Emmy Noether - Circle 3 for 2009-2010

Part I: Problems

Problem 1: What? Chips, but no Dip?

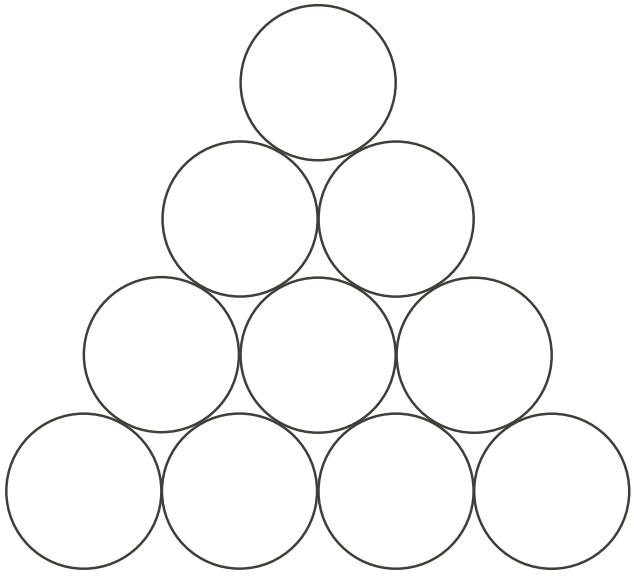
a) Colour the circles in the trianglular stack at right below using three different colours, with no two adjacent circles of the same colour.

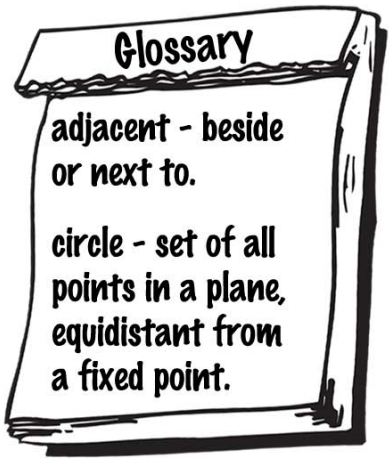
b) How many circles are there of each of the three colours in the stack a)?

c) Suppose the stack had only 3 rows of circles instead of 4. How many circles would there be of each colour? What if the stack had 5 rows?

d) Fill in the table below, and hence decide when there will be the same number of circles of each colour. Predict the first number of rows greater than 7 for which there is NOT the same number of circles of each colour.

No. of rows	No. of Circles	No. of Colour 1	No. of Colour 2	No. of Colour 3
2				
3				
4				
5				
6				
7				





Problem 2

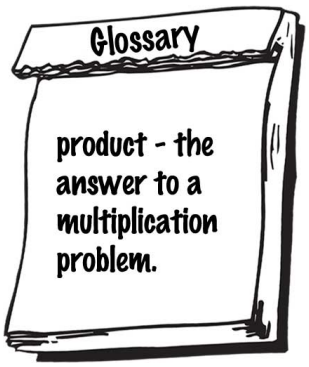
One day in class, instead of listening to your teacher, Mr.B.O.Ring, you are playing with your calculator when you notice that the number 9 key is not working. Use 'mental math' to describe how you could find the following products using the broken calculator, and state the answers.

a) 9×23

b) 6×99

c) 11×998

d) 9×750



***Extension* :**

1. Suppose the $\times$ sign was also not working. Describe how you could find the above products using mental math without the number 9 nor calculator multiplication.

Problem 3

As Bailey rows her sturdy boat across the lake one sunny morning, alas a submerged log makes a small crack in the hull. Water begins to leak into the boat at 3 litres per minute. If Bailey alternates rowing and bailing, the boat travels 2 kilometres per hour, and she bails out 1.5 litres per minute. The boat will sink if it takes on 135 litres of water.

a) If Bailey is 4 kilometres from the shore of the lake when the boat springs the leak, can she make it to shore before the boat sinks? Explain your reasoning.

b) If your answer to part a) is 'No', how far is Bailey from shore when the boat sinks?

c) Does your answer to part a) change if Bailey is only 3 kilometres from shore when she hits the log? Explain your reasoning.

Complete the table below to help discover the answers.

Time (in hours)	Distance (km) $= 2 \times$ time	Water In (litres) $= 180 \times$ time	Water Out (litres) $= 90 \times$ time	Accumulated Water in Boat
$\frac{1}{2}$	1	90	45	45
1				
$1\frac{1}{2}$				
2				

Problem 4

Someone's dog goes around the neighborhood every night getting into people's garbage pails and making a mess. Four of the neighborhood kids say they know what the guilty dog looks like, but the culprit strikes at night, so it's hard to see it clearly. In their descriptions, each of the four witnesses has one and only one detail right, and each detail is described correctly by only one witness.

- Dan says the dog is white, fluffy, wears a red collar, and has a long tail.
- Karen says the dog is black, has short hair, wears a red collar, and has a long tail.
- Max says the dog is brown, has long, silky hair, wears a blue collar, and has a long tail.
- Emma says the dog is spotted, fluffy, wears a red collar, and has a short tail.

Fill in the table below to help sort out which details are correct, and hence describe the guilty pooch.

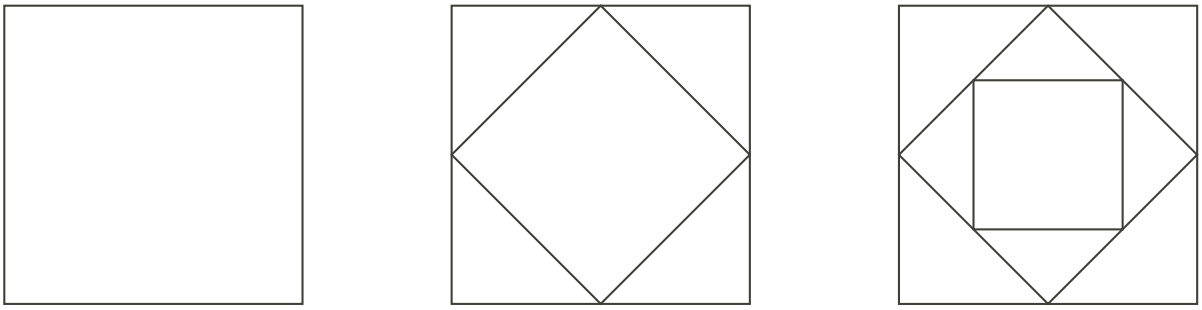
Child's Name	Colour	Hair Type	Collar Colour	Tail Length
Dan				
Karen				
Max				
Emma				

***Extension* :**

1. Work with a friend to make up another set of four descriptions which could also be used to correctly describe the guilty pooch.

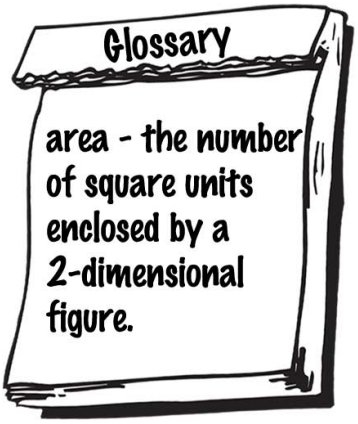
Problem 5

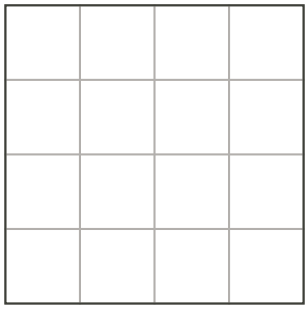
a) Count the number of non-overlapping sections in each figure. Following this pattern, how many figures would you have to draw to have 41 sections?



b) In the second figure, each of the triangles has area 1/8 of the larger (outer) square. In the third figure, there are two sizes of triangles. What fraction of the larger (outermost) square is the area of each of the smaller triangles?

c) Draw the next figure in the fourth square below. What fraction of the largest (outermost) square is the area of the smallest triangle?





d) For what figure will the fraction of the area of the smallest triangle to the whole square be 1/1024?

Problem 6: Plasticine Geometry (Suggested for pairs or groups of students)

For this investigation, you will need some plasticine and a piece of fishing line (or thin straight wire, such as a cheese cutter, or a straightened paper clip) for each student.
First, use plasticine to make one of each of the following three-dimensional solid figures:

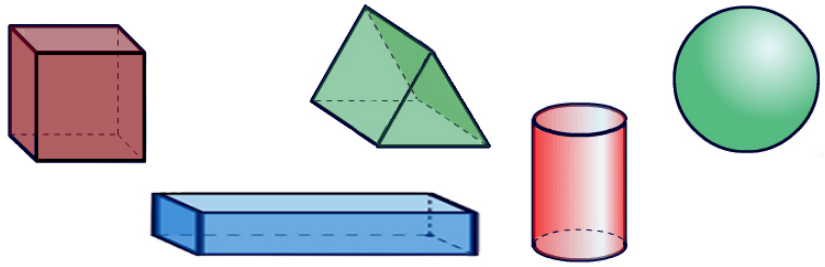
a) a cube;

b) a rectangular prism;

c) a sphere;

d) a cylinder;

e) a triangular prism.



Now, follow the instructions below:

1. Imagine slicing each solid into two pieces with a thin cutting tool. While slicing, your tool can be held at an angle, but it must move in a straight line from one face or side of the solid to another face or side. The cut face on each piece is called a *cross-section*, or a *section* of the solid.

2. Predict what two-dimensional figures the cross sections will be, **before** you cut; enter your guesses in the table below for each three-dimensional solid figure.

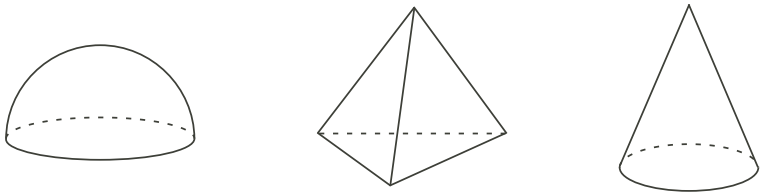
3. Then, using the fishing line like a thin knife, carefully make each of the cuts you thought about, and describe the actual resulting cross section in the appropriate row of the table.

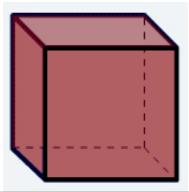
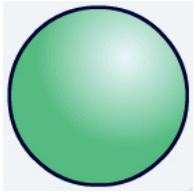
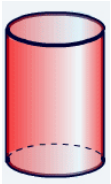
How many different cross sections can you find for each solid?

Use the table on the following page to show your answers.

***Extension* :**

1. Repeat the suggestions above for a hemisphere (half a sphere), for a triangular-based pyramid, and for a cone.



Solid	Guesses for Cross Sections	Actual Cross Sections	# of Cross Sections
1. Cube 			
2. Rectangular Prism 			
3. Sphere 			
4. Cylinder 			
5. Triangular Prism