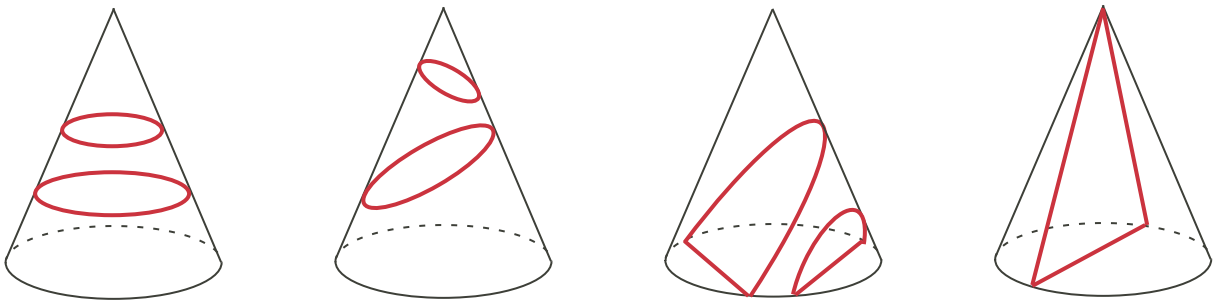
circle truncated circle

2. Cross sections of a triangular-based pyramid:



triangle trapezoid

3. Cross sections of a cone:



circle oval (ellipse) truncated ovals triangle

Comment: An excellent web reference showing cross sections of solids is http://www.learner.org/courses/learningmath/geometry/session9/