Emmy Noether - Circle 2 for 2009-2010

Part I: Problems

Problem 1: That's fast!

In 2008, Usain Bolt from Jamaica was the "fastest person in the world". He ran 100 metres in a world record time of 9.69 seconds.

a) If he could maintain that pace over 1000 metres, what would his time be?

b) The current world record for the marathon is 2 hours, 3 minutes, 59 seconds. If Usain Bolt could maintain his pace (9.69 seconds for 100 metres) over 42.2 kilometres (the length of a marathon), what would his time be? Give your answer in hours and minutes.

c) A cheetah is the fastest land mammal and can reach a speed of 120 kilometres per hour over short distances. If a cheetah could maintain this speed over 42.2 kilometres, what would be its time? Give your answer in minutes.

Extension :

1. Search the web for some recent data on the time taken by runners in 400 metre races. Then calculate their mean times for 100 metres, and compare them to Usain Bolt's time for 100 metres. Would you expect them to be greater or less? Explain your answer.

Problem 2

A 'Lorna' number has 3 digits, and the tens (middle) digit equals the hundreds (left) digit minus the units or ones (right) digit. For example, 752 is a 'Lorna' number, since $5 = 7 - 2$ (which could also be written $7 = 5 + 2$).

a) If the hundreds digit is a 3, what are the possible 'Lorna' numbers?

b) If the hundreds digit is a 5, what are the possible 'Lorna' numbers?

c) What is the least (smallest) possible 'Lorna' number? What is the greatest 'Lorna' number?

d) How many 'Lorna' numbers are there in total?

Part I – For the Student

Extension :

1. A 'Dennis' number also has 3 digits, but the tens digit is the units digit minus the hundreds digit. Is there the same number of 'Dennis' numbers as 'Lorna' numbers? Explain your answer.
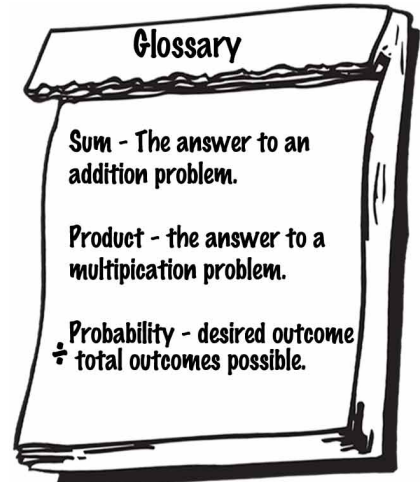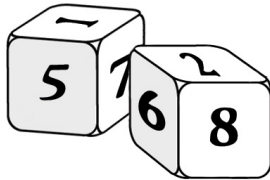
Problem 3

Hamed has a pair of dice (number cubes). One has the odd numbers $1, 3, 5, 7, 9, 11$ on its faces, and the other, the even numbers $2, 4, 6, 8, 10, 12$.

a) If he rolls the dice and adds the two numbers on the top faces:

    (i) What is the probability that the sum is *odd*?

    (ii) What is the probability that the sum is less than 15?

b) Suppose instead that Hamed multiplies the two numbers:

    (i) What is the probability that the product is *odd*?

    (ii) What is the probability that the product is equal to 18?

Glossary

Sum - The answer to an addition problem.

Product - the answer to a multipication problem.

Probability - desired outcome ÷ total outcomes possible.

Numbers on each die:

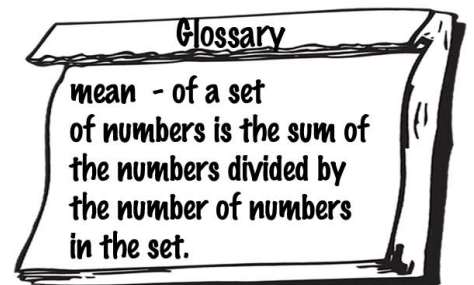| Die 1	1	3	5	7	9	11
Die 2	2	4	6	8	10	12

Extensions :

1. In a game, Hamed and Beth take turns rolling the two dice. Hamed gets a point every time the number on one die is exactly divisible by the number on the other die. Beth gets a point if the sum of the two numbers has a digit sum divisible by 4. (For example, if she rolls a 6 and an 11, with sum 17, then its digit sum is 1+7=8, which is divisible by 4, so she get a point. But she gets no point if she rolls a 3 and an 8, since their sum is 11, with digit sum 2). Is this a fair game (i.e., do both Hamed and Beth have an equal chance to get a point)?

Problem 4

Buster, Billy, Bobby, and Belinda have been collecting Nascar action figures. Buster has 23 figures, Billy has 36, Bobby has 15, and Belinda has 34.

a) What is the mean number of action figures owned by the four people?

b) Some time later, they have collected more figures. Now Buster has 27, Billy has 38, and Bobby has 20. If the mean number of figures is now 30, how many figures does Belinda have?

Glossary

mean - of a set of numbers is the sum of the numbers divided by the number of numbers in the set.

c) How many more figures will they have to collect in total to have a mean of 32 figures?
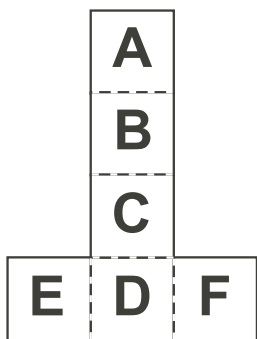
Extension :

1. Suppose the four friends have a mean of 27 figures.
   Which of the following statements could be true? (Explain your answers.)

   a) Two of them have over 50 figures each.

   b) Two of them each have an even number of figures.
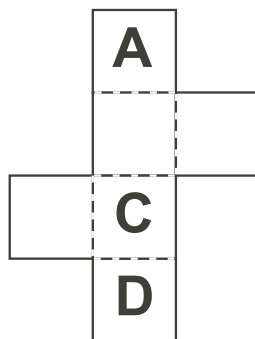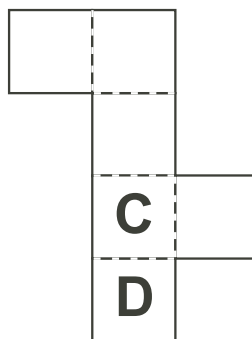
   c) One of them has only 9 figures.

Problem 5

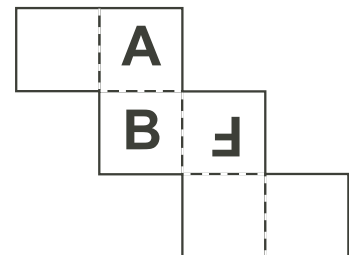Below is a set of four nets, all for the same die.
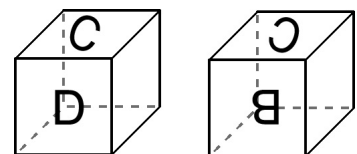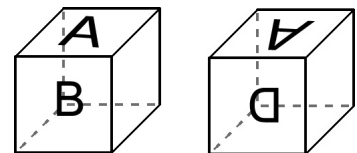
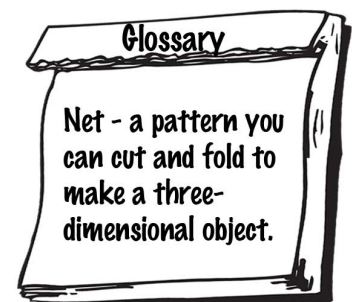| Net 1	Net 2	Net 3	Net 4

a) Both cubes at the right were formed using Net 1. Fill in the appropriate letter as it would appear on the blank (right) face of each cube.

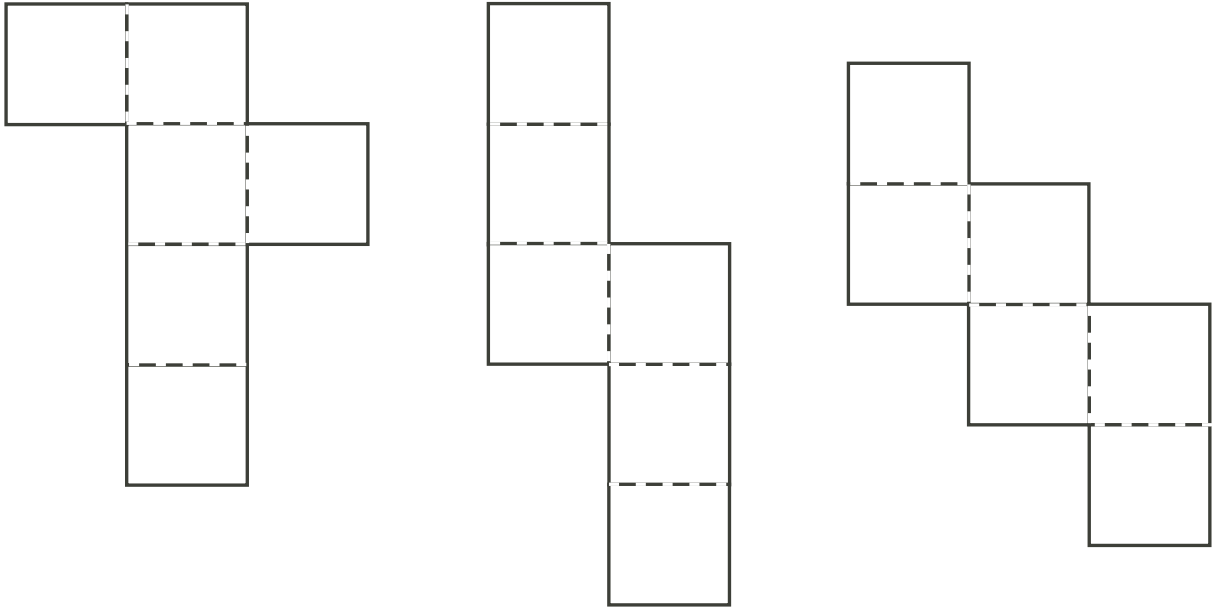b) Repeat part a) for the cubes shown at the right, which were also formed from Net 1.

Glossary

Net - a pattern you can cut and fold to make a three-dimensional object.

c) Complete Nets 2, 3, and 4 so that the letters are oriented the correct way to make exactly the same die as Net 1 would make.

Part I – For the Student

***Extensions* :**

1. Use each of the blank nets below to design a net for a die so that, from one point of view, the four vertical sides of the resulting cube form the word **MATH**.

Problem 6: A 'Timely' Offer! (Suggested for pairs or groups of students)

Sarah's Dad offers her the choice of a weekly allowance of $ 1.00 for every time the hands of her clock make a right angle between the hours of noon and midnight, OR $ 2.00 for every time they make an angle of 180°. Sarah decides on the first choice, believing it will give her a larger allowance. Explain why she is right (or wrong).

To help you decide, here is a useful tool. Place this page on a piece of cardboard. Cut out the hands for the clock and use a stick pin or thumbtack to place them at the center of the clock, pinning the diagram to the cardboard.

Part I – For the Student

long hand

short hand