Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Data Management

Problem 3 - Measurement

Problem 4 - Number Sense

Problem 5 - Logic and Pattern

Problem 6 - Geometry

Hints and Suggestions:

Problem 1a)

Hint 1 - How does the problem change once you've combined two digits? For example, if you combine 2 and 3 to form 23, what must be the sum of all the other numbers?

Hint 2 - What combinations of two digits are impossible? Why?

Problem 1b)

Hint 1 - What combinations of two digits reversed will not work? Why?

Problem 2a)

Hint 1 - Ali has chosen cheese as his first condiment. If he chooses lettuce as his second condiment, what choices does he have for his third condiment? How many combinations are there with lettuce as his second condiment?

Hint 2 - For each choice of his second condiment, what choices does Ali have for his third condiment? Are all the resulting sets of three condiments different?

Problem 2b)

Hint 1 - In what way is this question different from part a)? In what way is it the same?

Suggestions:

1. If students find this problem too complicated, start them off with fewer choices, e.g., cheese, onions, and tomatoes only.

2. Students could use a tree diagram in parts a) and b) of this problem. But they may need a suggestion to look for overlapping combinations, since trees give permutations rather than combinations.

Problem 3

Suggestion: Teachers may wish to suggest students make a chart in order to record the perimeter and area of squares with side 1, 2, 3, etc.

Problem 4

Hint 1 - How many games would be played in the closest possible match?

Hint 2 - What would be the final score in the first game of the closest possible match? And in the second game?

Hint 3 - How many games will be played if one team scores 5 points for every 2 points of the other team?

Hint 4 - Would the winning team win all three games if they score the minimum number of points in total?

Suggestion: Students may find it helpful to make a chart for each match, with two columns, one for each of the two teams, and three rows, one for each game.

	Team A	Team B
Game 1		
Game 2		
Game 3		

Problem 5

Hint 1 - Which coin is easiest to move?

Hint 2 - Which coin is hardest to move?

Suggestion: This is a 'guess and check' activity. Substitutes for the coins could be used, or the images cut out from the given problem, in order for each student to be able to give it a try.

Problem 6

Hint 1 - Must the plots be rectangular?

Hint 2 - Do you need more than one fence along common boundaries?

Suggestion: You may wish to have a class discussion about whether needing sections 'the same size and shape' permits sections that are reflections or rotations of one another.

Solutions

Problem 1

a) The possible two-digit combinations are 12, 23, 34, 45, 56, 67, 78, and 89. The first five are too small (e.g., for 56, the sum of the remaining digits is $1+2+3+4+7+8+9 = 34$, and $56+34 = 90$), so only 78 and 89 are possible. For 78, two solutions are:

$$1+2+3-4+5+6+78+9 = 100, \text{ or } -1+2-3+4+5+6+78+9 = 100$$

(there may be others).

The larger combination, 89, leaves a remainder of 11, which must be constructed from sums and/or differences of 1, 2, 3, 4, 5, 6, 7. Since there are 4 odd numbers and 3 even numbers, only an even sum or difference can occur, because sum and differences of two odd numbers are always even. Thus 11 cannot be constructed so 89 is not possible from these numbers.

b) With the digits reversed, the same conditions hold as in a). The combinations 54 and 32 are too small, but there are two combinations of adjacent digits that work:

$$9+8+76+5-4+3+2+1 = 100, \text{ and } 98+7-6+5-4+3-2-1 = 100$$

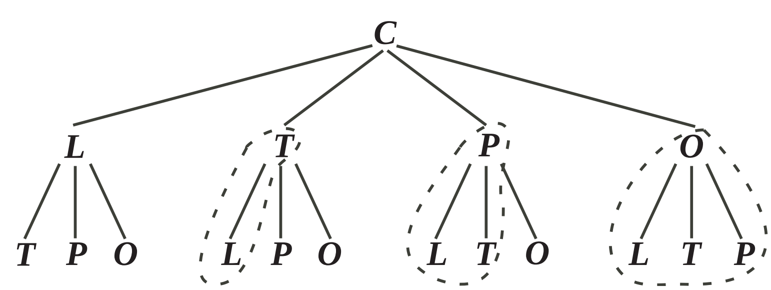
***Extension* :** Here are several solutions.

$$1\times 2\times 3-4+5+6+78+9 = 100$$
$$1\times 2-3+4-5+6+7+89 = 100$$
$$123-4+5-6-7-8-\sqrt{9} = 100$$
$$1+2^3-4+5-6+7+89 = 100$$
$$\tfrac{1}{2}\times 34+(5\times 6)+(7\times 8)-\sqrt{9} = 100$$

Problem 2

a) With cheese for sure as his first choice, Ali can choose any two of lettuce, tomatoes, pickles, or onions. So he has 4 choices for his second condiment. For each of those, he has 3 possible choices for his third condiment. In tree form, with C for cheese, L for lettuce, T for tomato, P for pickle, and O for onion, a 'tree' to illustrate all these choices looks like this:

Tree Diagram



BUT…some of these choices give identical combinations! For example, CLT and CTL give the same combination, cheese, lettuce, and tomato. (The dotted ovals enclose combinations which already exist to the left in the tree.) Thus there are only 6 unique combinations for Ali to choose.

b) If Tanya wants tomatoes for sure, she also has 4 choices for her second condiment and 3 for her third. Thus she has the same number of choices as Ali, namely 6. (This is the same problem as a), with different names!)

c) Since Xiao wants either cheese or tomatoes, he has 2 choices for his first condiment. Using the abbreviations above, the unique combinations with either cheese or tomato but not both are: CPL, CLO, CPO, TLP, TLO, TPO, a total of 6.

d) No matter what choice is made for the first condiment, there will always be 6 unique combinations as in a) above. Since there are 5 different possible choices for the first condiment, the total number of possible combinations is $5 \times 6 = 30$.

Comment: Part c) could be illustrated by combining five separate trees like the one in part a), but with different entries for each tree.

Problem 3

a) Here is a chart for squares of side length 1, 2, 3, 4, 5, 6, 7, 8.

Square Side	1	2	3	4	5	6	7	8
Perimeter	4	8	12	16	20	24	28	32
Area	1	4	9	16	25	36	49	64

Notice that the perimeter increases by 4 units of length for an increase of 1 unit in side length, but the area increases as the square numbers.

(i) From the chart, any of the squares with the side length 1, 2, or 3 has units of perimeter greater than units of area.

(ii) Any square with side 5 units or greater has units of perimeter less than units of area. Thus there is an infinite number of such squares.

(iii) A square of side 4 has perimeter 16 units, and area 16 units. It is the only such square.

b) The squares in a)(i) are the only possible such squares, so there are three of them.

Problem 4

a) Think of the two teams as Team A and Team B. The closest possible match won by Team A would occur with Team A winning one of the first two games with a score of 21 to 19, Team B winning the other with a score of 21 to 19, and then Team A winning the final game 15 to 13. Thus the total point difference for the match would be 2 points.

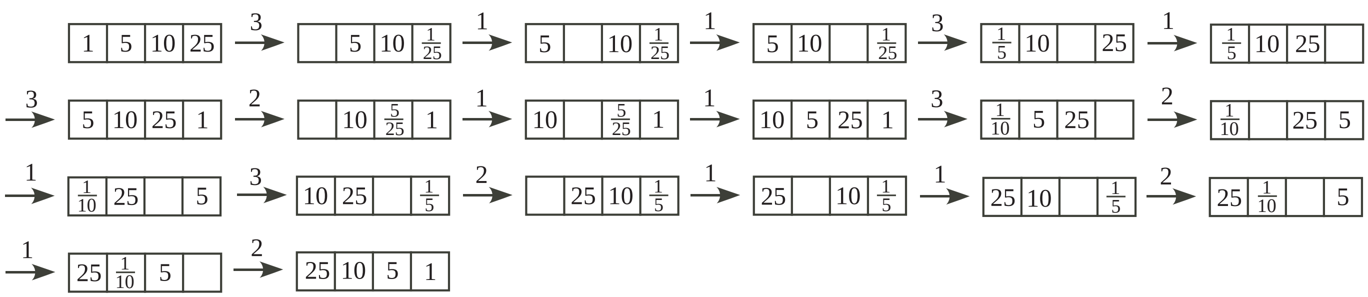
b) From a), the closest possible match would give the losers, Team B, a total of $19 + 21 + 13 = 53$ points.

c) The match will be won in 2 games, in which case the winning team would have $21 + 21 = 42$ points. If the losers only score 2 points in each game for every 7 points scored by the winners, then they would have scored 6 points in each game, giving a total of 12 points.

d) The minimum number of points Team A could score to win a match would be if they win the first game (21 points), score no points in the second game, and then win the third game (15 points). Thus is would take at least 36 points in total.

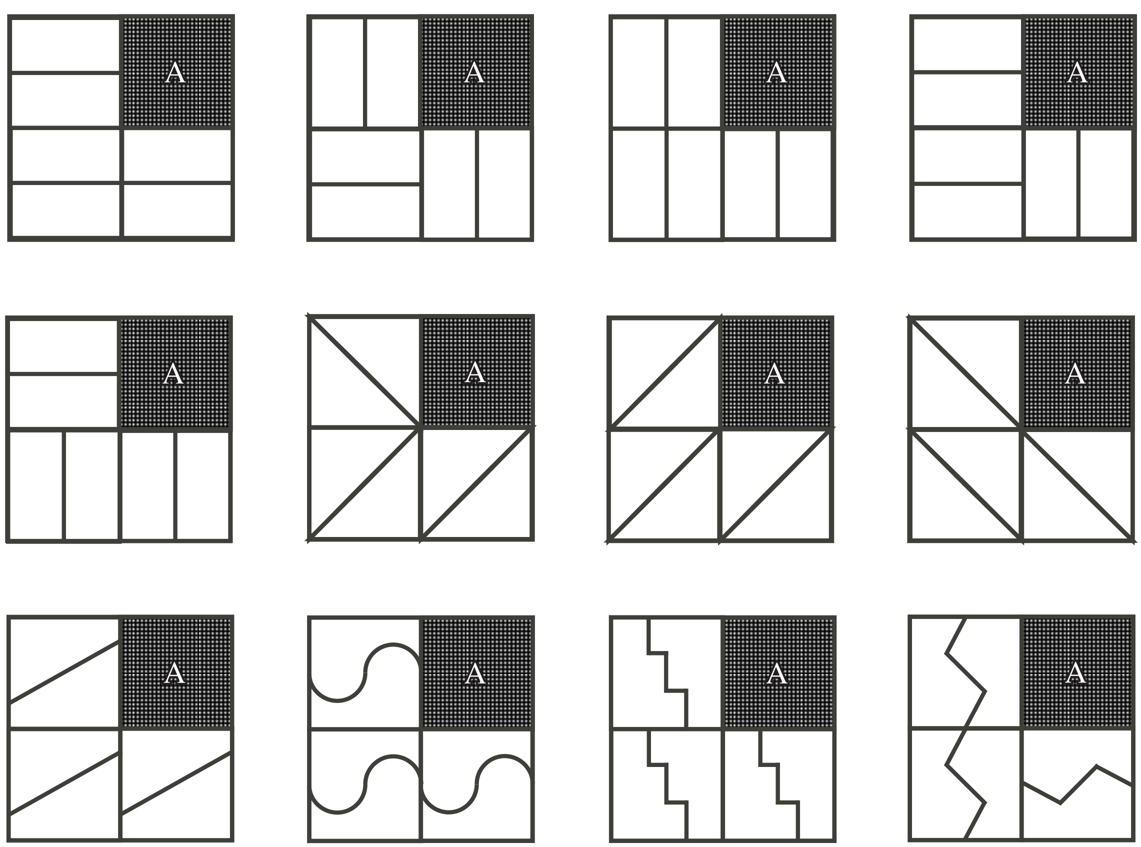
Problem 5

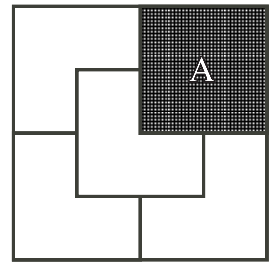
The steps of one possible solution are shown in the sequence of diagrams below, with the coins indicated by their numeric values in cents. Stacks of two coins are shown as fractions. Each grid gives moves for only one coin; the number of moves is shown above the arrow (e.g., the penny moves 3 to the right at first). There may be more efficient solutions.



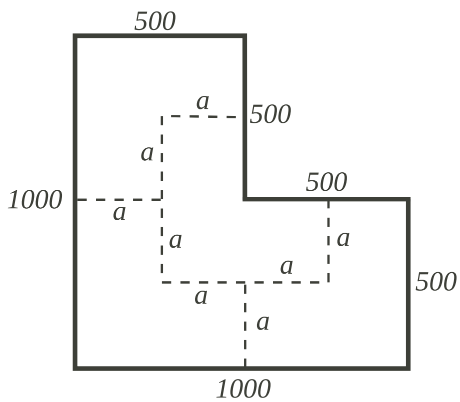
Problem 6

a) There are infinitely many solutions. Here is a selection; you may wish to have students contribute to a set of solutions for the whole class to critique, as to shape and equal size.





b) Here is a solution. An interesting question for class discussion is whether there are any other solutions.



c) If each side of section A has length 500 m, then the outside boundary of the four plots would require $1000 + 1000 + 500 + 500 + 500 + 500 = 4000$ m of fencing. The interior (dashed) boundaries would require another $8 \times a = 8 \times 250 = 2000$ m of fencing. Thus, in total, you would need $4000 + 2000 = 6000$ m of fencing.